Grand Round 2005

Odds

2005/5/19

Odds is a singular word, not the plural of ‘odd’

Although odds are mainly used in gambling, they give us a way to make very small probabilities clearer. “Odds of 999 to 1” may be easier to understand than “probability 0.001”

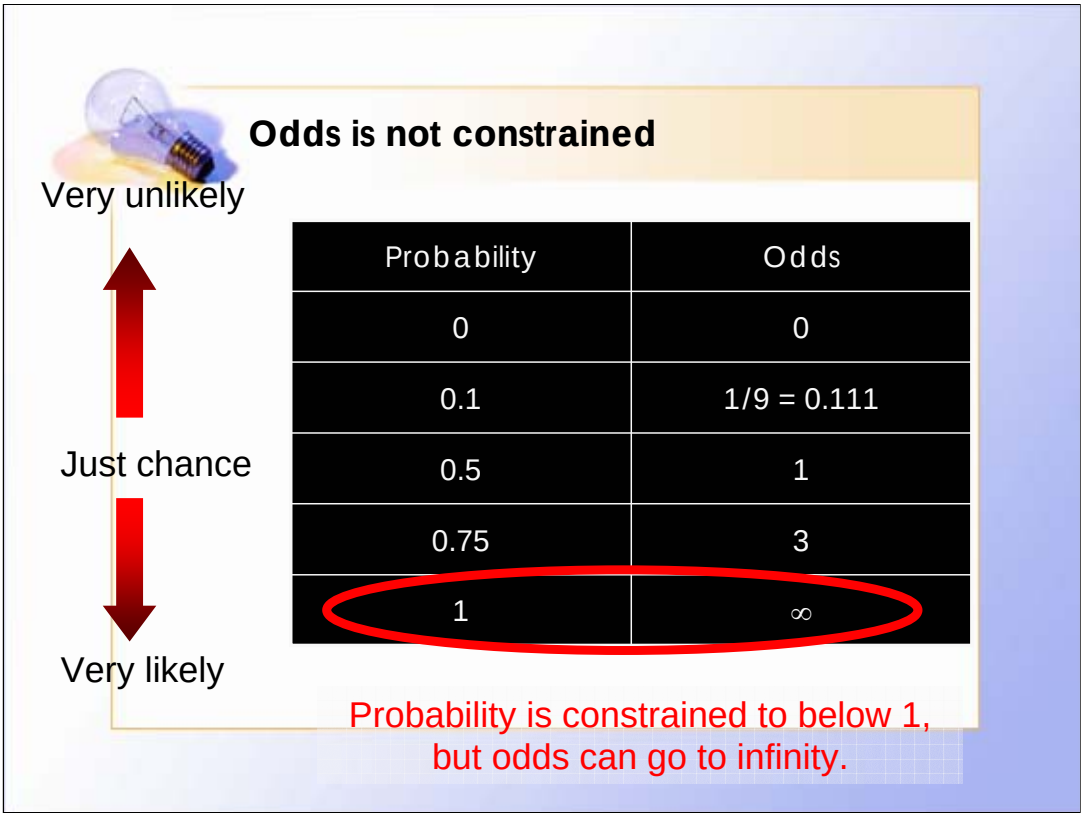
Odds and probability

$$\text{Odds} = \frac{p}{1 \quad p}$$

$$\text{probability} = \frac{\text{Odds}}{\text{Odds} + 1}$$

Odds Ratio

- Odds is ratio of probability
- Odds ratio is ratio of odds
- Odds ratio is ratio of ratio of probability



