# OSNOVE MEHANIKE FLUIDA

Pripremili: mr.sc. Davor Franjković, Jasna Sviben

**(Napomena: Za pregled formula potreban je program Rapid Pi, koji možete preuzeti na stranici www.rapid-pi.com – probna verzija traje 60 dana)**

*** 3 temeljna načela:**

1. masa ne može biti niti stvorena niti uništena, tj. masa je stalna → zakon održanja mase → jednadžba kontinuiteta

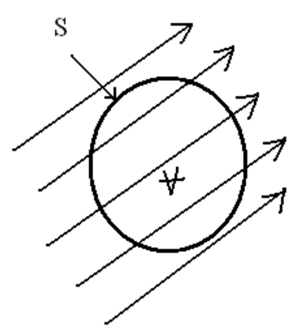
2. sila je jednaka umnošku mase i akceleracije
   F = ma → 2. Newtonov zakon

3. energija je stalna, može se samo mijenjati iz jednog oblika u drugi

*** odgovarajući model fluida**

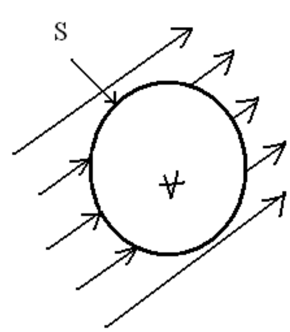
1. kontrolni volumen konačnih dimenzija

   a) kontrolni volumen fiksiran u prostoru → fluid prolazi kroz njega



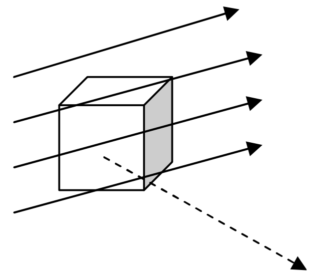

S = granična površina (kontrolna površina)

   b) kontrolni volumen se giba zajedno s fluidom; u njemu se stalno nalaze iste čestice fluida

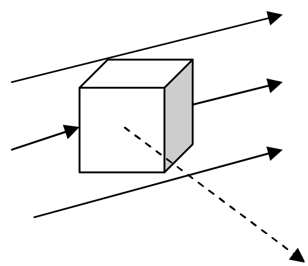


2. beskonačno (infinitezimalno) mali element fluida
   beskonačno mali, ali dovoljno velik da sadrži ogroman broj molekula, tako da vrijedi pretpostavka o kontinuiranosti fluida

   a) fiksiran u prostoru




   b) giba se zajedno s fluidom




3. molekularni pristup

*** 3 temeljna načela primijeniti na odgovarajući model fluida kako bi dobili izraze koji pravilno opisuju strujanje**

*** Kinematika**

Brzina

$$\vec{V} = u\,\vec{i} + v\,\vec{j} + w\,\vec{k} = v_x\,\vec{i} + v_y\,\vec{j} + v_z\,\vec{k}$$

Ubrzanje

$$\vec{a} = \frac{D\vec{V}}{Dt} = \underbrace{\frac{\partial \vec{V}}{\partial t}}_{\text{lokalno ubrzanje}} + \underbrace{u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}}_{\text{konvektivno ubrzanje}}$$

*** MATERIJALNA DERIVACIJA**

$$\frac{D}{Dt}=\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}+w\frac{\partial}{\partial z}$$

$$a_x=\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}$$

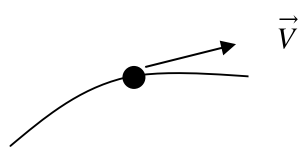
$$a_y=\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}$$

$$a_z=\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}$$

*** metode za proučavanje kinematike fluida**

| Langrangeova metoda | Eulerova metoda |
|---|---|
| $\downarrow$ | $\downarrow$ |
| promatra se čestica | promatra se točka u prostoru |

