DIMENZIJSKA ANALIZA

1. Aeroprofil duljine tetive ℓ nalazi se u struji zraka pod napadnim kutom α koji se mijenja u nekom rasponu. Odrediti funkcionalnu ovisnost sile uzgona na aeroprofil i ostalih relevantnih bezdimenzionalnih karakteristika.

Rješenje:

$1.\,korak \qquad F_z = f(v, \ell, \rho, \mu, E, \alpha)$

$2.\,korak$

	F	***v***	***ℓ***	***ρ***	***μ***	***E***	***α***
M	1	0	0	1	1	1	0
L	1	1	1	-3	-1	-1	0
T	-2	-1	0	0	-1	-2	0

$3.\,korak$

$$\mathbf{B} = \begin{array}{ccc} v & \ell & \rho \\ \multicolumn{3}{c}{\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & -3 \\ -1 & 0 & 0 \end{vmatrix}} \end{array} = 1 \cdot (0+1) = 1 \quad \neq 0 \qquad \surd$$

$k = n - r = 6 - 3 = 3$ $\quad\rightarrow\ rang\ bezdimenzionalne\ matrice$

$\rightarrow\ mogu\ se\ formirati\ tri\ bezdimenzionalna\ parametra$

$4.\,korak$

$\Pi_1 = F\, v^{a_1}\, \ell^{a_2}\, \rho^{a_3}$

$M^\circ L^\circ T^\circ = M^1 L^1 T^{-2} \cdot (L^1 T^{-1})^{a_1} \cdot (L^1)^{a_2} \cdot (M^1 L^{-3})^{a_3}$

$M^\circ L^\circ T^\circ = M^{1+a_3} \cdot L^{1+a_1+a_2-3a_3} \cdot T^{-2-a_1}$

$$\left.\begin{array}{lll} M:\ 1 + a_3 = 0 & \Rightarrow & a_3 = -1 \\ T:\ -2 - a_1 = 0 & \Rightarrow & a_1 = -2 \\ L:\ 1 + a_1 + a_2 - 3a_3 = 0 & \Rightarrow & a_2 = -2 \end{array}\right\} \quad \Pi_1 = \frac{F}{\rho\, v^2\, \ell^2}$$

$\Pi_2 = \mu\, v^{b_1}\, \ell^{b_2}\, \rho^{b_3}$

$M^\circ L^\circ T^\circ = M^1 L^{-1} T^{-1} \cdot (L^1 T^{-1})^{b_1} \cdot (L^1)^{b_2} \cdot (M^1 L^{-3})^{b_3}$

$M^\circ L^\circ T^\circ = M^{1+b_3} \cdot L^{-1+b_1+b_2-3b_3} \cdot T^{-1-b_1}$

$$
\left.\begin{aligned}
M&:\ 1 + b_3 = 0 && \Longrightarrow\ b_3 = -1\\
T&:\ -1 - b_1 = 0 && \Longrightarrow\ b_1 = -1\\
L&:\ -1 + b_1 + b_2 - 3b_3 = 0 && \Longrightarrow\ b_2 = -1
\end{aligned}\right\}
\quad \Pi_2 = \frac{\mu}{\rho\, v\, \ell} \qquad \Pi_2 = \frac{1}{\mathcal{R}e}
$$

$\Pi_3 = E\, v^{c_1}\, \ell^{c_2}\, \rho^{c_3}$

$M^\circ L^\circ T^\circ = M^1 L^{-1} T^{-2} \cdot (L^1 T^{-1})^{c_1} \cdot (L^1)^{c_2} \cdot (M^1 L^{-3})^{c_3}$

$M^\circ L^\circ T^\circ = M^{1+c_3} \cdot L^{-1+c_1+c_2-3c_3} \cdot T^{-2-c_1}$

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\left.\begin{aligned}
M&:\ 1 + c_3 = 0 && \Longrightarrow\ c_3 = -1\\
T&:\ -2 - c_1 = 0 && \Longrightarrow\ c_1 = -2\\
L&:\ -1 + c_1 + c_2 - 3c_3 = 0 && \Longrightarrow\ c_2 = 0
\end{aligned}\right\}
\quad \Pi_3 = \frac{E}{\rho\, v^{2}}
$$

- za izentropski proces iz jedandžbe stanja:

$$\frac{p}{\rho^\kappa} = K = konst. \quad /\ln \qquad \& \qquad c = \sqrt{\kappa\, R\, T}$$

$$\ln\frac{p}{\rho^\kappa} = \ln K \qquad\qquad p = \rho R T \ \Longrightarrow\ RT = \frac{p}{\rho}$$

$$\ln p - \ln \rho^\kappa = \ln K \qquad\qquad c = \sqrt{\kappa\,\frac{p}{\rho}}$$

$$\ln p - \kappa \ln \rho = \ln K \quad /d \qquad\qquad c^2 \rho = \kappa p$$

$$\frac{dp}{p} - \kappa\frac{d\rho}{\rho} = 0 \qquad\qquad \Pi_3 = \frac{\kappa p}{\rho\, v^{2}} = \frac{c^2}{v^2} \ \Longrightarrow\ \Pi_3 = \frac{1}{Ma}$$

$$\frac{dp}{p} = \kappa\frac{d\rho}{\rho} \qquad\qquad \Phi\left(\frac{F}{\rho\, v^2\, \ell^2}, \mathcal{R}e, Ma, \alpha\right) = 0$$

$$\frac{dp}{d\rho/\rho} = \kappa p$$

$$E = \frac{dp}{d\rho/\rho} = \kappa p$$

$$\frac{F}{\rho\, v^2\, \ell^2} = \Phi_1(\mathcal{R}e, Ma, \alpha)$$

2. Metodom dimenzionalne analize odredi bezdimenzionalne parametre o kojima ovisi snaga propelera, ako se u obzir trebaju uzeti slijedeće veličine:

