OSNOVE MEHANIKE FLUIDA

1. Na ispitnom stolu istražuje se horizontalna sila koju stvara ispuh mlaznog motora na vertikalno postavljenu ravnu ploču. Brzina ispuha je 150 m/s pri standardnim uvjetima na razini mora, a promjer ispuha je 0,3 m. Odredi veličinu horizontalne sile u osloncu.
   Ispuh ima karakteristike zraka.

Pretpostavke:
- stacionarno strujanje
- nestlačivo
- ravnomjeran raspored brzina po presjeku gdje fluid sječe granicu kontrolnog volumena
- promatramo sile u x-smjeru

I način:

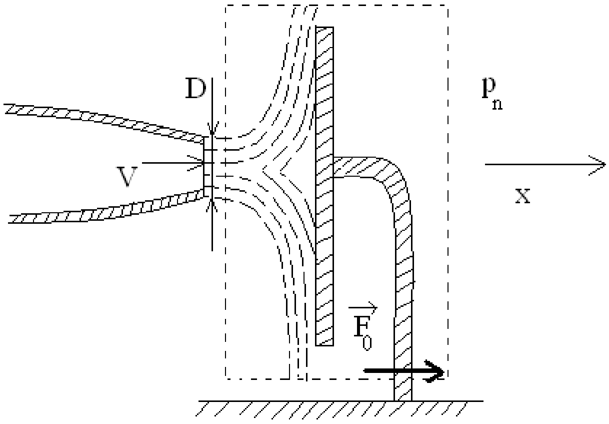
Odabran je kontrolni volumen tako da je površina na lijevoj strani jednaka površini na desnoj. Kontrolni volumen sječe oslonac. Silu kojom oslonac djeluje na kontrolni volumen $\vec{F}_o$ pretpostavljamo pozitivnom, pa će sila kontrolnog volumena na oslonac biti po iznosu jednaka,ali suprotno usmjerena.

$$\underbrace{\frac{\partial}{\partial t}\int_{GV}\rho\vec{V}d\forall}_{\varnothing} + \int_{GP}(\rho\vec{V}d\vec{A})\,\vec{V} = \underbrace{\vec{F}_G}_{\varnothing} + \vec{F}_p + \underbrace{\vec{F}_K}_{\varnothing} + \vec{F}_o$$

$$-\rho VVA_1 = p_n A - p_n A + \vec{F}_o$$

$$\vec{F}_o = -\rho V^2 A_1 = -1.225\cdot 150^2\,\frac{0.3^2\pi}{4} = -1948 \quad N$$

$$\vec{F}_x = -\vec{F}_o = 1948 \quad N$$



II način:

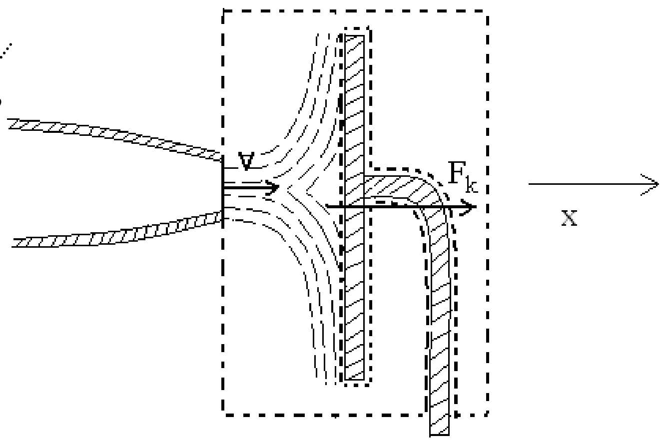
Odabran je kontrolni volumen tako da je površina na lijevoj strani jednaka površini na desnoj. Kontrolni volumen ne sječe oslonac. Međutim, kontrolni volumen je u dodiru sa osloncem preko nekoliko dijelova kontrolne površine. Oslonac/kontura na kontrolni volumen djeluje silom $\vec{F}_K$.

$$\underbrace{\frac{\partial}{\partial t}\int_{GV}\rho\vec{V}d\forall}_{\varnothing} + \int_{GP}(\rho\vec{V}d\vec{A})\,\vec{V} = \underbrace{\vec{F}_G}_{\varnothing} + \vec{F}_p + \vec{F}_K + \underbrace{\vec{F}_o}_{\varnothing}$$

$$-\rho VVA_1 = p_n A - p_n A + \vec{F}_K$$

$$\vec{F}_K = -\rho V^2 A_1 = -1948 \quad N$$

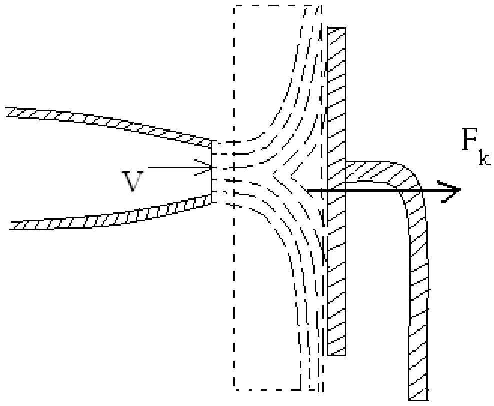
$$\vec{F}_X = -\vec{F}_K = 1948 \quad N$$



III način:

Odabran je kontrolni volumen tako da je površina na lijevoj strani jednaka površini na desnoj. Sila oslonca na kontrolni volumen označena je s $\vec{F}_K$.

