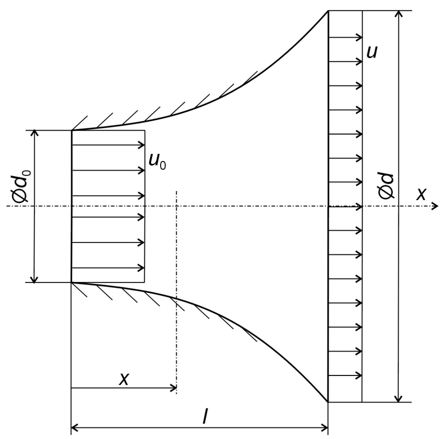
KINEMATIKA FLUIDA

1. Nestlačivi fluid struji kroz difuzor čiji se promjer mijenja prema jednadžbi $d = d_0 + k \cdot x^2$. Brzina na ulazu u difuzor mijenja se prema jednadžbi $u_0 = a + b \cdot t$. Strujanje se može smatrati jednodimenzijskim. Odredi brzinu i ubrzanje na izlazu iz difuzora u trenutku t_1.



Podaci:

d_0 = 100 mm = 0,1 m
l = 200 mm = 0,2 m
a = 2 m/s
b = 0,5 m/s^2
k = 1/400 mm^{-1} = 1/0,4 m^{-1} = 2,5 m^{-1}
t_1 = 10 s

Rješenje: J. K: $\rho_0 \cdot u_0 \cdot A_0 = \rho \cdot u \cdot A$

$$\rho_0 = \rho = \text{const.} \quad \rightarrow \quad u_0 \cdot A_0 = u \cdot A \quad \rightarrow \quad u = u_0 \cdot \frac{A_0}{A} = u_0 \cdot \frac{d_0^2}{d^2}$$

$$u = u_0 \cdot \frac{d_0^2}{d^2} = (a + b \cdot t) \cdot \frac{d_0^2}{\left(d_0 + k \cdot x^2\right)^2} = \frac{(a + b \cdot t) \cdot d_0^2}{\left\{d_0 \cdot \left[1 + (k/d_0) \cdot x^2\right]\right\}^2} = \frac{a + b \cdot t}{\left[1 + (k/d_0) \cdot x^2\right]^2}$$

$$t = t_1 = 10 \text{ s}; \quad x = l = 0{,}2 \text{ m} \quad \rightarrow \quad u = \frac{a + b \cdot t}{\left[1 + (k/d_0) \cdot x^2\right]^2} = \frac{2 + 0{,}5 \cdot 10}{\left[1 + (2{,}5/0{,}1) \cdot 0{,}2^2\right]^2} = 1{,}75 \text{ m/s}$$

– ubrzanje:
$$a_x = \underbrace{\frac{\partial u}{\partial t}}_{\substack{\text{lokalno}\\ \text{ubrzanje}}} + \underbrace{u \cdot \frac{\partial u}{\partial x} + v \cdot \frac{\partial u}{\partial y} + w \cdot \frac{\partial u}{\partial z}}_{\substack{\text{konvektivno}\\ \text{ubrzanje}}}$$

– za jednodimenzijsko strujanje: $a_x = \dfrac{\partial u}{\partial t} + u \cdot \dfrac{\partial u}{\partial x}$

