

# FORMULARI DE FÍSICA QUÀNTICA

Formulari elaborat per Raimon Sunyer i Borrell. Distribució exclusiva a <http://www.geocities.com/cyberfisica>

$$h = 6.62 \times 10^{-34} \text{ J} \cdot \text{s}$$

$$\hbar = 1.05459 \times 10^{-34} \text{ J} \cdot \text{s}$$

$$e = 1.60219 \times 10^{-19} \text{ C}$$

$$k = 1.38066 \times 10^{-23} \text{ J K}^{-1}$$

$$m_e = 9.10953 \times 10^{-31} \text{ kg} = 5.48580 \times 10^4 \text{ u.m.a.}$$

$$M_p = 1.67265 \times 10^{-27} \text{ kg} = 1.007276 \text{ u.m.a.}$$

$$R_H = 1.09678 \times 10^7 \text{ m}^{-1}$$

$$R(\infty) = \frac{m_e^4}{8\varepsilon_0^2 h^3 c} = 1.09737 \times 10^7 \text{ m}^{-1}$$

$$a_0 = \frac{4\pi\varepsilon_0 \hbar^2}{m_e e^2} = 5.29177 \times 10^{-11} \text{ m}$$

$$hc = 1.23985 \times 10^{-6} \text{ eV} \times m = 12398.5 \text{ eV} \text{ \AA}$$

$$c = 3 \times 10^{10} \text{ cm/s} \quad \frac{e^2}{\hbar c} = \frac{1}{137}$$

$$\hbar c = 197 \text{ MeV F} = 1.97 \text{ keV} \text{ \AA}$$

$$m_e c^2 = 0.511 \text{ MeV} \quad m_p c^2 = 938 \text{ MeV} \quad m_n c^2 = 939 \text{ MeV}$$

$$R_\nu = I_\nu a_\nu \quad a + r + t = 1$$

$$\rho(\nu, T) = \left( \frac{8\pi\nu^2}{c^3} \right) \frac{h\nu}{e^{\frac{h\nu}{k_B T}} - 1}$$

$$dE(\nu, T) = \rho(\nu, T) d\nu = \left( \frac{8\pi h}{c^3} \right) \frac{\nu^3 d\nu}{e^{\frac{h\nu}{k_B T}} - 1}$$

$$R(T) = \sigma T^4 \text{ [W m}^{-2}] \quad \sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$$

$$R(T) = \int_0^\infty R(\nu, T) d\nu \quad R_T(\nu) = \frac{c}{4} \rho_T(\nu)$$

$$\lambda_{max} T = b = 2.898 \times 10^{-3} \text{ mK}$$

$E = h\nu - \Phi$  energia per arrencar un electró

$$E_{cin} = h\nu - \Phi - eV \quad V_f = \frac{h}{e}\nu - \frac{\Phi}{e}$$

$$\lambda' = \lambda + \frac{h}{mc}(1 - \cos\theta) \quad K = \frac{E^2}{E + \frac{mc}{1 - \cos\theta}}$$

$$\text{amb } K \equiv \sqrt{c^2|p'|^2 + m^2c^4 - mc^2} \quad E \equiv h\nu$$

$$\frac{1}{\tan\varphi} = - \left( 1 + \frac{h\nu}{mc^2} \right) \tan \frac{\theta}{2}$$

$$N(\theta) d\theta = \left( \frac{1}{4\pi\varepsilon_0} \right)^2 \left( \frac{Ze^2}{2E} \right)^2 \frac{N_\perp \rho g 2\pi \sin\theta d\theta}{\sin^4 \theta/2}$$

$$dn(\theta, \varphi) = N_\perp \sigma(\theta) d\Omega \quad \sigma(\theta) = \left( \frac{qQ}{4E} \right)^2 \frac{1}{\sin^4 \theta/2}$$

$$L = m_e v r = n\hbar \quad \Delta E = h\nu = n \frac{hc}{\lambda}$$

$$r = \frac{4\pi\varepsilon_0 \hbar^2 n^2}{Z m_e e^2} \quad a_0 = \frac{4\pi\varepsilon_0 \hbar^2}{m_e e^2} = 0.529 \text{ \AA}$$

$$E_n = -\frac{m_e}{2\hbar^2} \left( \frac{Ze^2}{4\pi\varepsilon_0} \right)^2 \frac{1}{n^2}$$