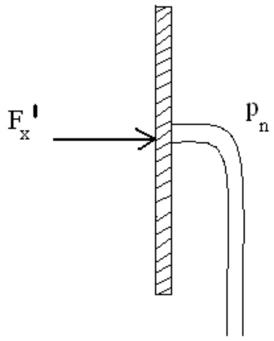
$$\underbrace{\frac{\partial}{\partial t}\int_{GV}\rho\vec{V}d\forall}_{\varnothing} + \int_{GP}(\rho\vec{V}d\vec{A})\cdot\vec{V} = \underbrace{\vec{F}_G}_{\varnothing} + \vec{F}_p + \vec{F}_K + \underbrace{\vec{F}_o}_{\varnothing}$$



$$-\rho VVA_1 = p_n A + \vec{F}_K$$

$$\vec{F}_K = -p_n A - \rho V^2 A_1$$

$$\vec{F}_x^{'} = -\vec{F}_K = p_n A + \rho V^2 A_1$$



$$\vec{F}_X = \vec{F}_x^{'} - p_n A = p_n A + \rho V^2 A_1 - p_n A$$

$$\vec{F}_X = \rho V^2 A_1 = 1948 \quad N$$

2. Zrakoplov leti na visini 3000 m u uvjetima standardne atmosfere. Brzinomjer koji je kalibriran prema nestlačivom strujanju na nultoj nadmorskoj razini pokazuje brzinu 280 km/h. Odredi stvarnu brzinu zrakoplova u slučajevima:
   a) idealan instrument (bez greške) i točno mjerenje statičkog tlaka
   b) idealan instrument, ali postoji pogreška pri mjerenju statičkog tlaka:

$$\varepsilon = \frac{p_{mjereno} - p_{st\,\mathrm{var}no}}{p_{st\,\mathrm{var}no}} = \pm 2\%$$

<u>H=3000 m</u>

$$p = p_n (1 - 2.256 \cdot 10^{-5} \cdot H)^{5.256} = 101325 \cdot (1 - 2.256 \cdot 10^{-5} \cdot 3000)^{5.256} = 70105 \quad Pa$$

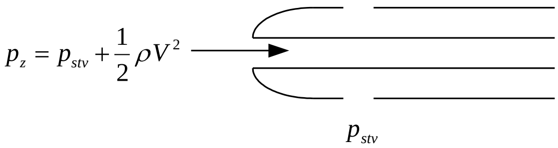
$$\rho = \rho_n (1 - 2.256 \cdot 10^{-5} \cdot H)^{4.256} = 1.225 \cdot (1 - 2.256 \cdot 10^{-5} \cdot 3000)^{4.256} = 0.9091 \quad kg/m^3$$

a) $V_i = V_e = 280 \quad km/h$

$$p_z - p_{stv} = p_d$$

$$p_{stv} + \frac{1}{2}\rho V^2 - p_{stv} = \frac{1}{2}\rho_n V_e^2$$

$$V = V_e \sqrt{\frac{\rho_n}{\rho}} = 280\sqrt{\frac{1.225}{0.9091}} = 325 \quad km/h = 90.3 \quad m/s$$

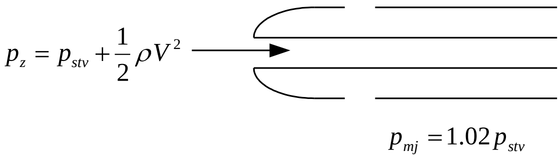


b)

$$\frac{p_{mj} - p_{stv}}{p_{stv}} = \pm 2\% = 0.02$$

$$p_{mj} - p_{stv} = 0.02 \cdot p_{stv}$$

$$p_{mj} = 1.02 \cdot p_{stv}$$



$$p_{stv} + \frac{1}{2}\rho V^2 - p_{mj} = \frac{1}{2}\rho_n V_e^2$$

$$p_{stv} + \frac{1}{2}\rho V^2 - 1.02\, p_{stv} = \frac{1}{2}\rho_n V_e^2$$

$$\frac{1}{2}\rho V^2 = \frac{1}{2}\rho_n V_e^2 + 0.02\, p_{stv} \Big/ \frac{2}{\rho}$$

$$V = \sqrt{\frac{2}{\rho}\left(\frac{1}{2}\rho_n V_e^2 + 0.02 p_{stv}\right)} = \sqrt{\frac{2}{0.9091}\left[\frac{1}{2} \cdot 1.225 \cdot \left(\frac{280}{3.6}\right)^2 + 0.02 \cdot 70105\right]} = 106 \quad m/s = 381.6 \quad km/h$$

3. Zrakoplov leti u uvjetima standardne atmosfere. Visinomjer zrakoplova pokazuje visinu po tlaku od 5200 m. Indicirana brzina na idealnom instrumentu (bez greške) koji je baždaren u uvjetima standardne atmosfere na H = 0 m pokazuje brzinu 200 km/h. Odrediti:
   a) uvjete okolne atmosfere
   b) stvarnu brzinu
   c) brzinu zvuka
   d) Machov broj

a) $p = p_n(1 - 2.256\cdot 10^{-5}\cdot H)^{5.256} = 101325\cdot(1 - 2.256\cdot 10^{-5}\cdot 5200)^{5.256} = 52587 \quad Pa$

$T = T_n - 6.5\cdot 10^{-3}\cdot H = 288.15 - 6.5\cdot 10^{-3}\cdot 5200 = 254.35 \quad K \qquad (t = -18.8 \quad °C)$

$$\rho = \frac{p}{RT} = \frac{52587}{287.053\cdot 254.35} = 0.720 \quad kg/m^3$$

b)

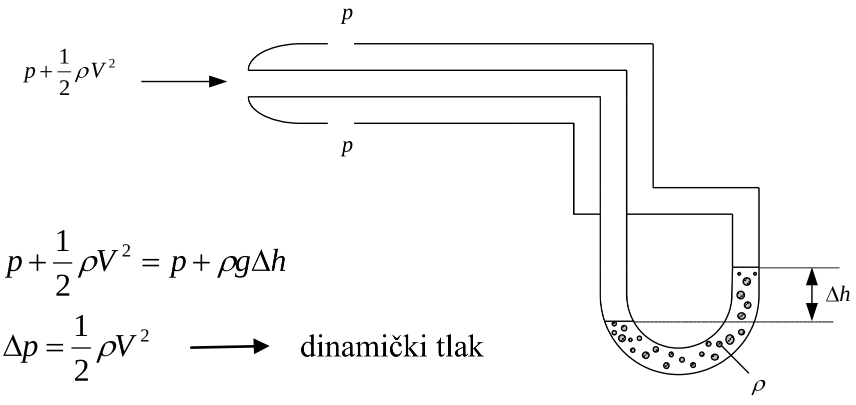
$$\frac{1}{2}\rho_n V_e^{\,2} = \frac{1}{2}\rho_H V_S^{\,2} \Rightarrow V_S = V_e\sqrt{\frac{\rho_n}{\rho_H}} = \frac{200}{3.6}\sqrt{\frac{1.225}{0.720}} = 72.45 \quad m/s$$