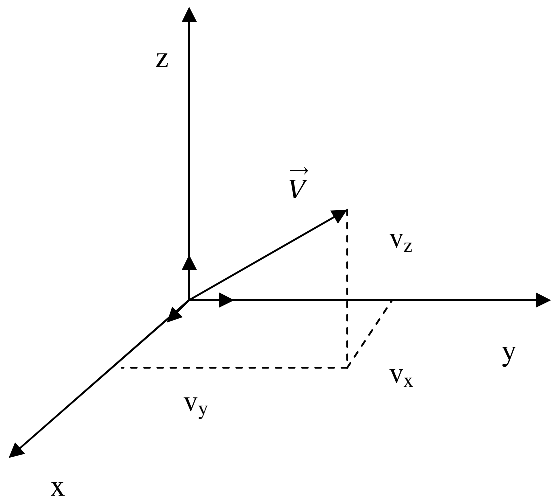
*** DIVERGENCIJA ILI GRADIJENT BRZINE**



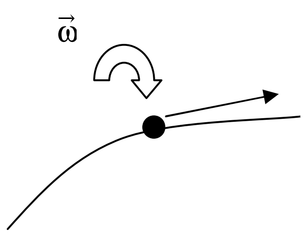

$\vec{V}=v_x\vec{i}+v_y\vec{j}+v_z\vec{k}$ $\longrightarrow$ brzina je vektor

$$\boxed{\nabla\vec{V}=\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}}$$ (gradijent brzine je skalarna veličina)

$\left(\nabla=\frac{\partial}{\partial x}\vec{i}+\frac{\partial}{\partial y}\vec{j}+\frac{\partial}{\partial z}\vec{k}\right)$ $\rightarrow$ Laplaceov operator (nabla)




## * VRTLOŽNOST BRZINE

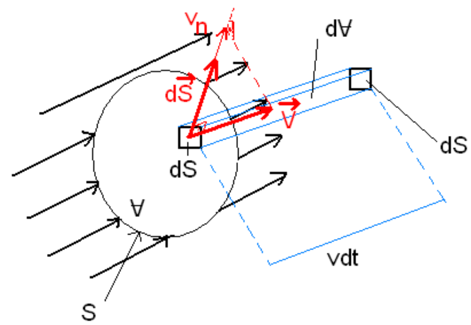



$$\nabla \times \vec{V} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_x & v_y & v_z \end{vmatrix} = \vec{i}\left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) + \vec{j}\left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) + \vec{k}\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)$$

(vektorska veličina)

*** JEDNADŽBA KONTINUITETA**

Masa ne može biti niti stvorena niti uništena.
Maseni protok koji napusti kontrolni volumen kroz graničnu površinu
= smanjenje mase s vremenom unutar kontrolnog volumena




$$d\vec{S} \cdot \vec{V} = dS \cdot V_n$$

$$d\forall = \left(\vec{V}\,\Delta t\right) \cdot d\vec{S} = V_n \cdot \Delta t \cdot dS$$

$$dm = \rho d\forall = \rho \cdot V_n \cdot \Delta t \cdot dS$$

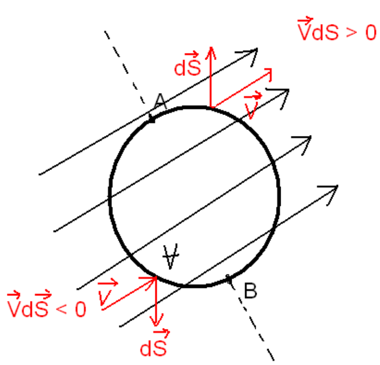
$$\Delta m = \int_S \rho V_n \cdot \Delta t \cdot dS$$

$$\Delta m = \rho V_n\, \Delta t \cdot S$$

$$\dot{m} = \frac{\Delta m}{\Delta t} = \rho V_n \cdot S$$ maseni protok

$$\int_S \rho \vec{V}\, d\vec{S} = -\frac{\partial}{\partial t} \int_\forall \rho d\forall$$

$$\boxed{\frac{\partial}{\partial t} \int_\forall \rho d\forall + \int_S \rho \vec{V} \cdot d\vec{S} = 0}$$ jednadžba kontinuiteta u integralnom obliku




(struja koja izlazi iz kontrolnog volumena)

(struja koja ulazi u kontrolni volumen)

$$\int_{\forall} \frac{\partial \rho}{\partial t} d\forall + \int_{S} \rho \vec{V} \cdot d\vec{S} = 0$$

Teorem divergencije: $\int_{S} \vec{V}\, d\vec{S} = \int_{\forall} \left(\nabla \cdot \vec{V}\right) d\forall$

$$\int_{\forall} \frac{\partial \rho}{\partial t} d\forall + \int_{\forall} \nabla \left(\rho \cdot \vec{V}\right) d\forall = 0$$