$$E_1 = -I_p = -\frac{m}{2\hbar^2} \left( \frac{Ze^2}{4\pi\varepsilon_0} \right)^2 = hc R(\infty) Z^2 = -13.6 \text{ eV}$$

$$\nu_{ab} = \frac{m_e}{4\pi\hbar^3} \left( \frac{Ze^2}{4\pi\varepsilon_0} \right)^2 \left( \frac{1}{n_b^2} - \frac{1}{n_a^2} \right)$$

$$\mu = \frac{mM}{m+M} \quad L = \mu v r = n\hbar$$

$$E_n = -\frac{\mu}{2\hbar^2} \left( \frac{Ze^2}{4\pi\varepsilon_0} \right)^2 \frac{1}{n^2} \quad r = \frac{4\pi\varepsilon_0 \hbar^2}{Z\mu e^2} n^2$$

$$E_{K\alpha} = h\nu_{K\alpha} = E_L - E_K = h(\nu_K - \nu_L)$$

$$\frac{1}{\lambda_\alpha} = R_H (Z-1)^2 \left( 1 - \frac{1}{2^2} \right)$$

$$\nu = \frac{E}{h} \quad \lambda_{dB} = \frac{h}{p} \quad L = n \frac{\lambda}{2}$$

$$\Delta x \Delta p \geq \frac{\hbar}{2} \quad \Delta E \Delta t \geq \hbar \quad \Delta \ell \Delta \phi \geq \frac{\hbar}{2}$$

Fotó

$$E = h\nu \quad p = \frac{h\nu}{c}$$

Partícula

$$E = \sqrt{m^2 c^4 + c^2 |\vec{p}|^2} = mc^2 + T \quad p = mv$$

$$i\hbar \frac{\partial}{\partial t} \psi(x, t) = -\frac{\hbar^2}{2\mu} \frac{\partial^2}{\partial x^2} \psi(x, t) + V(x) \psi(x, t)$$

$$H\phi(x) = E\phi(x) \quad \frac{d^2}{dx^2} \phi(x) - \left[ \frac{2m}{\hbar} (V(x) - E) \right] \phi(x) = 0$$

$$\int_{-\infty}^{\infty} |\psi(\mathbf{r}, t)|^2 d\mathbf{r} = 1 \quad \sum_n |a_n|^2 = 1$$

$$\psi(x, t) = \sum_{n=1}^{\infty} a_n \Phi_n(x) e^{-i E_n t / \hbar} \quad a_n = \int_{-\infty}^{\infty} \Phi_n^* \psi(x, 0) dx$$

$$\langle E \rangle = \int_{-\infty}^{\infty} \psi^*(x, t) E \psi(x, t) dx \quad \langle E \rangle = \sum_n |a_n|^2 E_n$$

$$\langle E^\ell \rangle = \int_{-\infty}^{\infty} \psi^*(x, t) E^\ell \psi(x, t) dx \quad \langle E^\ell \rangle = \sum_n |a_n|^2 (E_n)^\ell$$

$$\langle f(E) \rangle = \sum_n |a_n|^2 f(E_n)$$

Pou de Parets infinites (amplada  $L$ )

$$\phi_n(x) = \begin{cases} \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) & \text{si } |x| \leq L/2 \\ 0 & \text{si } |x| \geq L/2 \end{cases}$$

$$k = \left[ \frac{2m(E - V_0)}{\hbar^2} \right]^{1/2} \quad E_n - V_0 = \frac{\hbar^2 \pi^2}{2mL^2} n^2$$

Pou quadrat (amplada  $L = 2a$ )

$$\alpha = \left[ \frac{2m}{\hbar^2} (V_0 - |E|) \right]^{1/2} \quad k = \left[ \frac{2m}{\hbar^2} |E| \right]^{1/2}$$

$$\tan \alpha a = \frac{k}{\alpha} \quad \frac{1}{\tan \alpha a} = -\frac{k}{\alpha}$$

$$\left( \underbrace{\alpha a}_{\xi} \right)^2 + \left( \underbrace{ka}_{\eta} \right)^2 = \theta_0^2 = \left( \frac{2mV_0 a^2}{\hbar^2} \right)$$

$$\begin{aligned} \text{Parells} & \quad \eta = \xi \tan \xi \\ \text{Senars} & \quad \eta = -\xi \frac{1}{\tan \xi} \end{aligned}$$

