MECÀNICA TEÒRICA

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0$$

$$L = T - U \qquad\qquad p_j = \frac{\partial L}{\partial \dot{q}_j}$$

$$\frac{\partial L}{\partial q_j} = 0 \implies \text{coordenada cíclica}$$

$$L \neq L(t) \text{ conservació de l'energia}$$

$$h(q,\dot{q},t) = \sum_j \dot{q}\frac{\partial L}{\partial \dot{q}_j} - L$$

$$\frac{dh}{dt} = -\frac{\partial L}{\partial t}$$

$$I_{ij} = \sum_\alpha \left\{ \delta_{ij} \sum_k x_{\alpha,k}^2 - x_{\alpha,i}x_{\alpha,j} \right\}$$

$$I_{ij} = \int_V \rho(\vec{r}) \left\{ \delta_{ij} \sum_k x_k^2 - x_i x_j \right\} dV$$

$$L_i = \sum_j I_{ij}\omega_j \qquad\qquad \vec{L} = \{\mathsf{I}\} \cdot \vec{\omega}$$

$$T_{rot} = \frac{1}{2}\sum_i \omega_i L_i = \frac{1}{2}\sum_{i,j} I_{ij}\omega_i\omega_j = \frac{1}{2}\vec{\omega}\cdot\vec{L}$$

Diagonalització dels eixos

$$I_{ij} = I_i\delta_{ij} \qquad L_i = I_i\omega_i \qquad T_{rot} = \frac{1}{2}\sum_i I_i\omega_i^2$$

Teorema d'Steiner

$$I_{ij} = J_{ij} - M\left[a^2\delta_{ij} - a_i a_j\right]$$

$$\mathsf{I}' = \lambda \cdot \mathsf{I} \cdot \lambda^\top$$

$$\vec{F}_{ef} = m\vec{a}_r = m\vec{a}_f - m\vec{\omega}\times(\vec{\omega}\times\vec{r}) - 2m\vec{\omega}\times\vec{v}_r$$

$$\vec{a}_r = \vec{g} - 2\vec{\omega}\times\vec{v}_r$$

$$(I_i - I_j)\omega_i\omega_j - \sum_k I_k\dot{\omega}_k\epsilon_{ijk} = 0$$

$$I_1\dot{\omega}_1 = \omega_2\omega_3(I_2 - I_3)$$
$$I_2\dot{\omega}_2 = \omega_3\omega_1(I_3 - I_1)$$
$$I_3\dot{\omega}_3 = \omega_1\omega_2(I_1 - I_2)$$

$$H(q_i,p_i,t) = \sum \dot{q}_i p_i - L(q_i,\dot{q}_i,t)$$

$$\dot{p}_i = \frac{\partial L}{\partial q_i} \qquad\qquad p_i = \frac{\partial L}{\partial \dot{q}_i}$$

$$\dot{q}_i = \frac{\partial H}{\partial p_i} \qquad\qquad \dot{p}_i = -\frac{\partial H}{q_i}$$

$$\frac{dH}{dt} = -\frac{\partial L}{\partial t}$$

en determinades circumstàncies $\qquad H = T + V$

$$\frac{\partial Q_i(q,p)}{\partial q_j} = \frac{\partial p_j(Q,P)}{\partial P_i} \qquad\qquad \frac{\partial Q_i(q,p)}{\partial p_j} = -\frac{\partial q_j(Q,P)}{\partial P_i}$$

$$\frac{\partial P_i(q,p)}{\partial q_j} = -\frac{\partial p_j(Q,P)}{\partial Q_i} \qquad\qquad \frac{\partial P_i(q,p)}{\partial p_j} = \frac{\partial q_j(Q,P)}{\partial Q_i}$$

$$[u,v]_{q,p} = \sum_i \left(\frac{\partial u}{\partial q_i}\frac{\partial v}{\partial p_i} - \frac{\partial u}{\partial p_i}\frac{\partial v}{\partial q_i}\right)$$

$[u,u] = 0 \qquad\qquad [u,v] = -[v,u]$

$[au+bv,w] = a\,[u,w] + b\,[v,w] \qquad\qquad [uv,w] = [u,w]v + u[v,w]$

$[u,[v,w]] + [v,[w,u]] + [w,[u,v]] = 0$

Si u és constant del moviment $[H,u] = \frac{\partial u}{\partial t}$

$$p_i\dot{q}_i - H = P_i\dot{Q}_i - K + \frac{dF}{dt}$$

- $F = F_1(q,Q,t)$

$$p_i = \tfrac{\partial F_1}{\partial q_i} \qquad P_i = -\tfrac{\partial F_1}{\partial Q_i} \qquad K = H + \tfrac{\partial F_1}{\partial t}$$