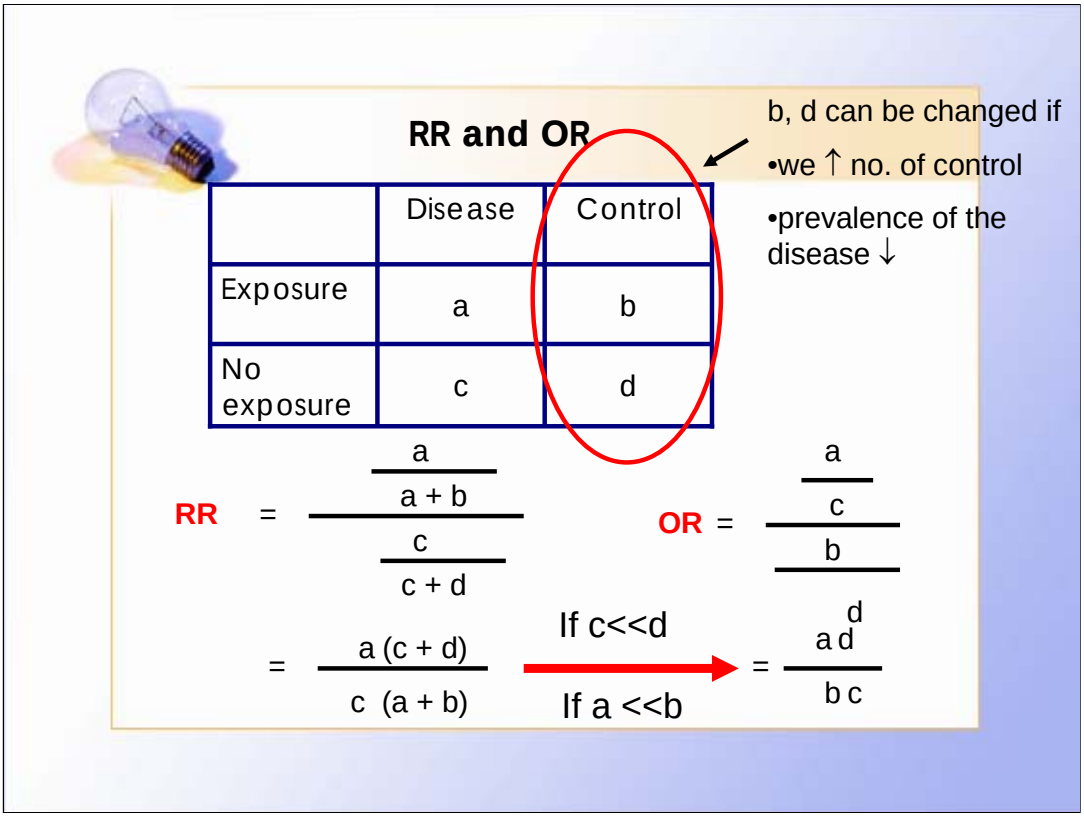
Odds, unlike probability, is **not constrained** to lie between 0 and 1

Log Odds (Logit)

$$\text{Log}_e(o) = \text{Log}_e\left(\frac{p}{1-p}\right)$$

Probability	Odds	Log Odds
0	0	-∞
0.1	1/9 = 0.111	-2.19
0.5	1	0
0.75	3	1.09
1	∞	+∞

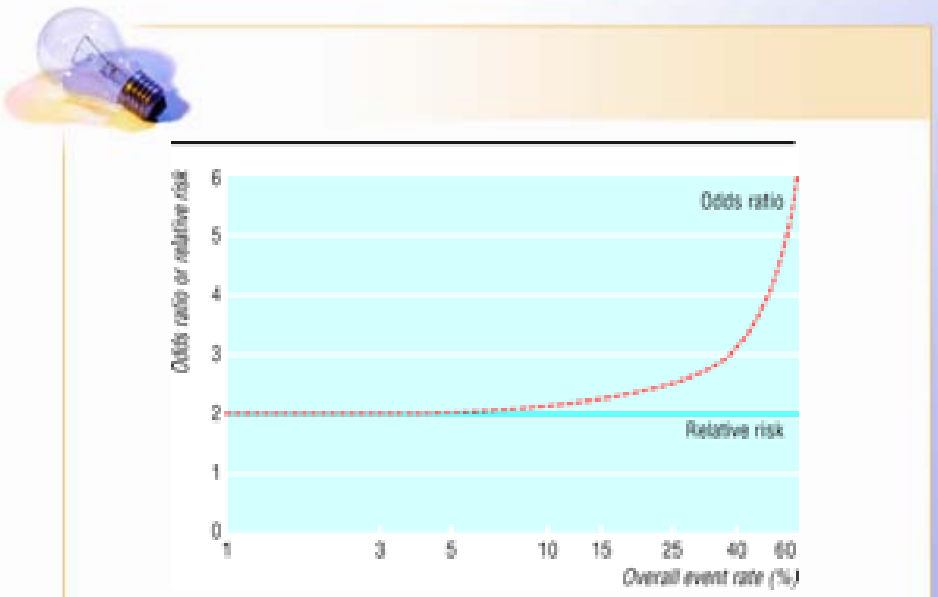
Logit has no lower bound or upper bound (i.e. unconstrained), a straight line can be drawn from -∞ to +∞ passing zero; thus good for model fitting