$V_S = 260.83 \quad m/s$

c) $a = \sqrt{\kappa RT} = \sqrt{1.4\cdot 287.053\cdot 254.35} = 319.71 \quad m/s^2$

d) $Ma = \frac{V}{a} = \frac{72.45}{319.71} = 0.227$

4. Zrakoplov leti na visini h=5000 m. Indikator brzine zrakoplova koji je umjeren (baždaren) na uvjetima standardne atmosfere na h = 0 m pokazuje brzinu V_i = 350 km/h. Ako je brzinomjer tipa Pitot-Prandtl odredi stvarnu brzinu zrakoplova i Machov broj.



$$p+\frac{1}{2}\rho V^2 = p+\rho g\Delta h$$

$$\Delta p = \frac{1}{2}\rho V^2 \longrightarrow \text{dinamički tlak}$$

<u>I način:</u>

h = 0 m

$$\rho_n = 1.225 \quad kg/m^3$$

$$V_i = V_e = 350\cdot\frac{1000}{3600} = 97.22 \quad m/s$$

$$\Delta p = \frac{1}{2}\rho_n V_i^2 = \frac{1}{2}1.225\cdot 97.22^2 = 5789.2 \quad Pa$$

h = 5000 m

$$\rho = 0.73643 \quad kg/m^3$$

$$\Delta p = \frac{1}{2}\rho V_S{}^2 \Rightarrow V_S = \sqrt{\frac{2\Delta p}{\rho}} = \sqrt{\frac{2\cdot 5789.2}{0.73643}} = 125.4 \quad m/s$$

<u>II način:</u>

$$V_e^2\rho_n = V_S^2\rho \qquad \Rightarrow \quad V_S = V_e\sqrt{\frac{\rho_n}{\rho}} = 97.22\sqrt{\frac{1.225}{0.73643}} = 125.4 \quad m/s$$

$$pogreška: \qquad \Delta V = V_i - V_S = 97.22 - 125.4 = -28.2 \quad m/s$$

$$\Delta V(\%) = \frac{\Delta V}{V_S} = \frac{-28.2}{125.4}\cdot 100 = -22.5\%$$

$$Ma = \frac{V}{a}\;; \quad \kappa = 1.4; \qquad R = 287.053 \quad J/kgK; \qquad T = \frac{p}{\rho R}$$

$$a = \sqrt{\kappa RT} = \sqrt{\kappa R\frac{p}{\rho R}} = \sqrt{\kappa\frac{p}{\rho}} = \sqrt{1.4\frac{54048.3}{0.73643}} = 320.55 \quad m/s^2$$

$$Ma = \frac{V}{a} = \frac{125.4}{320.55} = 0.391$$

5. Zrakoplov leti na visini H = 6000 m, brzinom V = 400 km/h. Odredi:

a) statički tlak okolne atmosfere
b) dinamički tlak
c) totalni tlak
d) ekvivalentnu brzinu
e) Machov broj

a) $p_a = p_n(1 - 2.256 \cdot 10^{-5} \cdot H)^{5.256} = 101325 \cdot (1 - 2.256 \cdot 10^{-5} \cdot 6000)^{5.256} = 47176 \quad Pa$

b)

$$p_d = \frac{1}{2}\rho_a V^2$$

$$\rho_a = \rho_n(1 - 2.256 \cdot 10^{-5} \cdot H)^{4.256} = 1.225 \cdot (1 - 2.256 \cdot 10^{-5} \cdot 6000)^{4.256} = 0.6596 \quad kg/m^3$$

$$p_d = \frac{1}{2}\rho_a V^2 = \frac{1}{2} 0.6596 \cdot \left(\frac{400}{3.6}\right)^2 = 4071.6 \quad Pa$$

c) $p = p_a + p_d = 47176 + 4071.6 = 51247.6 \quad Pa$

d)

$$V_e^2 \rho_n = V_S^2 \rho_a$$

$$V_e = V_S \sqrt{\frac{\rho_a}{\rho_n}} = \frac{400}{3.6}\sqrt{\frac{0.6596}{1.225}} = 81.53 \quad m/s$$

e)

$$Ma = \frac{V}{a}$$

$$T_a = T_n - 6.5 \cdot 10^{-3} \cdot H = 288.15 - 6.5 \cdot 10^{-3} \cdot 6000 = 249.15 \quad K \qquad (t = -24 \quad °C)$$