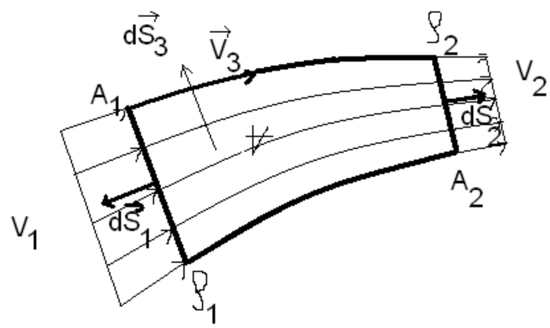
$$\int_{\forall} \left[\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \vec{V}\right)\right] d\forall = 0$$

$\boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \vec{V}\right) = 0}$ jednadžba kontinuiteta u diferencijalnom obliku

- neustaljeno strujanje: $\rho = \rho\,(x, y, z, t)$
- ustaljeno strujanje: $\frac{\partial}{\partial t} = 0 \rightarrow \rho = \rho\,(x, y, z)$

$$\int \rho \cdot \vec{V} \cdot d\vec{S} = 0 \quad \text{ili} \quad \nabla \cdot \left(\rho \vec{V}\right) = 0$$

*** Jednadžba kontinuiteta za strujnu cijev za ustaljeno gibanje**




$S_1 :\ \vec{V_1} \cdot d\vec{S_1} = -V_1 \cdot dS_1$

$S_2 :\ \vec{V_2} \cdot d\vec{S_2} = V_2 \cdot dS_2$

$S_3 :\ \vec{V_3} \cdot d\vec{S_3} = 0$

$$\frac{\partial}{\partial t} \int_{\forall} \rho d\forall + \int_{S} \rho \vec{V}\, d\vec{S} = 0$$

$\frac{\partial}{\partial t}=0 \;\rightarrow\;$ ustaljeno gibanje

$$\int_S \rho \vec{V}\, d\vec{S}=0 \quad \rightarrow \quad -\int_{S_1} \rho_1 V_1\, dS_1+\int_{S_2} \rho_2 V_2\, dS_2=0$$

$$-\rho_1 V_1 A_1+\rho_2 V_2 A_2=0$$

$$\boxed{\rho_1 V_1 A_1=\rho_2 V_2 A_2}$$

Za nestlačivi fluid: $\rho_1=\rho_2=\rho=\text{konst} \;\rightarrow\; V_1 A_1=V_2 A_2$

*** JEDNADŽBA ODRŽANJA KOLIČINE GIBANJA**

(Jednadžba momenta, osnovna jednadžba dinamike fluida)

$\vec{F}=m\cdot\vec{a} \;\rightarrow\;$ 2. Newtonov zakon

$$\vec{F}=\frac{d}{dt}\left(m\cdot\vec{V}\right)$$

$m\cdot\vec{V}$ - količina gibanja (moment)

Sila = promjena količine gibanja s vremenom

- volumenske sile: sila gravitacije, elektromagnetske sile...
- površinske sile: tlak i tangencijalno naprezanje na graničnoj površini

$$\vec{F}=\int_{\forall} \rho \vec{f}\, d\forall-\int_S p d\vec{S}+\vec{F}_{visc}+\vec{F}_{kont}+\vec{F}_{osl}$$

$\int_{\forall} \rho \vec{f}\, d\forall \;\rightarrow\;$ volumenske sile $\vec{f}\left[\frac{N}{kg}\right]$

$\int_S p d\vec{S} \;\rightarrow\;$ površinske sile

$\vec{F}_{visc} \;\rightarrow$ viskozne sile

$\vec{F}_{kont} \;\rightarrow$ sile konture

$\vec{F}_{osl} \;\rightarrow$ sile oslonaca

$$\frac{d}{dt}\left(m\vec{V}\right)=\int_S \underbrace{\left(\rho \vec{V}\, d\vec{S}\right)}_{d\dot{m}}\cdot\vec{V}+\frac{\partial}{\partial t}\int_{\forall} \rho \vec{V}\, d\forall$$

Promjena količine gibanja jednaka je zbroju količine gibanja koja izlazi iz kontrolnog volumena kroz graničnu površinu S i promjene količine gibanja unutar kontrolnog volumena s vremenom.

$$\frac{\partial}{\partial t}\int_{\forall} \rho \vec{V}\, d\forall + \int_{S} \left(\rho \vec{V}\, d\vec{S}\right)\vec{V} = \int_{\forall} \rho \vec{f}\, d\forall - \int_{S} p d\vec{S} + \vec{F}_{visc} + \vec{F}_{kont} + \vec{F}_{osl}$$

