Rough Intuitionistic Fuzzy Sets

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Abstract:
In this paper we define rough intuitionistic fuzzy sets (analogous to the definition of rough fuzzy sets introduced by Dubois and Prade [8]) and study their properties. Some propositions in this notion are proved.

Keywords:
Rough set, fuzzy set, intuitionistic fuzzy set, rough fuzzy set, rough intuitionistic fuzzy set.

Introduction
There are several non-classical and higher order fuzzy sets ([1], [6,7] ,[9-11]) all having very good application potential in the area of Computer Science. One of the interesting generalizations of the theory of fuzzy sets is the theory of intuitionistic fuzzy sets introduced by Atanassov [1]. Intuitionistic fuzzy sets are fuzzy sets described by two functions : a membership function and a non-membership function that are loosely related. While the fuzzy set is a powerful tool to deal with vagueness, the theory of rough sets introduced by Pawlak [15] is a powerful mathematical tool to deal with incompleteness. Fuzzy sets and rough sets are two different topics ([8],[16]), none conflicts the other. In [8], Dubois and Prade defined rough fuzzy sets and fuzzy rough sets providing hints on some research directions on them . Nanda [13], Nakamura [12] also defined fuzzy rough sets independently in different ways. Fuzzy rough sets and rough fuzzy sets are concerned with both of vagueness and incompleteness. In the present paper we define rough intuitionistic fuzzy sets and study their properties.

Preliminaries
In this section we present some preliminaries which will be useful to our work in the next section **.**

Definition 2.1

Let U be any non-empty set . Suppose R is an equivalence relation over U. For any non-null subset X of U, the sets

$A1(X) = \{ x : [x]_R \subseteq X \}$ and

$A2(X) = \{ x : [x]_R \cap X \neq \phi \}$

are called the lower approximation and upper approximation, respectively of X, where the pair S = (U, R) is called an approximation space. This equivalent relation R is called indiscernibility relation. The pair **A (X) = (A1(X) , A2(X))** is called the rough set of X in S. Here $[x]_R$ denotes the equivalence class of R containing x.

Definition 2.2

Let $A = (A_1, A_2)$ and $B = (B_1, B_2)$ be two rough sets in the approximation space S = (U , R) .

Then,

$A \cup B = (A_1 \cup B_1, A_2 \cup B_2)$,

$A \cap B = (A_1 \cap B_1, A_2 \cap B_2)$,

$A \subset B$ if $A \cap B = A$,

$-A = \{ U - A_2 , U - A_1 \}$.

For more details on the algebra and operations on rough sets [14], [15] may be seen.

Definition 2.3

Let E be a fixed universe . An intuitionistic fuzzy set

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(or IFS) A in E is an object having the form

$$A = \{ < x, \mu_A(x), \nu_A(x) > : x \in E \}$$

where the functions $\mu_A : E \rightarrow [0, 1]$ and $\nu_A : E \rightarrow [0, 1]$ define the degree of membership and the degree of non-membership respectively of the element $x \in E$ to the set A , and $\forall x \in E$

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

Fuzzy sets can be viewed as intuitionistic fuzzy sets but not conversely [1]. For various operations and relations on IFSs, one can see [1],[2],[3],[4] and these are not mentioned here.

Rough Intuitionistic Fuzzy Sets

The notion of rough fuzzy sets has been introduced by Dubois and Prade[8], giving few applications. In this section we define rough intuitionistic fuzzy sets and some operations viz. union, intersection , inclusion and equalities over them.

