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Basics of input-output analysis

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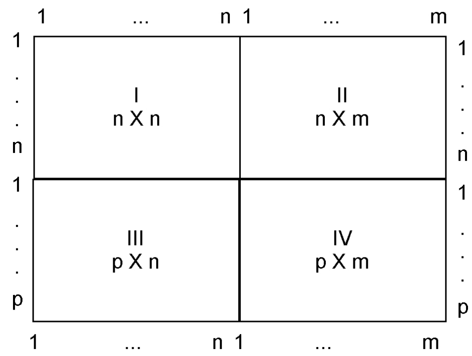

Figure 1: General structure of an input-output table

1 General

This is a basic introduction to the input-output analysis, which was founded by Vassilii Leontiev in the 1930es. This explanation is based upon O'Connor and Henry (1975).

2 The input-output table

The general structure of an input-output table is shown in figure 1. An input-output table is divided into four quadrants:

Intermediate demand Quadrant I represents flows of products, which are both produced and consumed in the process of production of goods. These flows are called *inter-industry flows* or *intermediate demand*.

Final demand Quadrant II contains data of *final demand* for the output of each producing industry, i.e. demand of non-industry consumers like households, government or exports. Final demand is the demand for goods, which are *not* used to produce other goods (as opposed to intermediate demand).

Primary inputs to industries These are inputs (e.g. raw materials) to producing industries, which are not produced by any industry like imported raw materials.

Primary inputs to direct consumption These are inputs (e.g. imported electricity), which are directly consumed, i.e. they are not used to produce other goods.

Input ↓ Output→	Agriculture	Industry	Services	Total outputs
Agriculture	2,180	81,687	1,143	200,345
Industry	27,709	98,036	25,457	538,119
Services	11,020	32,242	19,487	301,311
Total inputs	200,345	538,119	301,311	1,945,233

Table 1: Fragment of an input-output table

Input ↓ Output→	1	2	3	Total final demand	Total outputs
1	x_{11}	x_{12}	x_{13}	Y_1	X_1
2	x_{21}	x_{22}	x_{23}	Y_2	X_2
3	x_{31}	x_{32}	x_{33}	Y_3	X_3
All primary inputs	Z_1	Z_2	Z_3	-	-
Total inputs	X_1	X_2	X_3	-	-

Table 2: Fragment of an input-output table

3 Technical coefficients (cost structure)

Consider the fragment of an input-output table shown in table 1. The technical coefficient for a sector is calculated according to the following formula:

$$TC = \frac{F_i}{\sum_{j=1N} F_j} \tag{1}$$

$$F_i \ldots \text{Flow } i \tag{2}$$

$$N \ldots \text{Number of rows in column} \tag{3}$$

$$\sum_{j=1N} F_j \ldots \text{Column sum of the flow} \tag{4}$$

If, for instance, we want to determine the technical coefficient for agriculture, we obtain

$$TC = \frac{2,180}{200,345} = 0.0109 \tag{5}$$

Technical coefficients reflect the *direct* effects of change in final demand for a certain commodity. To measure indirect effects we need something else, namely interdependence coefficients.

	1	2	3
1	a_{11}	a_{12}	a_{13}
2	a_{21}	a_{22}	a_{23}
3	a_{31}	a_{32}	a_{33}

Table 3: Technical coefficients

4 Interdependence coefficients

Consider the fragment of an input-output table shown in figure 2. In order to determine the interdependence coefficients we first calculate the technical coefficients:

$$X_1 = x_{11} + x_{12} + x_{13} + Y_1 \tag{6}$$
$$X_2 = x_{21} + x_{22} + x_{23} + Y_2 \tag{7}$$
$$X_3 = x_{31} + x_{32} + x_{33} + Y_3 \tag{8}$$
$$a_{ij} = \frac{x_{ij}}{X_j} \tag{9}$$
$$i \ldots \text{row} \tag{10}$$
$$j \ldots \text{column} \tag{11}$$