$$\begin{aligned} 0 \leq \theta_0 \leq \frac{\pi}{2} & \implies 1 \text{ soluci\u00f3} \\ \frac{\pi}{2} \leq \theta_0 \leq \pi & \implies 2 \text{ solucions} \\ \pi \leq \theta_0 \leq \frac{3\pi}{2} & \implies 3 \text{ solucions} \end{aligned}$$

$$\beta = \left[ \frac{2m}{\hbar^2} (V_0 + E) \right]^{1/2}$$

$$R = \left[ 1 + \frac{4E(V_0 + E)}{V_0^2 \sin^2(\beta L)} \right]^{-1}$$

$$T = \left[ 1 + \frac{V_0^2 \sin^2(\beta L)}{4E(V_0 + E)} \right]^{-1}$$

Barrera de potencial (amplada  $a$ )

$$\alpha = \left[ \frac{2m(V_0 - E)}{\hbar^2} \right]^{1/2} \quad k = \left[ \frac{2mE}{\hbar^2} \right]^{1/2}$$

$$R = \frac{|B|^2}{|A|^2} = \left[ 1 + \frac{4E(V_0 - E)}{V_0^2 \sinh^2(\alpha a)} \right]^{-1}$$

$$T = \frac{|C|^2}{|A|^2} = \left[ 1 + \frac{V_0^2 \sinh^2(\alpha a)}{4E(V_0 - E)} \right]^{-1}$$

Funcions d'ona de l'oscil·lador harm\u00f2nic

$$\left[ -\frac{\hbar}{2m} \frac{d^2}{dx^2} + \frac{1}{2} m \omega^2 x^2 \right] \phi_n = E_n \phi_n(x)$$

$$\alpha \equiv \left( \frac{2m\omega}{\hbar} \right)^{1/2} \quad \xi \equiv \alpha x \quad E_n = \left( n + \frac{1}{2} \right) \hbar \omega$$

$$\phi_n(x) = \left( \frac{\sqrt{2\pi}}{\alpha} 2^n n! \right)^{-1/2} e^{-\frac{1}{4}\xi^2} H_n \left( \frac{\xi}{\sqrt{2}} \right)$$

$$\langle x \rangle = \int_{-\infty}^{\infty} \phi_n^*(x) x \phi_n(x) dx = 0$$

$$\int_{-\infty}^{\infty} \phi_n^*(x) \phi_m(x) dx = \delta_{nm}$$

$$n = 1, 2, 3, \dots$$

$$l = 0, 1, 2, 3, \dots, n-1$$

$$m = -l, -l+1, \dots, 0, \dots, l-1, l$$

$$\Psi(\vec{r}) = \frac{R_{nl}(r)}{r} Y_{lm}(\theta, \varphi) \quad T(t) = \exp\left(-i \frac{E_n}{\hbar} t\right)$$

$$\int_{\Omega} Y_{l'm'}^*(\theta, \varphi) Y_{lm}(\theta, \varphi) d\Omega = \delta_{l'l} \delta_{m'm} \quad \int_0^{\infty} |R_{nl}(r)|^2 dr = 1$$

$$P = \int_{R_1}^{R_2} r^2 \left| \frac{R_{nl}(r)}{r} \right|^2 dr \int_{\Omega} Y_{l'm'}^*(\theta, \varphi) Y_{lm}(\theta, \varphi) d\Omega$$

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$$

$$\int_{-\infty}^{\infty} e^{-ax^2+bx+c} dx = \sqrt{\frac{\pi}{a}} \exp\left(\frac{b^2}{4a} + c\right)$$

$$\int_0^{\infty} x^n e^{-ax} dx = \frac{\Gamma(n+1)}{a^{n+1}} \quad \Gamma(n+1) = n!$$

$$\int_0^{\infty} \frac{x^3}{e^x - 1} dx = \frac{\pi^4}{15}$$

$$\int_{-\infty}^{\infty} x^2 e^{-ax^2} dx = \frac{\sqrt{\pi}}{2a^{3/2}}$$

Frank Hertz

$$\lambda = \frac{hc}{eU_R}$$

$$\Delta E = \sqrt{\langle E^2 \rangle - \langle E \rangle^2}$$

$$p = -i\hbar \nabla$$

$$T = \frac{p^2}{2m} = -\frac{\hbar^2}{2m} \nabla^2$$

$$E = H = -\frac{\hbar^2}{2m} \nabla^2 + V(\vec{r}, t)$$