Jednadžba održanja količine gibanja u integralnom obliku

Teorem gradijenta $\int_{S} p d\vec{S} = \int_{\forall} \nabla p d\forall$

$$\int_{\forall} \frac{\partial \left(\rho \vec{V}\right)}{\partial t}\, d\forall + \int_{S} \left(\rho \vec{V}\, d\vec{S}\right)\vec{V} = \int_{\forall} \rho \vec{f}\, d\forall - \int_{S} \nabla p d\forall + \vec{F}_{visc} + \vec{F}_{kont} + \vec{F}_{osl}$$

Vektorska jednadžba $\longrightarrow$ 3 skalarne jednadžbe

$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k} = v_x\vec{i} + v_y\vec{j} + v_z\vec{k}$$

X: $$\int_{\forall} \frac{\partial (\rho u)}{\partial t}\, d\forall + \int_{S} \left(\rho \vec{V}\, d\vec{S}\right) u = \int_{\forall} \rho f_x\, d\forall - \int_{\forall} \frac{\partial p}{\partial x}\, d\forall + \left(F_x\right)_{visc} + \left(F_x\right)_{kont} + \left(F_x\right)_{osl}$$

Teorem divergencije: $\int_{S} \left(\rho \vec{V}\, d\vec{S}\right) u = \int_{S} \left(\rho u \vec{V}\right) d\vec{S} = \int_{\forall} \nabla \cdot \left(\rho u \vec{V}\right) d\forall$

$$\int_{\forall} \left[\frac{\partial (\rho u)}{\partial t} + \nabla \cdot \left(\rho u \vec{V}\right) - \rho f_x + \frac{\partial p}{\partial x} - \left(F_x\right)_{visc} - \left(F_X\right)_{kont} - \left(F_X\right)_{osl}\right] d\forall = 0$$

Jednadžbe održanja količine gibanja u diferencijalnom obliku $\longrightarrow$ Navier – Stokesove jednadžbe

$$\boxed{\frac{\partial (\rho u)}{\partial t} + \nabla \cdot \left(\rho u \vec{V}\right) = \rho f_x - \frac{\partial p}{\partial x} + \left(F_x\right)_{visc} + \left(F_x\right)_{kont} + \left(F_x\right)_{osl}}$$

$$\boxed{\frac{\partial (\rho v)}{\partial t} + \nabla \cdot \left(\rho v \vec{V}\right) = \rho f_y - \frac{\partial p}{\partial y} + \left(F_y\right)_{visc} + \left(F_y\right)_{kont} + \left(F_y\right)_{osl}}$$

$$\boxed{\frac{\partial (\rho w)}{\partial t} + \nabla \cdot \left(\rho w \vec{V}\right) = \rho f_z - \frac{\partial p}{\partial z} + \left(F_z\right)_{visc} + \left(F_z\right)_{kont} + \left(F_z\right)_{osl}}$$

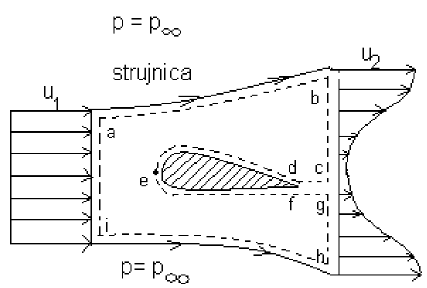
Za ustaljeno $\left(\frac{\partial}{\partial t}=0\right)$, neviskozno $\left(F_{visc}=0\right)$ strujanje u kojem se volumenske sile te sile konture i oslonca mogu zanemariti $\left(f=0,\, F_{kont}=0,\, F_{osl}=0\right)$ :

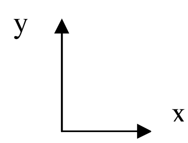
$$\boxed{\nabla\left(\rho u \vec{V}\right)=-\frac{\partial p}{\partial x}}$$

$$\boxed{\nabla\left(\rho v \vec{V}\right)=-\frac{\partial p}{\partial y}}$$ EULEROVE JEDNADŽBE