where a_{ij} is the technical coefficient. Therefore, technical coefficients for sectors 1, 2 and 3 are as shown in table 3. Then,

$$x_{ij} = a_{ij}X_j \tag{12}$$
$$\Rightarrow X_1 = a_{11}X_1 + a_{12}X_2 + a_{13}X_3 + Y_1 \tag{13}$$
$$\Rightarrow X_2 = a_{21}X1 + a_{22}X_2 + a_{23}X_3 + Y_2 \tag{14}$$
$$\Rightarrow X_3 = a_{31}X_1 + a_{32}X_2 + a_{33}X_3 + Y_3 \tag{15}$$

After some calculations omitted here and given in (O'Connor and Henry, 1975, p. 26-27), the following equation emerges:

$$X_1 = z_1Y_1 + z_2Y_2 + z_3Y_3 \tag{16}$$

where z_1, z_2 and z_3 are interdependence coefficients. The interdepenence coefficients are given by

$$(I - A)X = Y \tag{17}$$
$$X = (I - A)^{-1}Y \tag{18}$$

where the inverse of the matrix $(I - A)$ means division, and the term $(I - A)^{-1}$ are the interdependence coefficients.

Interpretation of interdependence coefficients

The equation

$$X_1 = z_1Y_1 + z_2Y_2 + z_3Y_3 \tag{19}$$

can be interpreted as

$$X_1 = f(Y_1, Y_2, Y_3), \tag{20}$$

i.e. the output of sector 1 depends on the final demand for products of sector 1, sector 2 and sector 3. The interdependence coefficients express the extent of this dependence. On also can interpret the interdependence coefficients in other ways. For one Euro of final demand for products of sector 1, the total output of sector 1 is z_1 Euros, total output of sector 2 is z_2 Euros and total output of sector 3 are z_3 Euros[1].

[1] See also O'Connor and Henry (1975, p. 29).

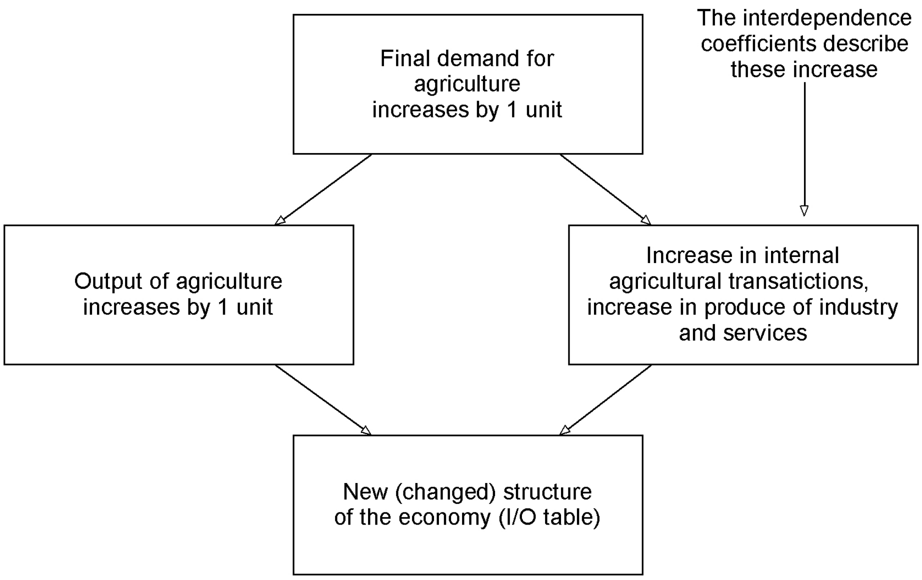

Figure 2: First-order effects of an increase in final demand for sector 1 (agricultural)