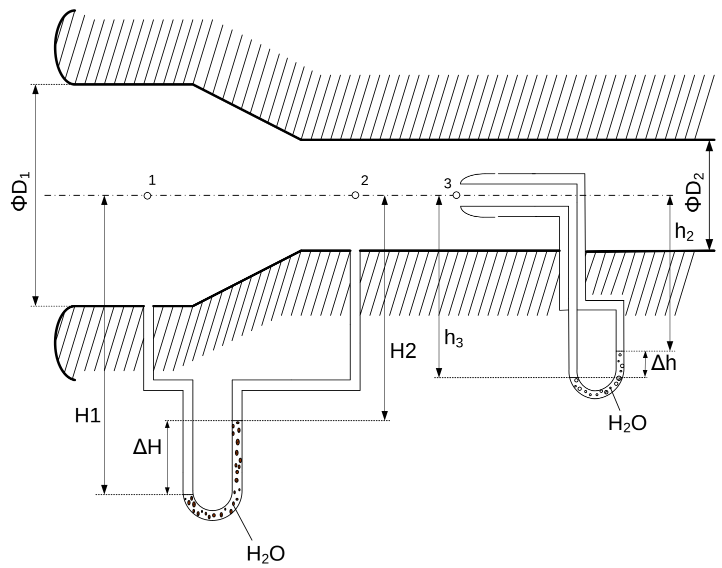
$$a = \sqrt{\kappa R T} = \sqrt{1.4 \cdot 287.053 \cdot 249.15} = 316.43 \quad m/s^2$$

$$Ma = \frac{V}{a} = \frac{400 \cdot \frac{1}{3.6}}{316.43} = 0.351$$

6. Na ulaznom suženju protočnog presjeka mlaznog motora izmjeren je pad tlaka od 500 mm H_2O. Ako je promjer ulazne cijevi smanjen sa D_1=250 mm na D_2 = 200 mm, a zrakoplov leti na visini H = 2000 m, odredi:

   a) protok zraka kroz motor
   b) brzinu zrakoplova
   c) dinamički tlak izmjeren pomoću Pitot-Prandtlove cjevčice postavljene u suženom dijelu

Strujanje smatrati izotermnim.



Rješenje:

a)

$$H = 2000\ m$$
$$\rho = \rho_n (1 - 2.256 \cdot 10^{-5} \cdot H)^{4.256} = 1.225 \cdot (1 - 2.256 \cdot 10^{-5} \cdot 2000)^{4.256} = 1.00646 \quad kg/m^3$$

Jednadžba kontinuiteta:

$$V_1 A_1 = V_2 A_2$$

$$V_1 D_1^2 = V_2 D_2^2$$

$$\frac{V_1}{V_2} = \frac{D_2^2}{D_1^2}$$

Jednadžba manometra od točke 1 do točke 2

$$p_1 + \rho_a g H_1 = p_2 + \rho_a g H_2 + \rho_v g \Delta H$$
$$p_1 - p_2 = \rho_v g \Delta H - \rho_a g (H_1 - H_2) = \rho_a g \Delta H \left(\frac{\rho_v}{\rho_a} - 1\right)$$

Bernoulli-eva jednadžba između točaka 1 i 2:

$$p_1 + \frac{1}{2}\rho_a V_1^2 = p_2 + \frac{1}{2}\rho_a V_2^2 \quad \rightarrow \qquad p_1 - p_2 = \frac{1}{2}\rho_a (V_2^2 - V_1^2)$$

$$\frac{1}{2}\rho_a (V_2^2 - V_1^2) = \rho_a g \Delta H \left(\frac{\rho_v}{\rho_a} - 1\right)$$

$$V_2^2 \left(1 - \frac{V_1^2}{V_2^2}\right) = 2g\Delta H \left(\frac{\rho_v}{\rho_a} - 1\right)$$

$$V_2^2 \left[1 - \left(\frac{D_2}{D_1}\right)^4\right] = 2g\Delta H \left(\frac{\rho_v}{\rho_a} - 1\right)$$

$$V_2 = \sqrt{\frac{2g\Delta H\left(\frac{\rho_v}{\rho_a} - 1\right)}{1 - \left(\frac{D_2}{D_1}\right)^4}} = \sqrt{\frac{2 \cdot 9.81 \cdot 0.5\left(\frac{1000}{1.00646} - 1\right)}{1 - \frac{0.2^4}{0.25^4}}} = 128.5 \quad m/s$$

$$Q = V_2 A_2 = V_2 \frac{D_2^2 \pi}{4} = 128.5 \cdot \frac{0.2^2 \pi}{4} = 4.04 \quad m^3/s$$

b) $\quad V_1 = V_2 \frac{D_2^2}{D_1^2} = 128.5 \frac{0.2^2}{0.25^2} = 82.2\, m/s$

c)

$p_2 + \frac{1}{2}\rho_a V_2^2 = p_3 \;\; \rightarrow \;\; p_2 - p_3 = \frac{1}{2}\rho_a V_2^2$ …jednadžba kontinuiteta

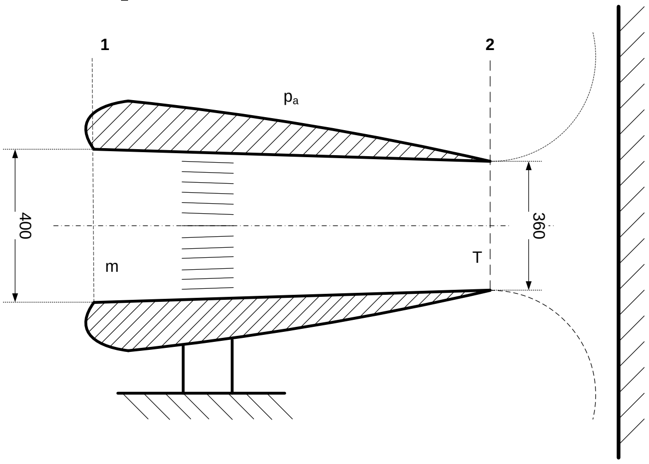
$p_3 + \rho_a g h_3 = p_2 + \rho_a g h_2 + \rho_v g \Delta h$ …jednadžba manometra od točke 2 do točke 3

$$p_3 - p_2 = \rho_v g \Delta h - \rho_a g (h_3 - h_2) = \rho_a g \Delta h \left(\frac{\rho_v}{\rho_a} - 1\right)$$

$$\frac{1}{2}\rho_a V_2^2 = \rho_a g \Delta h \left(\frac{\rho_v}{\rho_a} - 1\right)$$

$$\Delta h = \frac{V_2^2}{2g\left(\frac{\rho_v}{\rho_a} - 1\right)} = \frac{128.5^2}{2 \cdot 9.81\left(\frac{1000}{1.00646} - 1\right)} = 0.847\, m$$