$$\boxed{\nabla\left(\rho w \vec{V}\right)=-\frac{\partial p}{\partial z}}$$

*** Primjena jednadžbe održanja količine gibanja na određivanje otpora dvodimenzionalnog tijela**

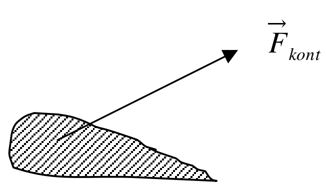


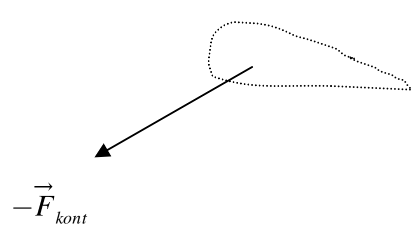

$u_1$ = const
$u_2$ = f(y)

$$\underbrace{\frac{\partial}{\partial t}\int_{\forall}\rho\vec{V}\,d\forall}_{\varnothing}+\int_{S}\left(\rho\vec{V}\,d\vec{S}\right)\vec{V}=\underbrace{\int_{\forall}\rho\vec{f}\,d\forall}_{\varnothing}-\int_{S}p d\vec{S}+\underbrace{\vec{F}_{visc}}_{\varnothing}+\vec{F}_{kont}+\underbrace{\vec{F}_{osl}}_{\varnothing}$$

$$\int_{S}\left(\rho\vec{V}\,d\vec{S}\right)\vec{V}+\int_{S}p d\vec{S}=\vec{F}_{kont}$$




sila na konturi tijela




sila na kontrolni volumen

3\. Newtonov zakon

$$\text{x:} \quad \int_S \left(\rho\,\vec{V}\,d\vec{S}\right)u + \underbrace{\cancel{\int_S \left(p\,d\vec{S}\right)_x}}_{\varnothing} = -\left(F_x\right)_{kont} = -F_D$$

$$F_D = -\int_S \left(\rho\,\vec{V}\,d\vec{S}\right)u$$

$$F_D = +\int_a^i \left(\rho_1\,u_1\,dy\right)u_1 - \int_b^h \left(\rho_2\,u_2\,dy\right)u_2$$

$$\text{J.K.} \qquad -\int_a^i \rho_1\,u_1\,dy + \int_b^h \rho_2\,u_2\,dy = 0$$

$$\int_a^i \rho_1\,u_1\,dy = \int_b^h \rho_2\,u_2\,dy \;\; /*\; u_1$$

$$\int_a^i \rho_1\,u_1^2\,dy = \int_b^h \rho_2\,u_1\,u_2\,dy \qquad \text{uvrstiti}$$

$$F_D = \int_b^h \rho_2\,u_1\,u_2\,dy - \int_b^h \rho_2\,u_2^2\,dy$$

$$\boxed{F_D = \int_b^h \rho_2\,u_2\,(u_1 - u_2)\,dy} \qquad\qquad \rho = const \;\rightarrow\; \boxed{F_D = \rho\int_b^h u_2\,(u_1 - u_2)\,dy}$$

*** JEDNADŽBA ENERGIJE (SNAGE)**

Energija ne može biti niti stvorena niti uništena, ona samo prelazi iz jednog oblika u drugi.
U integralnom obliku:

$$\underbrace{\frac{\partial}{\partial t}\int_{\forall} \rho\left(e + \frac{V^2}{2}\right)d\forall}_{\substack{\text{promjena ukupne}\\ \text{energije unutar}\\ \text{kontrolnog volumena}\\ \text{s vremenom}}} + \underbrace{\int_S \rho\left(e + \frac{V^2}{2}\right)\vec{V}\cdot d\vec{S}}_{\substack{\text{promjena energije}\\ \text{kroz graničnu}\\ \text{površinu}}} = \underbrace{\int_{\forall} \dot{q}\rho d\forall + \dot{Q}_{visc}}_{\substack{\text{toplina dovedena}\\ \text{fluidu unutar}\\ \text{kontrolnog}\\ \text{volumena}}} \underbrace{- \int_S p\,\vec{V}\cdot d\vec{S}}_{\substack{\text{rad izvršen na}\\ \text{fluid uslijed}\\ \text{sila tlaka na}\\ \text{graničnoj}\\ \text{površini}}} + \underbrace{\int_{\forall} \rho\left(\vec{f}\cdot\vec{V}\right)d\forall}_{\substack{\text{rad volumenskih}\\ \text{sila na fluid}\\ \text{u kontrolnom}\\ \text{volumenu}}} + \underbrace{\dot{W}_{visc}}_{\substack{\text{rad}\\ \text{viskoznih}\\ \text{sila}}}$$