Consider the following interdependence coefficients:

$$X_1 = z_1 Y_1 + z_2 Y_2 + z_3 Y_3 \tag{21}$$
$$X_2 = z_4 Y_1 + z_5 Y_2 + z_6 Y_3 \tag{22}$$
$$X_3 = z_7 Y_1 + z_8 Y_2 + z_9 Y_3 \tag{23}$$

For each Euro of final demand for products of sector 1, the total output of sector 1 is z_1, total output of sector 2 is z_4 and total output of sector 3 is z_7.
It should be noted that for any sector the output required exceeds final demand because indirect relationships are expressed in the system. In fact, the interdependence coefficients show both direct and indirect effects of increasing final demand for any sector by one unit of value.

5 First, second and third order effects

If final demand for a certain good increases, this leads also to an increase in the inter-industry demand, which in turn leads to increase in the output of the other sectors (see figure 2).

The changing of the economy (represented by the input-output) table induced by change in final demand for some of the goods is called first order effect. The effect of chaning the economy due to increase in final demand are only partially reflected by the first-order effects. There are also second and third order effects (while it is possible to calculate x-order effects for any x, they become negligible if $x > 3$).
Let Y describe a change in final demand, i.e. unit increase in final demand for agricul-

Input ↓ Output→	1	2	3
1	0.0109	0.1518	0.0038
2	0.1383	0.1822	0.0845
3	0.0550	0.0599	0.0647

Table 4: Technical coefficients

Input ↓ Output→	1	2	3
1	1.0394	0.1945	0.0218
2	0.1833	1.2652	0.1150
3	0.0729	0.0925	1.0778

Table 5: Interdependence coefficients

tural produce (sector 1):

$$Y = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{24}$$

Then the x-order effects are equal to

$$AY = X^{(1)} \tag{25}$$
$$AX^{(1)} = X^{(2)} \tag{26}$$
$$AX^{(2)} = X^{(3)} \tag{27}$$
$$X^{(n)} = Y + AY + A^2Y + A^3Y + \cdots + A^nY \tag{28}$$
$$X^{(n)} = (I + A + A^2 + A^3 + \cdots + A^n)Y \tag{29}$$
$$X^{(y)} \ldots \text{changes in output (effects of order } y) \tag{30}$$

6 Elementary input-output planning

Imagine the final demand for the produce of sector 1 increases to N_1, of sector 2 to N_2 and of sector 3 to N_3. We can obtain the outputs of these sectors by using the following equation system (O'Connor and Henry (1975)):

$$X_1 = z_1N_1 + z_2N_2 + z_3N_3 \tag{31}$$
$$X_2 = z_4N_1 + z_5N_2 + z_6N_3 \tag{32}$$
$$X_3 = z_7N_1 + z_8N_2 + z_9N_3 \tag{33}$$

where $z_1, \ldots, z_9$ are interdependence coefficients. $N_1, \ldots, N_3$ represent the planned levels of demand.

By means of technical coefficients the level of internal flows can be calculated. Let table 4 be the technical coefficients, and $N_1 = 140$, $N_2 = 447$, $N_3 = 278$. Interdependence coefficients are given by table 5. Then the new level of output is

$$X_1 = 1.0394 \times 140 + 0.1945 \times 447 + 0.0218 \times 278 = 238.5 \tag{34}$$
$$X_2 = 0.1833 \times 140 + 1.2652 \times 447 + 0.1150 \times 278 = 623.2 \tag{35}$$
$$X_3 = 0.0729 \times 140 + 0.0925 \times 447 + 1.0778 \times 278 = 351.2 \tag{36}$$

Input ↓ Output→	1	2	3	Final Demand	Output
1	2.6	94.6	1.3	140	238.5
2	33.0	113.5	29.7	447	623.2
3	13.1	37.3	22.7	278	351.2
All primary inputs	?	?	?	-	-
Total Inputs	238.5	623.2	351.2	-	-