7. Mlazni motor ispituje se na eksperimentalnom stolu u uvjetima standardne atmosfere na razini mora. Ulazni promjer motora je 400 mm, a izlazni 360 mm. Maseni protok zraka na ulazu iznosi 15.4 kg/s, a temperatura plinova izgaranja na izlazu iz motora (pretpostaviti da imaju svojstva zraka) je 560 ˚C. Odrediti:
   a) brzinu leta koju simuliraju gornji uvjeti (brzina zraka na ulazu u motor)
   b) brzinu plinova izgaranja na izlazu iz motora
   c) silu kojom plinovi djeluju na ploču postavljenu okomito na pravac strujanja, neposredno iza motora.



$$p_1 = p_2 = p_a = 101325 \quad Pa$$

$$\rho_1 = 1.225 \quad kg/m^3$$

$$T_2 = 560 \quad °C = 833.15 \quad K$$

$$\rho_2 = \frac{p_2}{RT_2} = \frac{101325}{287.053 \cdot 833.15} = 0.4237 \quad kg/m^3$$

a) $\dot{m}_1 = \rho_1 V_1 A_1 = \rho_1 V_1 \dfrac{D_1^2 \pi}{4}$

$$V_1 = \frac{4\dot{m}_1}{\rho_1 D_1^2 \pi} = \frac{4 \cdot 15.4}{1.225 \cdot 0.4^2 \cdot \pi} = 100 \quad m/s$$

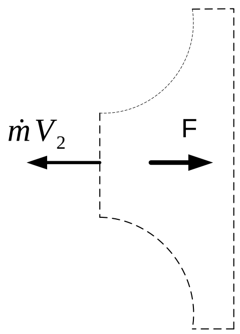
b) $\dot{m}_1 = \dot{m}_2$

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

$$\rho_1 V_1 \frac{D_1^2 \pi}{4} = \rho_2 V_2 \frac{D_2^2 \pi}{4}$$

$$V_2 = V_1 \frac{\rho_1}{\rho_2}\left(\frac{D_1}{D_2}\right)^2 = 100 \cdot \frac{1.225}{0.4237}\left(\frac{400}{360}\right)^2 = 357 \quad m/s$$

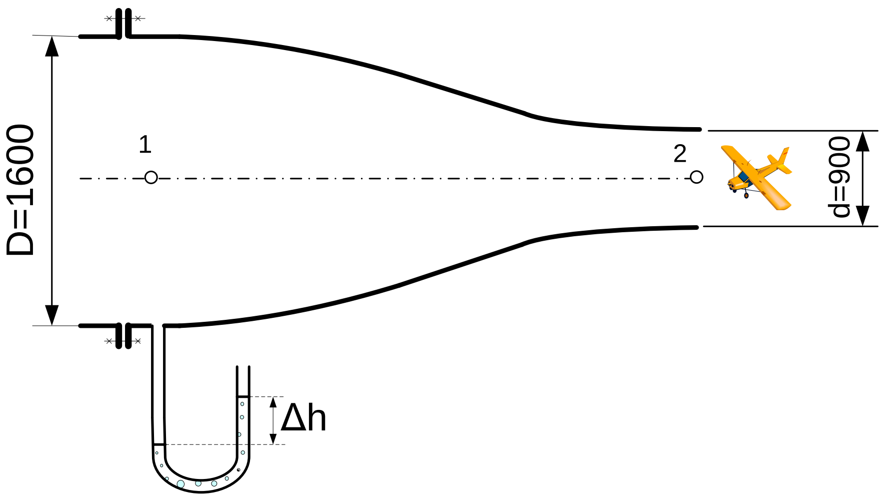
c)



$$F = \dot{m} \cdot V_2 = 15.4 \cdot 357 = 5497 \quad N$$

8. Mlaznica aerodinamičkog tunela izrađena je u obliku kružne redukcije s promjerom D=1600 mm do promjera d = 900 mm. Zrak iz mlaznice nastrujava na maketu zrakoplova u standardnim atmosferskim uvjetima na h = 0 m. ″U″ cijev ispunjena vodom pokazuje nadtlak od Δh = 150 mm H_2O. Odrediti:

   a) brzinu na izlazu iz mlaznice
   b) protok kroz mlaznicu
   c) silu u vijcima koji drže mlaznicu
   d) skicirati raspored tlaka duž mlaznice
   e) kolikom bi silom djelovao mlaz zraka na vertikalni zid koji bi stajao umjesto makete na razmaku većem od d/4



B.J. 1 – 2: $$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$$

$$\Delta p = p_1 - p_2 = \rho_v g h \quad \rightarrow \quad \rho_v g h = \frac{1}{2}\rho(V_2^2 - V_1^2)$$

J.K. 1 – 2: $$V_1 D^2 = V_2 d^2 \quad \rightarrow \quad V_1 = V_2\left(\frac{d}{D}\right)^2$$

$$2\frac{\rho_v}{\rho} g h = V_2^2 - V_2^2\left(\frac{d}{D}\right)^4$$

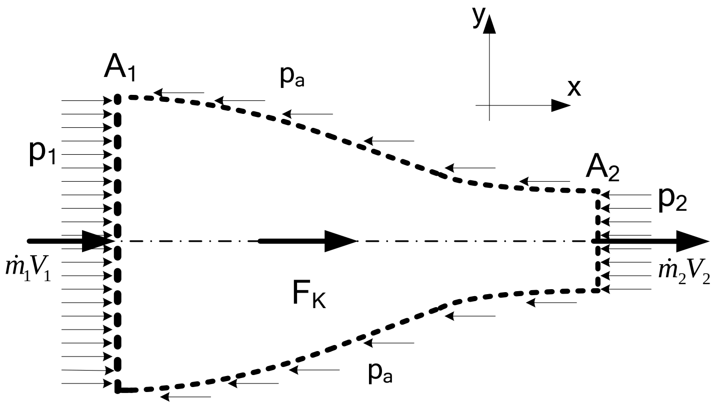
$$2\frac{\rho_v}{\rho} g h = V_2^2\left[1 - \left(\frac{d}{D}\right)^4\right]$$