Definition 3.1

Let X be a non-null set and R be an equivalence relation on X. Let F be an intuitionistic fuzzy set in X with the membership function μ_F and non-membership function ν_F. The lower and the upper approximations R1(F) and R2(F) respectively of the intuitionistic fuzzy set F are intuitionistic fuzzy sets of the quotient set X/R with

(i) Membership function defined by

$$\mu_{R1(F)}(X_i) = \inf \{ \mu_F(x) : x \in X_i \}$$

$$\mu_{R2(F)}(X_i) = \sup \{ \mu_F(x) : x \in X_i \}$$

(ii) and non-membership function defined by

$$\nu_{R1(F)}(X_i) = \sup \{ \nu_F(x) : x \in X_i \}$$

$$\nu_{R2(F)}(X_i) = \inf \{ \nu_F(x) : x \in X_i \}$$

We prove that R1 and R2 defined in this way are IFS. For $x \in X_i$, we obtain successively :

$$\mu_F(x) + \nu_F(x) \leq 1$$

$$\mu_F(x) \leq 1 - \nu_F(x)$$

$$\sup \{ \mu_F(x) \mid x \in X_i \} \leq \sup \{ 1 - \nu_F(x) \mid x \in X_i \}$$

$$\sup \{ \mu_F(x) \mid x \in X_i \} \leq 1 - \inf \{ \nu_F(x) \mid x \in X_i \}$$

$$\sup \{ \mu_F(x) \mid x \in X_i \} + \inf \{ \nu_F(x) \mid x \in X_i \} \leq 1$$

Hence R1 is an IFS. Similarly we can prove that R2 is an IFS. The rough intuitionistic fuzzy set of F is R(F) given by the pair

$$R(F) = < R1(F), R2(F) >.$$

Definition 3.2

If $R(F) = < R1(F), R2(F) >$ is a rough intuitionistic fuzzy set F in (X,R), the rough complement of R(F) is the rough intuitionistic fuzzy set denoted by $-R(F)$ and is defined by

$$-R(F) = < (R2(F))^C, (R1(F))^C > ,$$

where $(R2(F))^C$, $(R1(F))^C$ are the complements of the intuitionistic fuzzy sets R2(F)and R1(F) respectively.

Definition 3.3

If $R(F_1)$ and $R(F_2)$ are two rough intuitionistic fuzzy sets of the intuitionistic fuzzy sets F_1 and F_2 respectively in X, then we define the following :

(i) $R(F_1) = R(F_2)$

iff $R1(F_1) = R1(F_2)$ and $R2(F_1) = R2(F_2)$

(ii) $R(F_1) \subseteq R(F_2)$

iff $R1(F_1) \subseteq R1(F_2)$ and $R2(F_1) \subseteq R2(F_2)$

(iii) $R(F_1) \cup R(F_2) = < R1(F_1) \cup R1(F_2), R2(F_1) \cup R2(F_2) >$

(iv) $R(F_1) \cap R(F_2) = < R1(F_1) \cap R1(F_2), R2(F_1) \cap R2(F_2) >$

(v) $R(F_1) + R(F_2) = < R1(F_1) + R1(F_2), R2(F_1) + R2(F_2) >$

(vi) $R(F_1) \bullet R(F_2) = < R1(F_1) \bullet R1(F_2), R2(F_1) \bullet R2(F_2) >$

(vii) $\Box R(F_1) = < \Box R1(F_1), \Box R2(F_1) >$

(viii) $\Diamond R(F_1) = < \Diamond R1(F_1), \Diamond R2(F_1) >.$

If R, S, T are rough intuitionistic fuzzy sets in (X,R), then the results in the following proposition are straightforward from definitions.

PROPOSITION 3.1

(i) $-(-R) = R$
(ii) $R \cup S = S \cup R$, $R \cap S = S \cap R$
(iii) $(R \cup S) \cup T = R \cup (S \cup T)$,
$(R \cap S) \cap T = R \cap (S \cap T)$
(iv) $(R \cup S) \cap T = (R \cup S) \cap (R \cup T)$
(v) $(R \cap S) \cup T = (R \cap S) \cup (R \cap T)$

De Morgan's laws are satisfied for rough intuitionistic fuzzy sets :