U diferencijalnom obliku:

$$\boxed{\frac{\partial}{\partial t}\left[\rho\left(e+\frac{V^2}{2}\right)\right]+\nabla\cdot\left[\rho\left(e+\frac{V^2}{2}\right)\cdot\vec{V}\right]=\rho\dot{q}-\nabla\cdot\left(p\vec{V}\right)+\rho\left(\vec{f}\cdot\vec{V}\right)+\dot{Q}_{visc}+\dot{W}_{visc}}$$

**MATERIJALNA DERIVACIJA (STOKESOVA, STVARNA)**

$$\frac{D}{Dt}=\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}+w\frac{\partial}{\partial z}$$

$$\frac{D}{Dt}=\underbrace{\frac{\partial}{\partial t}}_{\text{lokalna}}+\underbrace{\vec{V}\cdot\nabla}_{\text{konvektivna (prijenosna)}}$$

derivacija

$$\boxed{\frac{D\rho}{Dt}+\rho\,\vec{\nabla}\cdot\vec{V}=0}$$ jednadžba kontinuiteta

$$\boxed{\rho\frac{Du}{Dt}=-\frac{\partial p}{\partial x}+\rho f_x+\left(F_x\right)_{visc}+\left(F_x\right)_{kont}+\left(F_x\right)_{osl}}$$

$$\boxed{\rho\frac{Dv}{Dt}=-\frac{\partial p}{\partial y}+\rho f_y+\left(F_y\right)_{visc}+\left(F_y\right)_{kont}+\left(F_y\right)_{osl}}$$ jednadžba održanja količine gibanja

$$\boxed{\rho\frac{Dw}{Dt}=-\frac{\partial p}{\partial z}+\rho f_z+\left(F_z\right)_{visc}+\left(F_z\right)_{kont}+\left(F_z\right)_{osl}}$$

$$\boxed{\rho\frac{D\left(e+\frac{V^2}{2}\right)}{Dt}=\rho\dot{q}-\nabla\cdot\left(p\vec{V}\right)+\rho\left(\vec{f}\cdot\vec{V}\right)+\dot{Q}_{visc}+\dot{W}_{visc}}$$ jednadžba energije

*** BERNOULLIJEVA JEDNADŽBA**

x: $\rho\frac{Du}{Dt}=-\frac{\partial p}{\partial x}$

$$\rho\frac{\partial u}{\partial t}+\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}+\rho w\frac{\partial u}{\partial z}=-\frac{\partial p}{\partial x}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial x}\qquad /*dx$$

$$u\frac{\partial u}{\partial x}dx+v\frac{\partial u}{\partial y}dx+w\frac{\partial u}{\partial z}dx=-\frac{1}{\rho}\frac{\partial p}{\partial x}dx$$

Strujnica $udz-wdx=0$

$vdx-udy=0$

$$u\frac{\partial u}{\partial x}dx + u\frac{\partial u}{\partial y}dy + u\frac{\partial u}{\partial z}dz = -\frac{1}{\rho}\frac{\partial p}{\partial x}dx$$

$$u\left(\frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz\right) = -\frac{1}{\rho}\frac{\partial p}{\partial x}dx$$

$$u = u\,(x,y,z) \qquad du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz$$

$$udu = -\frac{1}{\rho}\frac{\partial p}{\partial x}dx$$

$$\left.\begin{aligned} \frac{1}{2}d\left(u^2\right) &= -\frac{1}{\rho}\frac{\partial p}{\partial x}dx \\ \frac{1}{2}d\left(v^2\right) &= -\frac{1}{\rho}\frac{\partial p}{\partial y}dy \\ \frac{1}{2}d\left(w^2\right) &= -\frac{1}{\rho}\frac{\partial p}{\partial z}dz \end{aligned}\right\} \quad +$$

$$\frac{1}{2}d\underbrace{\left(u^2+v^2+w^2\right)}_{V^2 = u^2+v^2+w^2} = \underbrace{-\frac{1}{\rho}\left(\frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy + \frac{\partial p}{\partial z}dz\right)}_{dp = \frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy + \frac{\partial p}{\partial z}dz}$$