a) $$V_2 = \sqrt{\frac{2\rho_v g h}{\rho\left[1-\left(\frac{d}{D}\right)^4\right]}} = \sqrt{\frac{2\cdot 1000\cdot 9.81\cdot 0.15}{1.225\left(1-\frac{0.9^4}{1.6^4}\right)}} = 51.67 \quad m/s$$

$$V_1 = V_2\left(\frac{d}{D}\right)^2 = 51.67\left(\frac{0.9}{1.6}\right)^2 = 16.35 \quad m/s$$

b) $$Q = V_1 A_1 = V_1\frac{D^2\pi}{4} = 16.35\cdot\frac{1.6^2\pi}{4} = 32.87 \quad m^3/s$$

c)



J.K.G. 1 − 2: $$\underbrace{\frac{\partial}{\partial t}\int_{\forall}\rho\vec{V}d\forall}_{\varnothing} + \int_{A}(\rho\vec{V}d\vec{A})\cdot\vec{V} = \underbrace{\vec{F}_G}_{\varnothing} + \vec{F}_p + \vec{F}_K + \underbrace{\vec{F}_o}_{\varnothing}$$

$$-\rho V_1^2 A_1 + \rho V_2^2 A_2 = p_1 A_1 - p_2 A_2 - p_a(A_1 - A_2) + \vec{F}_K$$

$$-\rho V_1^2 A_1 + \rho V_2^2 A_2 = p_1 A_1 - p_2 A_2 - p_a A_1 + p_a A_2 + \vec{F}_K$$

$$-\rho V_1^2 A_1 + \rho V_2^2 A_2 = p_1 A_1 - p_2 A_1 + \vec{F}_K$$

$$\rho V_1^2 A_1 = \dot{m}_1 V_1 = \rho V_1^2\frac{D^2\pi}{4} = 1.225\cdot 16.35^2\frac{1.6^2\pi}{4} = 658.42 \quad N$$

$$\rho V_2^2 A_2 = \dot{m}_2 V_2 = \rho V_2^2\frac{d^2\pi}{4} = 1.225\cdot 51.67^2\frac{0.9^2\pi}{4} = 2080.60 \quad N$$

$$p_2 = 101325 \quad Pa$$

$$\Delta p = \rho_v g h = 1000\cdot 9.81\cdot 0.15 = 1471.5 \quad Pa$$

$$p_1 = p_2 + \Delta p = 101325 + 1471.5 = 102796.5 \quad Pa$$

$$-\rho V_1^2 A_1 + \rho V_2^2 A_2 = p_1 A_1 - p_2 A_1 + \vec{F}_K$$

$$p_1 A_1 = p_1 \frac{D^2 \pi}{4} = 102796.5 \frac{1.6^2 \pi}{4} = 206685 \quad Pa$$

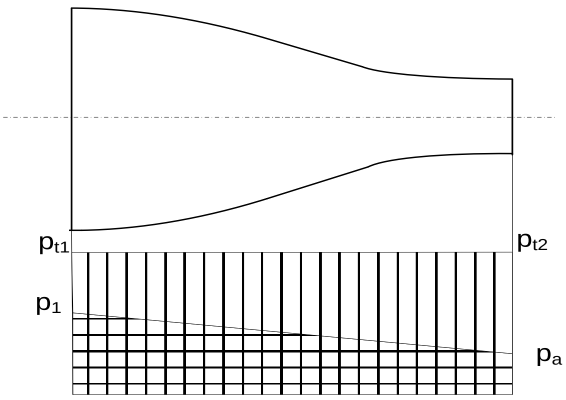
$$p_2 A_1 = p_2 \frac{D^2 \pi}{4} = 101325 \frac{1.6^2 \pi}{4} = 203726 \quad Pa$$

$$\vec{F}_K = -\rho V_1^2 A_1 + \rho V_2^2 A_2 - p_1 A_1 + p_2 A_1$$

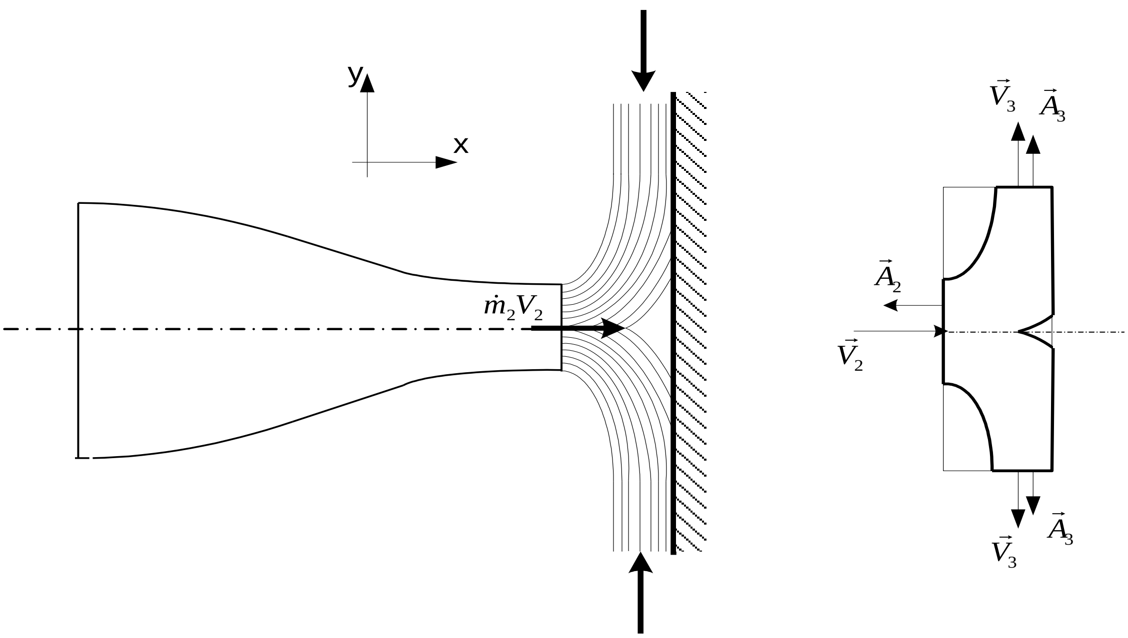
$$F_K = -658.42 + 2080.6 - 206685 + 203726 = -1536.8 \quad N$$