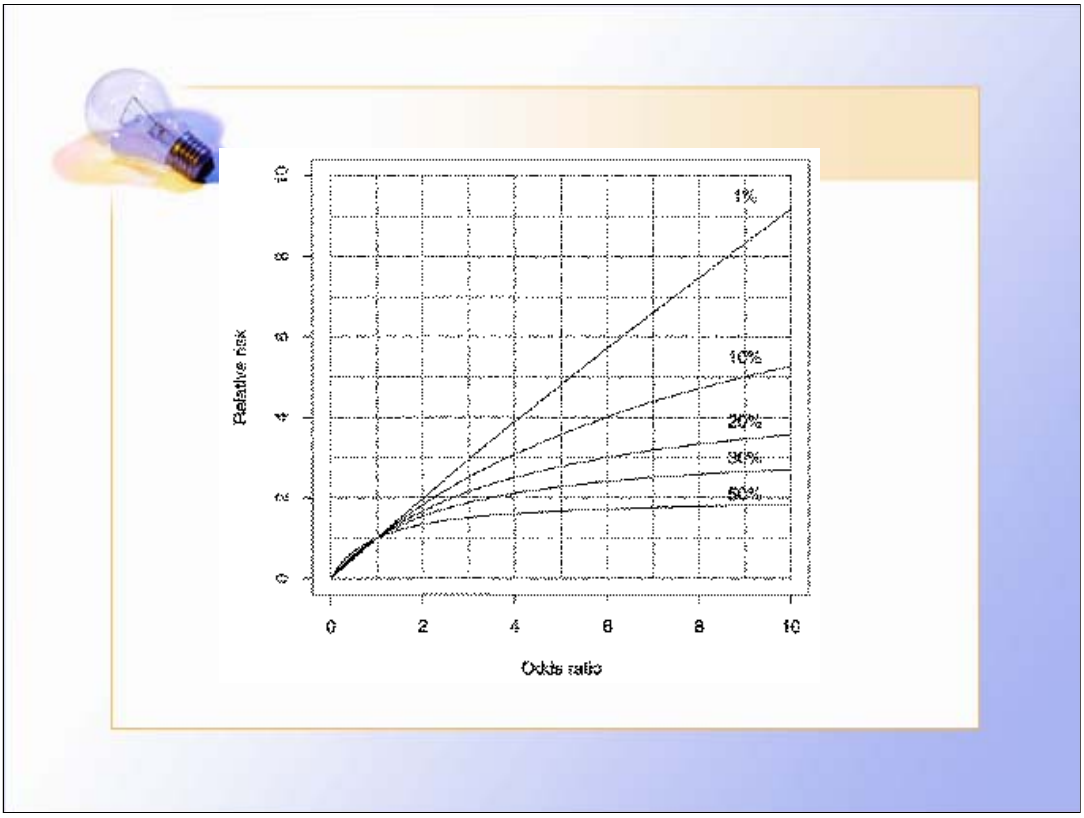
Odds ratios are always used in case-control studies where disease prevalence is not known: the apparent prevalence there depends solely on the ratio of sampling cases to controls, which is totally artificial. To use an effect measure which is altered by prevalence in these circumstances would obviously be wrong, so odds ratios are the ideal choice.

Mathematical advantage of OR

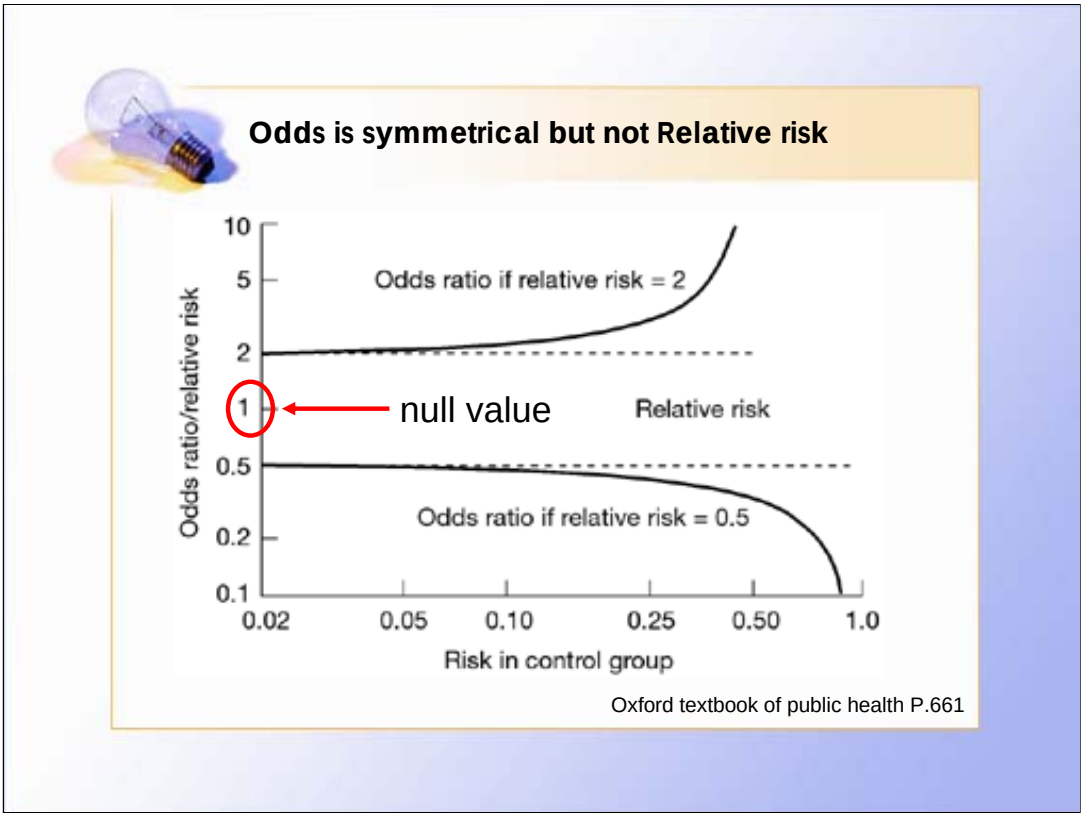
- the estimate of the effect is unaltered regardless of the number of controls in the study



