Ⅶ. 본시 교수-학습 과정안

주제(단원명)	합의 기호 $\sum$				
학습 목표	1. $\sum$의 뜻과 그 성질을 이해하고, 이를 활용할 수 있다. 2. $\sum$의 기본 성질을 이해하여 이를 활용할 수 있게 한다.				
지도상의 유의점	1. 합의 기호$\sum$가 매우 편리한 기호임을 이해하며, 수열의 합 $a_1+a_2+\cdots+a_n$을 $\sum$로 표현하는데 익숙해지도록 강조하여 지도한다. 2. $\sum_{k=1}^{n} a_k$에서 k 대신에 다른 문자, 예를 들면 i 또는 j를 써서 $\sum_{k=1}^{n} a_k = \sum_{i=1}^{n} a_i = \sum_{j=1}^{n} a_j$와 같이 표현할 수 있다는 것을 강조한다.				
단계	학습 흐름	교수-학습 활동		학습형태 및 자료	시간
		교사	학생		
도입	인사	Hello, How are you? Let's start. Let's do the Korean Style Greeting. Do you have any question from yesterday's class?	Fine Yes / No	설 명	5 분
	오늘 수업 계획 제시	Today's my plan is the followings. First, we will review yesterday's class. Then we will finally start a new section which is about the Sigma notation and its applications and examples.			
	지난 수업 복습	Ok, let's start to review what we have learned last time. Any one can tell me what we have learned last time? Yes, we have studied geometric sequences and their sums. We know how to derive the general formula for a_n for a geometric sequence and its sum S_n.	Geometric sequences and their general formula and sum $a_n = a+(n-1)d$ $S_n = \frac{n(a_1+a_n)}{2}$	질의 응답	5 분

단계	학습 흐름	교 수 - 학 습 활 동		학습형태 및 자료	시간
		교 사	학 생		
	교 과 활 동	As I've mentioned before a Series is the sum of terms of a sequence. People use a special notation for sums which is the sigma notation.			
		Why do we need a new notation for summation?	It's convenient.		
		Yes, and it also make things easier.		설명	
		For the sequence $a_1, a_2, a_3, \ldots, a_n, \ldots$ the partial sum up to nth term is $S_n = a_1 + a_2 + \cdots + a_n$ This is called the nth partial sum.			
전개		Now I am giving you the definition of summation notation.			35분
		The sum of the first n terms of a sequence is represented by $\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n$ where i is called the index of summation, n is the upper limit of summation, and 1 is the lower limit of summation.	This notation is convenient and simple.		
		By using this definition, you can solve any kind of problem is this section.			
	Examples	Do you have any question about the definition?	Yes / No	질의 응답	
		Let's do some examples to get used to the definition.			

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도입	examples	ex) Rewrite the summation using the sigma notation. $1+2+3+\cdots+10$ Now find the sum ex) $\sum_{i=3}^{6}(1+i^2)$	$=\sum_{k=1}^{10} a_k$ (Students Solve the problem.) $= (1+3^2)+(1+4^2)+(1+5^2)+(1+6^2)$ = 90	질의 응답	35분
	Some properties of Sigma	Properties of Sums. $\sum_{k=1}^{n} c = cn$ $\sum_{k=1}^{n} ca_k = c\sum_{k=1}^{n} a_k$ $\sum_{k=1}^{n}(a_k \pm b_k) = \sum_{k=1}^{n} a_k \pm \sum_{k=1}^{n} b_k$ They are easy to derive. (Show derivations of those formula)	(Students understand the derivations of the formulas.)		
	examples	ex) When $\sum_{i=1}^{20}(a_{2k-1}+a_{2k}) = 47$, find the value of $\sum_{k=1}^{40} a_k$ ex) The sequence $\{a_n\}$ is 1, 3, 7, 9, 13, 15, 19, 21, 25, 27,$\cdots$. What is the value of $\sum_{n=1}^{20} a_n$?	sol) $\sum_{i=1}^{20}(a_{2k-1}+a_{2k})$ $=$ $(a_1+a_2)+(a_3+a_4)+\cdots+($ $=\sum_{k=1}^{40} a_k$ (Students solve the problem)		

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		교 사	학 생		
전개	Examples and deriving formula	ex) $\sqrt{2}+\sqrt{2\sqrt{2}}+\sqrt{2\sqrt{2\sqrt{2}}}+\cdots$ Find the formula for nth term. Use the Sigma notation. Now, think about the following summation. $\sum_{j=1}^{n} j^2 = 1^2+2^2+3^2+\cdots+n^2$ I will show you how to derive the formula for this sum. We know that $(x+1)^3 = x^3+3x^2+3x+1$ so, $(x+1)^3-x^3 = 3x^2+3x+1$ when x=1, $2^3-1^3 = 3\times1^2+3\times1+1$ when x=2, $3^3-2^3 = 3\times2^2+3\times2+1$ $\vdots$ when x=n $(n+1)^3-n^3 = 3\times n^2+3\times n+1$ after adding all those equations, we get, $(n+1)^3-1 = 3\sum_{k=1}^{n}k^2+3\sum_{k=1}^{n}k+n$ Finally, we get $\sum_{k=1}^{n}k^2 = \frac{n(n+1)(2n+1)}{6}$ Now, can you derive $\sum_{k=1}^{n}k^3 = (\frac{n(n+1)}{2})^2$ formula using $(x+1)^4 = x^4+4x^3+6x^2+4x+1$?	$a_n = 2^{\frac{2n-1}{2}}$ $\sum_{k=1}^{n}2^{\frac{2n-1}{2}}$ Yes, (Students try to derive the formula.)	설명	35분

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및 자료</th><th rowspan="2">시간</th></tr>
<tr><th>교 사</th><th>학 생</th></tr>
<tr><td>전
개</td><td></td><td>ex) Consider the following sequence.
1,3,6,10,15,...
What kind of sequence is this?

I can see recursive relation for the sequence.
$a_{n+1} = a_n + (n+1)$

How about the general formular for nth term?

Find the formula for nth partial sum of this sequence.

How about the sum of the reciprocals of all the triangular numbers?
$1 + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \cdots = ?$</td><td>Triangular sequence.

$a_n = \frac{n^2 + n}{2}$

$\sum_{k=1}^{n} a_k = \sum_{k=1}^{n} \frac{k^2}{2} + \sum_{k=1}^{n} \frac{k}{2}$
$= \frac{n(n+1)(n+2)}{6}$

$= 2\sum_{k=1}^{\infty} \frac{1}{n(n+1)}$
=2*1=2</td><td>설명</td><td>35분</td></tr>
<tr><td>차
시
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습
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고</td><td></td><td>Can you derive
$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$ formula?
This is homework.

Next time, we will talk about finding $\sum_{i=1}^{n} i^2$, $\sum_{i=1}^{n} i^3$.

Please review today's class.

See you tomorrow.</td><td>Yes.

Thank you, bye.</td><td>설 명</td><td>5분</td></tr>
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