Table 6: New internal flows

Using the technical coefficients, the internal flows are calculated as shown in table 6 and in equations below:

$$0.0109 \times 238.5 = 2.6 \tag{37}$$
$$0.1383 \times 238.5 = 32.98455 \approx 33 \tag{38}$$
$$0.0550 \times 238.5 = 13.1175 \approx 13.1 \tag{39}$$
$$0.1518 \times 623.2 = 94.60176 \approx 94.6 \tag{40}$$
$$0.1822 \times 623.2 = 113.54704 \approx 113.5 \tag{41}$$
$$0.0599 \times 623.2 = 37.32968 \approx 37.3 \tag{42}$$
$$0.0038 \times 351.2 = 1.33456 \approx 1.3 \tag{43}$$
$$0.0845 \times 351.2 = 29.6764 \approx 29.7 \tag{44}$$
$$0.0647 \times 351.2 = 22.72264 \approx 22.7 \tag{45}$$

Using this technique it is possible to answer the question *How will the output change if the final demand reaches a certain level?*. It can serve as a tool for formulating a state policy to attain certain goal.

7 Impact analysis

Every one Euro of final demand for the products of a sector generates indirect as well as direct income effects on the economy as a whole. The relationship between the initial spending and the total effects generated by the spending is known as the *multiplier effect* of the sector, or more generally as the *impact* of the sector on the economy as a whole. For this reason the study of multipliers has come to be called *impact analysis*.
A unit increment of "autonomous" investment causes an initial increase in income which generates successive rounds of consumer spending and incomes, each round producing numerically smaller increments until the process has fully worked itself out, i.e. has reached equilibrium.
The fully worked out response to the stimulus produces

1. savings equal to the initial unit increment at investment
2. consumer spending (household consumption) considerably larger than the initial unit increment of investment.

There are several possibilities to model multipler effects in scope of an input-output table:

- partial multipliers

- complete multipliers
- Moore Type-2 multiplier

The applications of impact analysis include

- Examining the effect of starting a new industry or discontinuing an existing one
- Comparison of industries by income and employment multipliers

The drawback of impact analysis is that it can be easily manipulated, so that great carefulness is necessary when working with it.

8 Partial multipliers

Partial multiplier is calculated by multiplying the row of technical coefficient of income arising and the column of interdependence coefficients of the sector. Consider the following technical and interdependence coefficients:

Unit increase in final demand for agricultural produce (sector 1) increases the output of agriculture by 1.0394, that of industry (sector 2) by 0.1833 and that of services (sector 3) by 0.0729. Increase of 1.0394 units in agricultural output will increase the income arising in that sector by 0.6931 units (1.0394×0.6668). An increase in the output of the industry of 0.1833 will increase the industrial income by 0.0512 (0.1833×0.2795), while an increase of 0.0729 in output of services will increase the income of that sector by 0.0555 (0.0729×0.7619). The benefit to the whole economy of a unit increase in final demand for the products of agriculture is therefore an increase of 0.7998 units in the income of the nation ($0.6931 + 0.0512 + 0.0555$). For a more detailed description see (O'Connor and Henry, 1975, p. 42).
Partial multipliers can be calculated for:

- imports (show the import requirements of a unit of final demand for the produce of each sector and how the balance of trade is affected)
- subsidies
- indirect taxes
- depreciation
- employment
- capital

9 Complete multipliers

Partial multipliers are always less than unity, hence less than the Keynesian multipliers because household income is regarded as exogenous to the input-output table. In orer to derive proper Keynesian-like multipliers it is necessary to incorporate the households into the intermediate matrix. For a more detailed description see (O'Connor and Henry, 1975, pp. 46-50).

10 Concluding remarks

Using the input-output technique it is possible to:

- analyze the effectiveness of government's attempts to promote economic growth
- analyze the labour market
- forecast the economic development of a nation
- analyze the economic development of various regions

For more information on input-output analysis refer to Fleissner et al. (1993) and Holub and Schnabl (1994).

References

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