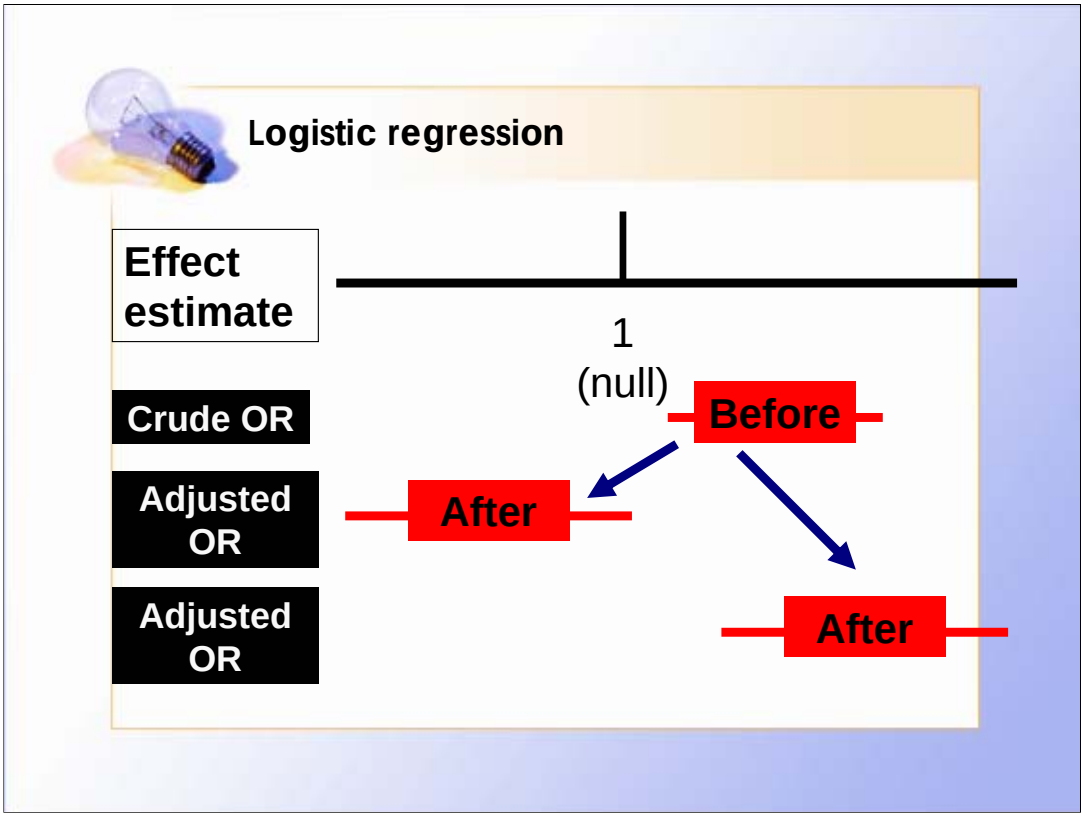
BMJ 1997; 315:1533



The first figure shows the relationship between ORs and RRs for studies which are assessing harm. Each line on the graph relates to a different baseline prevalence, or event rate in the control group. We can use this graph to get a grasp of how misleading it could be to interpret ORs as if they were RRs.



The range that relative risk can take therefore depends on the baseline event rate. This could obviously cause problems if we were performing a meta-analysis of relative risks in trials with greatly different event rates. Odds ratios also possess a symmetrical property: if you reverse the outcomes in the analysis and look at good outcomes rather than bad, the relationships will have reciprocal odds ratios. This again is not true for relative risks.



A fourth point of convenience occurs if we need to make adjustments for confounding factors using multiple regression. When we are measuring event rates the correct approach is to use logistic regression models which work in terms of odds, and report effects as odds ratios.