$$\frac{1}{2}d\left(V^2\right) = -\frac{dp}{\rho}$$

$\boxed{dp = -\rho V dV}$ Eulerova jednadžba

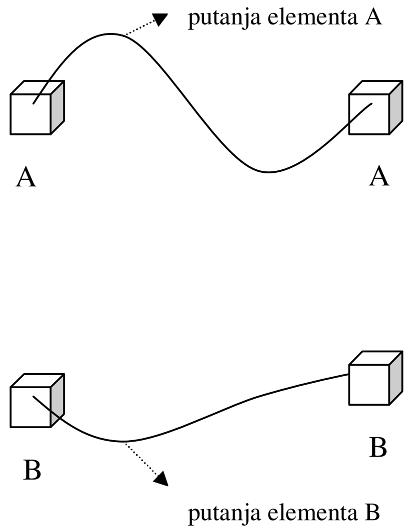
$$\int_{p_1}^{p_2} dp = -\rho\int_{V_1}^{V_2} VdV$$

$$p_2 - p_1 = -\rho\left(\frac{V_2^2}{2} - \frac{V_1^2}{2}\right)$$

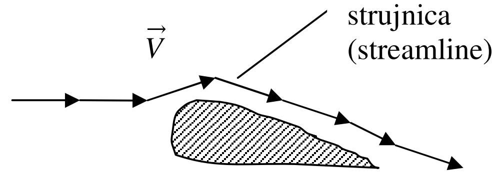
$\boxed{p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2}$ Bernoullijeva jednadžba

$p + \frac{1}{2}\rho V^2 = const$ $\rightarrow$ duž strujnice

## * STRUJNICA, PUTANJA, STRUJNA CIJEV



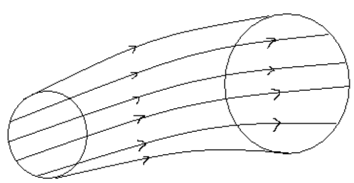

(staza, path)

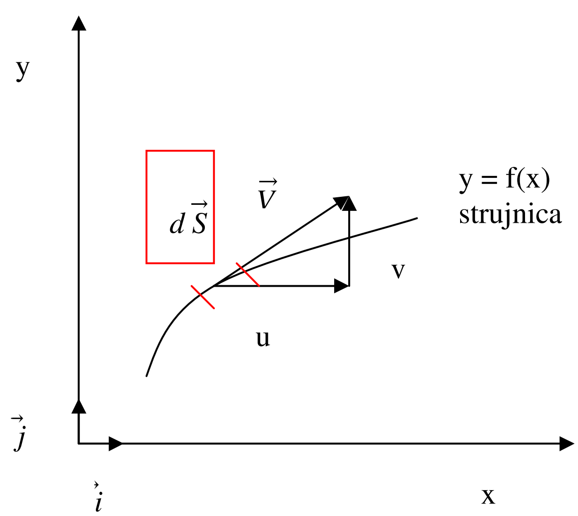



nema protoka kroz strujnice

kod ustaljenog strujanja strujnica i putanja se podudaraju

strujna cijev – skupina strujnica





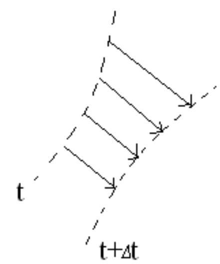
$$d\vec{S}\times\vec{V}=0$$
$$d\vec{S}=dx\,\vec{i}+dy\,\vec{j}+dz\,\vec{k}$$
$$\vec{V}=u\,\vec{i}+v\,\vec{j}+w\,\vec{k}$$

$$d\vec{S}\times\vec{V}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ dx & dy & dz\\ u & v & w\end{vmatrix}=\vec{i}\left(wdy-vdz\right)+\vec{j}\left(udz-wdx\right)+\vec{k}\left(vdx-udy\right)=0$$

$$\left.\begin{aligned} wdy-vdz&=0\\ udz-wdx&=0\\ vdx-udy&=0\end{aligned}\right\}\quad\rightarrow\ \frac{dy}{v}=\frac{dz}{w}\,;\,\frac{dz}{w}=\frac{dx}{u}\,;\,\frac{dx}{u}=\frac{dy}{v}$$

Trag je određen pozicijom svih čestica koje su prošle kroz određenu točku u prostoru (streamline).