- snaga propelera P
- broj okretaja propelera n
- gustoća zraka ρ
- viskoznost zraka μ
- promjer propelera D
- brzina zrakoplova V

Rješenje:

$1.\,korak \qquad f(\mathrm{P}, \mathrm{n}, \rho, \mu, \mathrm{D}, \mathrm{V}) = 0$

$2.\,korak$

	P	***n***	***ρ***	***μ***	***D***	***V***
M	1	0	1	1	0	0
L	2	0	-3	-1	1	1
T	-3	-1	0	-1	0	-1

$3.\,korak$

$$\mathbf{B} = \begin{array}{c} \begin{matrix} n & \rho & D \end{matrix} \\ \begin{vmatrix} 0 & 1 & 0 \\ 0 & -3 & 1 \\ -1 & 0 & 0 \end{vmatrix} \end{array} = -1 \cdot (0+1) = -1 \quad \neq 0 \quad \surd$$

$k = n - r = 6 - 3 = 3 \qquad \rightarrow\ rang\ bezdimenzionalne\ matrice$
$\rightarrow\ mogu\ se\ formirati\ tri\ bezdimenzionalna\ parametra$

$4.\,korak$

$\Pi_1 = P\, n^{a_1}\, \rho^{a_2}\, D^{a_3}$

$\mathrm{M}^{\circ}\mathrm{L}^{\circ}\mathrm{T}^{\circ} = \mathrm{ML}^2\mathrm{T}^{-3} \cdot (\mathrm{T}^{-1})^{a_1} \cdot (\mathrm{ML}^{-3})^{a_2} \cdot \mathrm{L}^{a_3}$

$\mathrm{M}^{\circ}\mathrm{L}^{\circ}\mathrm{T}^{\circ} = \mathrm{M}^{1+a_2} \cdot \mathrm{L}^{2-3a_2+a_3} \cdot \mathrm{T}^{-3-a_1}$

$$\left.\begin{array}{lcl} M:\ 1 + a_2 = 0 & \Rightarrow & a_2 = -1 \\ T:\ -3 - a_1 = 0 & \Rightarrow & a_1 = -3 \\ L:\ 2 - 3a_2 + a_3 = 0 & \Rightarrow & a_3 = -5 \end{array}\right\} \quad \Pi_1 = \frac{P}{\rho\, n^3\, D^5} = C_p$$

$\Pi_2 = \mu\, n^{b_1}\, \rho^{b_2}\, D^{b_3}$

$$M^\circ L^\circ T^\circ = ML^{-1}T^{-1}\cdot(T^{-1})^{b_1}\cdot(ML^{-3})^{b_2}\cdot L^{b_3}$$

$$M^\circ L^\circ T^\circ = M^{1+b_2}\cdot L^{-1-3b_2+b_3}\cdot T^{-1-b_1}$$

$$\left.\begin{array}{lcl} M:\ 1 + b_2 = 0 & \Longrightarrow & b_2 = -1 \\ T:\ -1 - b_1 = 0 & \Longrightarrow & b_1 = -1 \\ L:\ -1 - 3b_2 + b_3 = 0 & \Longrightarrow & b_3 = -2 \end{array}\right\}$$

$$\Pi_2 = \frac{\mu}{\rho\, n\, D^2} = \frac{\mu}{\rho\, n\, 4R^2}$$

$$\Pi_2 = \frac{\mu}{4\rho\cdot 2\pi\omega R\cdot R} = \frac{\mu}{8\pi\rho V R} = \frac{1}{8\pi}\cdot\frac{1}{Re}$$

$$\Pi_3 = V\, n^{c_1}\,\rho^{c_2}\, D^{c_3}$$

$$M^\circ L^\circ T^\circ = LT^{-1}\cdot(T^{-1})^{c_1}\cdot(ML^{-3})^{c_2}\cdot L^{c_3}$$

$$M^\circ L^\circ T^\circ = M^{c_2}\cdot L^{1-c_2+c_3}\cdot T^{-1-c_1}$$

$$\left.\begin{array}{lcl} M:\ c_2 = 0 & & \\ T:\ -1 - c_1 = 0 & \Longrightarrow & c_1 = -1 \\ L:\ 1 - 3c_2 + c_3 = 0 & \Longrightarrow & c_3 = -1 \end{array}\right\} \quad \Pi_3 = \frac{V}{n\, D} = J$$

$$\Phi\left(\frac{P}{\rho\, n^3\, D^5}, \frac{1}{Re}, \frac{V}{n\, D}\right) = 0$$

$$\frac{P}{\rho\, n^3\, D^5} = \Phi_1(\mathcal{R}e, \mathrm{J})$$