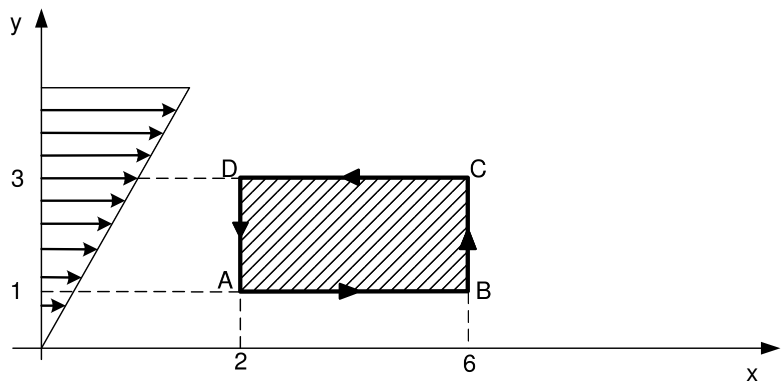
$$a_x = \frac{\partial u}{\partial t} + u \cdot \frac{\partial u}{\partial x} = \frac{\partial}{\partial t}\left\{\frac{a + b \cdot t}{\left[1 + (k/d_0) \cdot x^2\right]^2}\right\} + \frac{a + b \cdot t}{\left[1 + (k/d_0) \cdot x^2\right]^2} \cdot \frac{\partial}{\partial x}\left\{\frac{a + b \cdot t}{\left[1 + (k/d_0) \cdot x^2\right]^2}\right\}$$

$$a_x = \frac{b}{\left[1 + (k/d_0) \cdot x^2\right]^2} + \frac{a + b \cdot t}{\left[1 + (k/d_0) \cdot x^2\right]^2} \cdot \frac{-(a + b \cdot t) \cdot 2 \cdot \left(1 + (k/d_0) \cdot x^2\right) \cdot 2 \cdot (k/d_0) \cdot x}{\left[1 + (k/d_0) \cdot x^2\right]^4}$$

$$a_x = \frac{b}{\left[1 + (k/d_0) \cdot x^2\right]^2} - \frac{4 \cdot (k/d_0) \cdot x \cdot (a + b \cdot t)^2}{\left[1 + (k/d_0) \cdot x^2\right]^5}$$

$$t = 10 \text{ s}; \quad x = 0{,}2 \text{ m} \quad \rightarrow \quad a_x = \frac{0{,}5}{\left[1 + (2{,}5/0{,}1) \cdot 0{,}2^2\right]^2} - \frac{4 \cdot (2{,}5/0{,}1) \cdot 0{,}2 \cdot (2 + 0{,}5 \cdot 10)^2}{\left[1 + (2{,}5/0{,}1) \cdot 0{,}2^2\right]^5} = -30{,}5 \text{ m/s}^2$$

2. Za ravninsko strujanje fluida dan je raspored brzine sa $u = Cy$ i $v = 0$. Odredi :
   a) je li strujanje vrtložno,
   b) vrijednost cirkulacije za krivulju na slici.



a)

$$\vec{\omega} = rot\vec{V} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ u & v & w \end{vmatrix} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\vec{i} - \left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right)\vec{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k}$$

$$\vec{\omega} = -\frac{\partial u}{\partial y}\vec{k} = -\frac{\partial}{\partial y}(Cy)\vec{k} = -C\vec{k}$$

$$\vec{\omega} \neq 0 \quad \Rightarrow \quad strujanje \quad je \quad vrtložno$$

b)

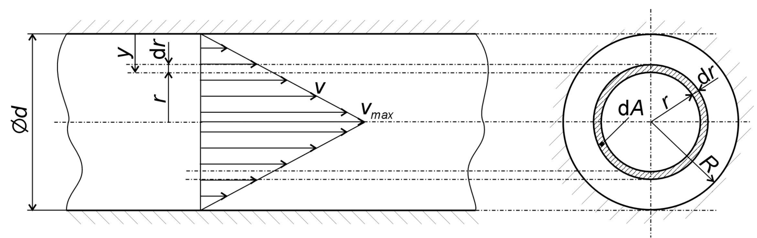
$$\Gamma = \oint_L \vec{V} \cdot d\vec{L} = \int (udx + vdy + wdz) = \int_A^B Cydx + \int_B^C 0 \cdot dy + \int_C^D Cydx + \int_D^A 0 \cdot dy =$$