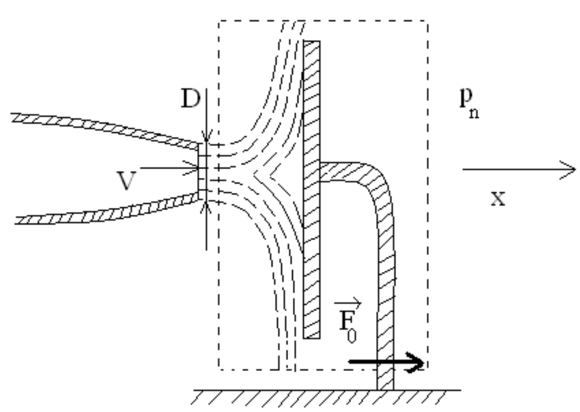
Vremenska linija (timeline) – susjedne čestice nakon određenog vremena




*** PRIMJER - ZADATAK**

Na ispitnom stolu istražuje se horizontalna sila koju stvara ispuh mlaznog motora na vertikalno postavljenu ravnu ploču. Brzina ispuha je 150 m/s pri standardnim uvjetima na razini mora, a promjer ispuha je 0,3 m. Odredi veličinu horizontalne sile.
Ispuh ima karakteristike zraka.

**1. način**



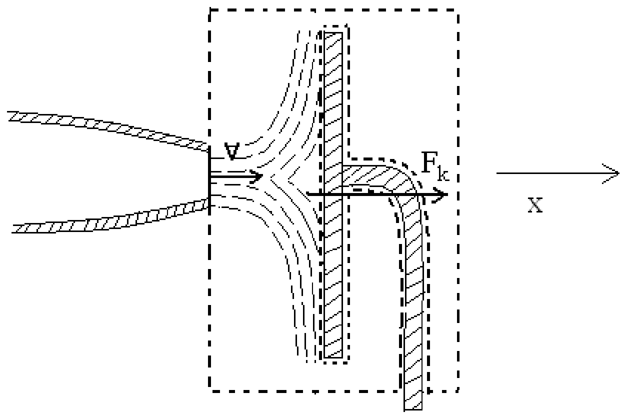
$$\underbrace{\frac{\partial}{\partial t}\int_{GV}\rho\vec{V}\,d\forall}_{\varnothing}+\int_{GP}\rho\vec{V}\,\vec{V}\,d\vec{A}=\underbrace{\vec{F}_G}_{\varnothing}+\vec{F}_P+\underbrace{\vec{F}_K}_{\varnothing}+\vec{F}_O$$

$$-\rho V\cdot VA_1=p_n\cdot A-p_n\cdot A+F_o$$

$$F_o=-\rho\cdot V^2\cdot A_1=-1{,}225\cdot 150^2\cdot\frac{0{,}3^2\,\pi}{4}=-1948\,N$$

$$F_x=-F_o=1948\,N$$

**2. način**



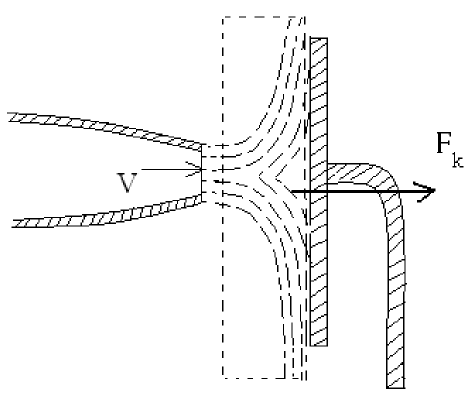

$$\underbrace{\frac{\partial}{\partial t}\int_{GV}\rho\vec{V}\,d\forall}_{\varnothing}+\int_{GP}\rho\vec{V}\,\vec{V}\,d\vec{A}=\underbrace{\vec{F}_G}_{\varnothing}+\vec{F}_P+\vec{F}_K+\underbrace{\vec{F}_O}_{\varnothing}$$

$$-\rho VVA_1=p_n\cdot A-p_n\cdot A+F_k$$

$$F_k=-\rho V^2A_1$$

$$F_x=-F_k=1948\,N$$

**3. način**



$$\cancelto{\varnothing}{\frac{\partial}{\partial t}\int_{GV} \rho \vec{V}\, d\forall} + \int_{GP} \rho \vec{V}\vec{V}\, d\vec{A} = \cancelto{\varnothing}{\vec{F}_G} + \vec{F}_P + \vec{F}_K + \cancelto{\varnothing}{\vec{F}_O}$$

$$-\rho V V A_1 = p_n \cdot A + F_k$$

$$F_k = -p_n A - \rho V^2 A_1$$

$$F_x{}' = -F_k = p_n A + \rho V^2 A_1$$

$$F_x = F_x{}' - p_n A = p_n A + \rho V^2 A_1 - p_n A$$

$$F_x = \rho V^2 A$$