$$\vec{F}_X = -\vec{F}_K = 1536.8 \quad N$$

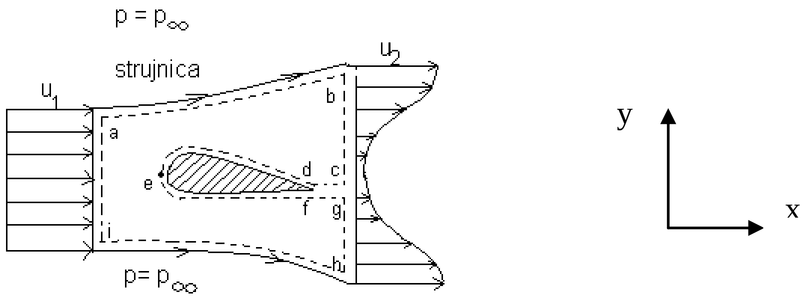
d)



e) $F = \dot{m}_2 V_2 = 2080.6 \quad N$



9. Primjena jednadžbe održanja količine gibanja na određivanje sile otpora F_D dvodimenzionalnog tijela.



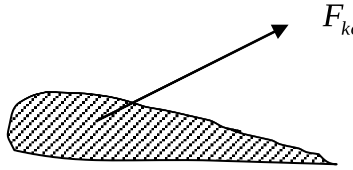
$u_1 = konst$
$u_2 = f(y)$

$$\underbrace{\frac{\partial}{\partial t}\int_{\forall} \rho \vec{V} d\forall}_{\varnothing} + \int_S (\rho \vec{V} d\vec{A}) \cdot \vec{V} = \underbrace{\int_{\forall} \rho \vec{f} d\forall}_{\varnothing} - \int_S p d\vec{A} + \underbrace{\vec{F}_{visc}}_{\varnothing} + \vec{F}_{kont} + \underbrace{\vec{F}_{osl}}_{\varnothing}$$

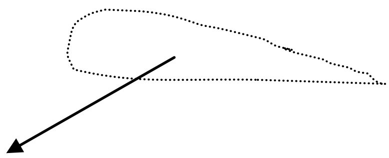
$$\int_S (\rho \vec{V} d\vec{A}) \cdot \vec{V} + \int_S \vec{p} d\vec{A} = \vec{F}_{kont}$$

3. Newton-ov zakon

$\vec{F}_{kont}$



sila na konturi tijela



$-\vec{F}_{kont}$ sila na kontrolni volumen

x: $$\int_S (\rho \vec{V} d\vec{A}) \cdot u + \underbrace{\int_S (pdA)_x}_{\varnothing} = -(F_x)_{kont} = F_D$$

$$F_D = +\int_S (\rho \vec{V} d\vec{A}) \cdot u$$

$$F_D = +\int_a^i (\rho_1 u_1 dy) u_1 - \int_b^h (\rho_2 u_2 dy) u_2$$

$$J.K.\qquad -\int_a^i \rho_1 u_1 dy + \int_b^h \rho_2 u_2 dy = 0$$

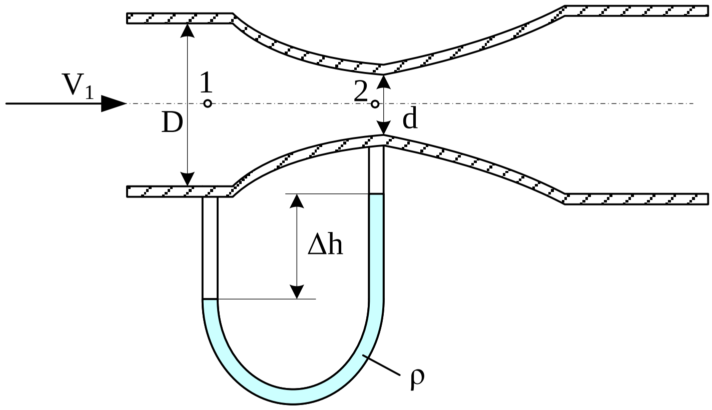
$$\int_a^i \rho_1 u_1 dy = \int_b^h \rho_2 u_2 dy \;/ * u_1$$

$$\int_a^i \rho_1 u_1^2 dy = \int_b^h \rho_2 u_1 u_2 dy$$ uvrstiti

$$F_D = \int_b^h \rho_2 u_1 u_2 dy - \int_b^h \rho_1 u_1^2 dy$$

$$\boxed{F_D = \int_b^h \rho_2 u_2 (u_1 - u_2) dy} \quad \rho = konst \rightarrow \boxed{F_D = \rho \int_b^h u_2 (u_1 - u_2) dy}$$

10. Ulazni promjer Venturijeve cijevi iznosi D = 250 mm, a promjer na najužem dijelu je d = 100 mm. Ako U cijev mjeri razliku u visini vode Δh = 450 mm i nalazi se na visini H = 6000 m potrebno je odrediti:
   a) maseni protok kroz Venturijevu cijev
   b) volumni protok
   c) brzinu leta, brzina na ulazu u cijev V_1



a)