$$= \int_2^6 C \cdot 1 \cdot dx + \int_6^2 C \cdot 3dx = C \cdot x\Big|_2^6 + 3C \cdot x\Big|_6^2 =$$

$$= C \cdot (6-2) + 3C(2-6) = 4C - 12C = -8C$$

- teorem Kutta-Zhukowski $\longrightarrow$ $F_Z = \rho V_\infty \times \Gamma$

3. Raspored brzina strujanja viskoznog fluida u cijevi kružnog poprečnog presjeka je linearan. Odredi:
   a) protok u funkciji maksimalne brzine v_m i promjera d,
   b) srednju brzinu strujanja u cijevi,
   c) udaljenost y od stjenke cijevi na kojoj je brzina jednaka srednjoj brzini.



Rješenje: – raspored brzina: $v = v_{max} \cdot \left(1 - \dfrac{r}{R}\right)$

– površina dA: $\mathrm{d}A = 2 \cdot r \cdot \pi \cdot \mathrm{d}r$

– protok kroz površinu dA: $\mathrm{d}Q = v \cdot \mathrm{d}A = v_{max} \cdot \left(1 - \dfrac{r}{R}\right) \cdot 2 \cdot r \cdot \pi \cdot \mathrm{d}r$

a) $$Q = \int_A v \cdot \mathrm{d}A = \int_0^R v_{max} \cdot \left(1 - \frac{r}{R}\right) \cdot 2 \cdot r \cdot \pi \cdot \mathrm{d}r = 2 \cdot \pi \cdot v_{max} \cdot \int_0^R \left(r - \frac{r^2}{R}\right) \cdot \mathrm{d}r$$

$$Q = 2 \cdot \pi \cdot v_{max} \cdot \left(\frac{r^2}{2} - \frac{r^3}{3 \cdot R}\right)\Bigg|_0^R = 2 \cdot \pi \cdot v_{max} \cdot \left(\frac{R^2}{2} - \frac{R^3}{3 \cdot R}\right) = 2 \cdot \pi \cdot v_{max} \cdot \frac{R^2}{6}$$

$$Q = \frac{1}{3} \cdot \pi \cdot R^2 \cdot v_{max}$$

b) $$Q = \bar{v} \cdot A \quad \rightarrow \quad \bar{v} = \frac{Q}{A} = \frac{Q}{R^2 \cdot \pi} = \frac{\pi \cdot R^2 \cdot v_{max}}{3 \cdot R^2 \cdot \pi} = \frac{v_{max}}{3}$$

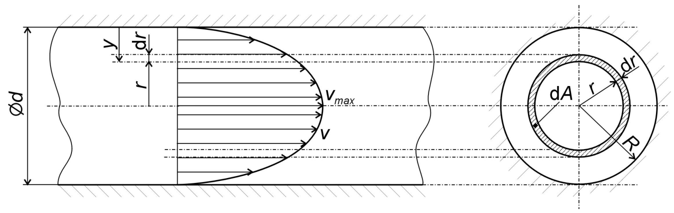
$$\bar{v} = \frac{v_{max}}{3}$$

c) $$v = v_{max} \cdot \left(1 - \frac{r}{R}\right) \quad \rightarrow \quad \frac{v}{v_{max}} = 1 - \frac{r}{R} \quad \rightarrow \quad r = R \cdot \left(1 - \frac{v}{v_{max}}\right)$$

$$v = \bar{v} = \frac{v_{max}}{3} \quad \rightarrow \quad r = R \cdot \left(1 - \frac{v_{max}}{3 \cdot v_{max}}\right) = \frac{2}{3} \cdot R$$

$$y = R - r \quad \rightarrow \quad y = R - \frac{2}{3} \cdot R = \frac{1}{3} \cdot R$$