PROPOSITION 3.2

(i) $-(R(F_1) \cup R(F_2)) = (-R(F_1)) \cap (-(R(F_2))$
(ii) $-(R(F_1) \cap R(F_2)) = (-R(F_1)) \cup (-(R(F_2))$

Proof:

$-(R(F_1) \cup R(F_2))$

$= -(\{R1(F_1) \cup R1(F_2)\}, \{R2(F_1) \cup R2(F_2)\})$

$= -(\{R2(F_1) \cup R2(F_2)\}^C, \{R1(F_1) \cup R1(F_2)\}^C)$

$= -(\{R2(F_1)\}^C \cap \{R2(F_2)\}^C, \{R1(F_1)\}^C \cap \{R1(F_2)\}^C)$

$= (\{R2(F_1)\}^C, \{R1(F_1)\}^C) \cap (\{R2(F_2)\}^C, \{R1(F_2)\}^C)$

$= (-R2(F_1)) \cap (-R2(F_2))$. Hence Proved.

(ii) Similar to the proof of (i).

PROPOSITION 3.3

If F_1 and F_2 are two intuitionistic fuzzy sets in X such that $F_1 \subseteq F_2$, then $R(F_1) \subseteq R(F_2)$ in (X,R).

Proof: Straightforward

PROPOSITION 3.4

$R(F_1 \cup F_2) \supseteq R(F_1) \cup R(F_2)$

$R(F_1 \cap F_2) \subseteq R(F_1) \cap R(F_2)$

Proof:

$\mu_{R1(F1 \cup F2)}(X_i)$

$= \inf \{ \mu_{F1 \cup F2}(x) \mid x \in X_i \}$

$= \inf (\max \{ \mu_{F1}(x), \mu_{F2}(x) \} \mid x \in X_i)$

$\geq \max \{ \inf \{ \mu_{F1}(x) \mid x \in X_i \},$

$\inf \{ \mu_{F2}(x) \mid x \in X_i \} \}$

$= \max \{ \mu_{R1(F1)}(X_i), \mu_{R1(F2)}(X_i) \}$

$= (\mu_{R1(F1)} \cup \mu_{R1(F2)})(X_i)$

Similarly ,

$\nu_{R1(F1 \cup F2)}(X_i) \leq (\nu_{R1(F1)} \cup \nu_{R1(F2)})(X_i)$

Thus , $R1(F_1 \cup F_2) \supseteq R1(F_1) \cup R1(F_2)$.

We can also see that

$R2(F_1 \cup F_2) = R2(F_1) \cup R2(F_2)$.

Hence, $R(F_1 \cup F_2) \supseteq R(F_1) \cup R(F_2)$.

(ii) Proof is similar to the proof of (i).

The following proposition relates the rough intuitionistic fuzzy set of an intuitionistic fuzzy set with the rough intuitionistic fuzzy set of its complement.

PROPOSITION 3.5

Rough complement of the rough intuitionistic fuzzy set of an intuitionistic fuzzy set is the rough intuitionistic fuzzy set of its complement.

Proof:

We see that for $X_i \in X/R$

$\mu_{R1(F^c)}(X_i) = \inf \{ \mu_{(F^c)}(x) : x \in X_i \}$

$= \inf \{ \nu_{F(x)} : x \in X_i \}$

$= \nu_{R2(F)}(X_i)$

$= \mu_{(R2(F))^c}(X_i)$

And similarly for ν.

Therefore,

$$R(F^C) = -R(F). \quad \text{Proved.}$$

Conclusion

In this paper we have defined the notion of rough intuitionistic fuzzy sets. We have also studied some properties on them and proved some propositions. The concept combines two different theories which are rough sets theory and intuitionistic fuzzy set theory. While Intuitionistic fuzzy set theory is mainly concerned with vagueness, rough set theory is with incompleteness; but both the theories deal with imprecision. Consequently , by the way they are defined, it is clear that rough intuitionistic fuzzy sets can be utilized for dealing with both of vagueness and incompleteness.

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