H = 6000 m

$$\rho = \rho_n (1 - 2.256 \cdot 10^{-5} \cdot H)^{4.256} = 1.225 \cdot (1 - 2.256 \cdot 10^{-5} \cdot 6000)^{4.256} = 0.6596 \quad kg/m^3$$

Jednadžba kontinuiteta:

$$V_1 A_1 = V_2 A_2$$

$$V_1 D^2 = V_2 d^2$$

$$\frac{V_1}{V_2} = \frac{d^2}{D^2}$$

Jednadžba manometra od točke 1 do točke 2

$$p_1 + \rho g \Delta h = p_2 + \rho_{H_2O} g \Delta h$$

$$p_1 - p_2 = \rho_{H_2O} g \Delta h - \rho g \Delta h = \rho g \Delta h \left(\frac{\rho_{H_2O}}{\rho} - 1 \right)$$

Bernoulli-eva jednadžba između točaka 1 i 2:

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2 \quad \rightarrow \qquad p_1 - p_2 = \frac{1}{2}\rho (V_2^2 - V_1^2)$$

$$\frac{1}{2}\rho (V_2^2 - V_1^2) = \rho g \Delta h \left(\frac{\rho_{H_2O}}{\rho} - 1 \right)$$

$$V_2^2 \left(1 - \frac{V_1^2}{V_2^2} \right) = 2 g \Delta h \left(\frac{\rho_{H_2O}}{\rho} - 1 \right)$$

$$V_2^2\left[1-\left(\frac{d}{D}\right)^4\right]=2g\Delta h\left(\frac{\rho_{H_2O}}{\rho}-1\right)$$

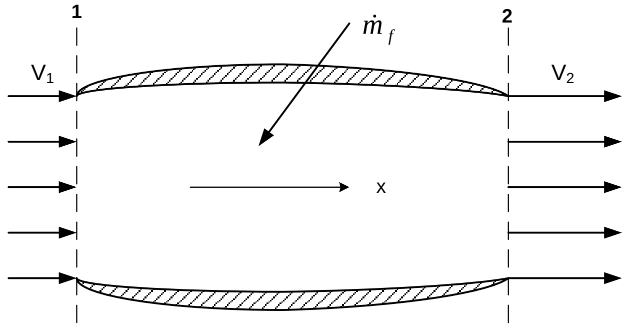
$$V_2=\sqrt{\frac{2g\Delta h\left(\frac{\rho_{H_2O}}{\rho}-1\right)}{1-\left(\frac{d}{D}\right)^4}}=\sqrt{\frac{2\cdot 9.81\cdot 0.45\left(\frac{1000}{0.6596}-1\right)}{1-\frac{0.10^4}{0.25^4}}}=117.2 \quad m/s$$

$$\dot{m}=\rho V_2 A_2=\rho V_2\frac{d^2\pi}{4}=0.6596\cdot 117.2\cdot\frac{0.1^2\pi}{4}=0.607 \quad kg/s$$

b) $Q=V_2 A_2=V_2\frac{d^2\pi}{4}=78.11\cdot\frac{0.1^2\pi}{4}=0.920 \quad m^3/s$

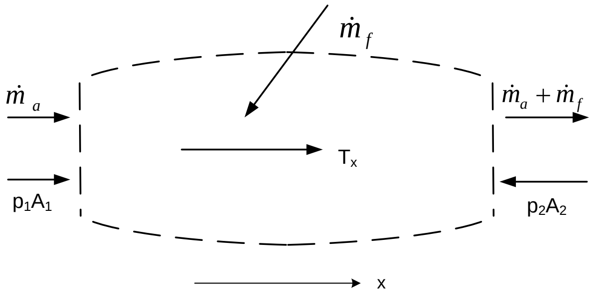
c) $V_1=V_2\frac{d^2}{D^2}=117.2\cdot\frac{0.1^2}{0.25^2}=18.75\, m/s$ (67.5 km/h)

11. Mlazni motor prikazan na slici testira se na ispitnom stolu. Brzina zraka na ulazu je 152.5 m/s, dok ispušni plinovi postižu brzinu od 1066.8 m/s. Tlak zraka na ulazu i tlak ispušnih plinova na izlazu, jednak je atmosferskom tlaku zraka. Odnos količine gorivo/zrak je 1/50, a površina ulazne i izlazne površine motora je jednaka i iznosi 0.186 m^2. Gustoća zraka na ulazu je 1.237 kg/m^3. Potrebno je odrediti silu kojom je potrebno držati motor da bi bio u ravnoteži.



Rješenje:

$$A_1 = A_2 = A$$
$$p_1 = p_2 = p_a$$



$$J.K.G. \quad \rightarrow \qquad F_x + \dot{m}_a V_1 = (\dot{m}_a + \dot{m}_f) V_2$$

F_x → ukupna vanjska sila koja djeluje na motor, a uključuje sile tlaka i silu reakcije na podlogu T_x

$$p_1 A_1 - p_2 A_2 + T_x + \dot{m}_a V_1 = (\dot{m}_a + \dot{m}_f) \cdot V_2$$

$$T_x = (\dot{m}_a + \dot{m}_f) \cdot V_2 - \dot{m}_a V_1 = \left(1 + \frac{\dot{m}_f}{\dot{m}_a}\right) \dot{m}_a V_2 - \dot{m}_a V_1$$

$\dot{m}_a = \rho_1 A_1 V_1$ … maseni protok zraka kroz motor

$$T_x = \left[\left(1 + \frac{\dot{m}_f}{\dot{m}_a}\right) V_2 - V_1\right] \dot{m}_a = \left[\left(1 + \frac{\dot{m}_f}{\dot{m}_a}\right) V_2 - V_1\right] \rho_1 A_1 V_1 =$$

$$= \left[\left(1 + \frac{1}{50}\right) 1066.8 - 152.4\right] \cdot 1.237 \cdot 0.186 \cdot 152.4 = 32811 \quad N$$

Statički potisak motora bio bi jednak po iznosu i suprotno usmjeren.