- $F = F_2(q,P,t) - Q_iP_i$

$$p_i = \tfrac{\partial F_2}{\partial q_i} \qquad Q_i = -\tfrac{\partial F_2}{\partial P_i} \qquad K = H + \tfrac{\partial F_2}{\partial t}$$

- $F = F_3(p,Q,t) + q_ip_i$

$$q_i = -\tfrac{\partial F_3}{\partial p_i} \qquad P_i = -\tfrac{\partial F_3}{\partial Q_i} \qquad K = H + \tfrac{\partial F_3}{\partial t}$$

- $F = F_4(p,P,t) + q_ip_i - Q_iP_i$

$$q_i = -\tfrac{\partial F_4}{\partial p_i} \qquad Q_i = \tfrac{\partial F_4}{\partial P_i} \qquad K = H + \tfrac{\partial F_4}{\partial t}$$

Teoria de Hamilton-Jacobi

$$H\left(q_1,\ldots,q_n;\frac{\partial F_2}{\partial q_1},\ldots,\frac{\partial F_2}{\partial q_n};t\right)+\frac{\partial F_2}{\partial t}=0$$

$$F_2\equiv S=S(q_1,\ldots,q_n;\alpha_1,\ldots,\alpha_{n+1};t)$$

$$P_i=\alpha_i \qquad p_i=\frac{\partial S(q,\alpha,t)}{\partial q_i} \qquad Q_i=\beta_i=\frac{\partial S(q,\alpha,t)}{\partial \alpha_i}$$

$$q_j=q_j(\alpha,\beta,t) \qquad p_i=p_i(\alpha,\beta,t)$$

$$\delta J=\int_{x_1}^{x_2}\left\{\frac{\partial f}{\partial y}-\frac{d}{dx}\frac{\partial f}{\partial \dot{y}}\right\}\delta y\;dx=0$$

$$\left(\frac{dJ}{d\alpha}\right)_0=d\alpha=\delta J \qquad \left(\frac{\partial y}{\partial \alpha}\right)_0=\delta y$$

$$f=f(y(\alpha,x),\dot{y}(\alpha,x),x)$$

$$y(x,\alpha)=y(x,0)+\alpha\eta(x)$$

$$dS=dx\sqrt{1+\dot{y}^2}$$

Matriu de canvi de coordenades

$$\begin{pmatrix}x_1'\\x_2'\\x_3'\end{pmatrix}=\begin{pmatrix}\cos\theta & \sin\theta & 0\\-\sin\theta & \cos\theta & 0\\0 & 0 & 1\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$$

Angles d'Euler

$$\begin{aligned}\omega_1&=\dot{\varphi}_1+\dot{\theta}_1+\dot{\psi}_1=\dot{\varphi}\sin\theta\,\sin\psi+\dot{\theta}\cos\psi\\ \omega_2&=\dot{\varphi}_2+\dot{\theta}_2+\dot{\psi}_2=\dot{\varphi}\cos\theta\,\sin\psi-\dot{\theta}\sin\psi\\ \omega_3&=\dot{\varphi}_3+\dot{\theta}_3+\dot{\psi}_3=\dot{\varphi}\cos\theta+\dot{\psi}\end{aligned}$$

Solucions per coordenades esfèriques

$$\begin{aligned}\dot{x}&=R\dot{\theta}\cos\theta\cos\phi-R\dot{\phi}\sin\theta\sin\phi\\ \dot{y}&=R\dot{\theta}\cos\theta\sin\phi+R\dot{\phi}\sin\theta\cos\phi\\ \dot{z}&=-R\dot{\theta}\sin\theta\end{aligned}$$

$$T=\frac{1}{2}mR^2(\dot{\theta}^2+\dot{\phi}^2\sin^2\theta)$$

Solucions per a coordenades cilindriques

$$T=\frac{1}{2}m(\dot{r}^2+r^2\dot{\theta}^2)$$

$$\vec{v}_\alpha=\vec{V}+\vec{\omega}\times\vec{r}_\alpha$$

$$\left(\frac{d\vec{L}}{dt}\right)_{fix}=\left(\frac{d\vec{L}}{dt}\right)_{cos}+\vec{\omega}\times\vec{L}=\vec{N}$$

$$\vec{A}\times(\vec{B}\times\vec{A})=A^2\cdot\vec{B}-\vec{A}\cdot(\vec{A}\cdot\vec{B})$$

$$(\vec{A}\times\vec{B})^2=A^2B^2-(\vec{A}\cdot\vec{B})^2$$