4. Raspored brzina strujanja viskoznog fluida u cijevi kružnog poprečnog presjeka dan je jednadžbom. Odredi:
   a) protok u funkciji maksimalne brzine v_m i promjera d,
   b) srednju brzinu strujanja u cijevi,



$$\frac{v}{v_{max}} = \left(1 - \frac{r}{R}\right)^{\frac{1}{n}}$$

$$(6 \le n \le 10)$$

<u>Rješenje:</u> $\frac{v}{v_{max}} = \left(1 - \frac{r}{R}\right)^{\frac{1}{n}}$; $\mathrm{d}A = 2 \cdot r \cdot \pi \cdot \mathrm{d}r$; $\mathrm{d}Q = v \cdot \mathrm{d}A = v_{max} \cdot \left(1 - \frac{r}{R}\right)^{\frac{1}{n}} \cdot 2 \cdot r \cdot \pi \cdot \mathrm{d}r$

a) $$Q = \int_A v \cdot \mathrm{d}A = \int_0^R v_{max} \cdot \left(1 - \frac{r}{R}\right)^{\frac{1}{n}} \cdot 2 \cdot r \cdot \pi \cdot \mathrm{d}r = 2 \cdot \pi \cdot v_{max} \cdot \int_0^R \left(1 - \frac{r}{R}\right)^{\frac{1}{n}} \cdot r \cdot \mathrm{d}r$$

– supstitucija: $k = 1 - \frac{r}{R} \quad \rightarrow \quad r = R \cdot (1 - k) \quad \rightarrow \quad \mathrm{d}r = R \cdot (-\mathrm{d}k) = -R \cdot \mathrm{d}k$

$r = 0 \quad \rightarrow \quad k = 1$; $r = R \quad \rightarrow \quad k = 0$

$$Q = 2 \cdot \pi \cdot v_{max} \cdot \int_1^0 k^{\frac{1}{n}} \cdot R \cdot (1 - k) \cdot (-R) \cdot \mathrm{d}k = 2 \cdot \pi \cdot R^2 \cdot v_{max} \cdot \int_0^1 \left(k^{\frac{1}{n}} - k^{\frac{1+n}{n}}\right) \cdot \mathrm{d}k$$

$$Q = 2 \cdot \pi \cdot R^2 \cdot v_{max} \cdot \left.\left(\frac{k^{\frac{1+n}{n}}}{\frac{1+n}{n}} - \frac{k^{\frac{1+2\cdot n}{n}}}{\frac{1+2 \cdot n}{n}}\right)\right|_0^1 = 2 \cdot \pi \cdot R^2 \cdot v_{max} \cdot \left(\frac{1}{\frac{1+n}{n}} - \frac{1}{\frac{1+2 \cdot n}{n}}\right)$$

$$Q = 2 \cdot \pi \cdot R^2 \cdot v_{max} \cdot \left(\frac{n}{1+n} - \frac{n}{1+2 \cdot n}\right) = 2 \cdot \pi \cdot R^2 \cdot v_{max} \cdot \frac{n + 2 \cdot n^2 - n - n^2}{(1+n) \cdot (1+2 \cdot n)} = \frac{2 \cdot \pi \cdot R^2 \cdot n^2 \cdot v_{max}}{(1+n) \cdot (1+2 \cdot n)}$$

$$Q = \frac{2 \cdot \pi \cdot R^2 \cdot n^2 \cdot v_{max}}{(1+n) \cdot (1+2 \cdot n)}$$

b) $$Q = \overline{v} \cdot A \quad \rightarrow \quad \overline{v} = \frac{Q}{A} = \frac{2 \cdot \pi \cdot R^2 \cdot n^2 \cdot v_{max}}{(1+n) \cdot (1+2 \cdot n) \cdot R^2 \cdot \pi} = \frac{2 \cdot n^2 \cdot v_{max}}{(1+n) \cdot (1+2 \cdot n)}$$

$$\overline{v} = \frac{2 \cdot n^2 \cdot v_{max}}{(1+n) \cdot (1+2 \cdot n)}$$