Insolvency Testing: An Empirical Analysis of the Generalized Beta Type 2 Distribution, Quantile Regression, and a Resampled Extreme Value Technique

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Abstract

Robbins et al. [79] use the Generalized Beta Type 2 distribution (GB2) in surplus modeling. Maintaining the historical approach of fitting a distribution, they point out the power and flexibility of this distribution. Craighead [14, 15] utilizes a resampled extreme value statistical (REV) technique (developed by Zelterman [110]) for insolvency modeling of surplus by using the fact that the distribution of either tail of a distribution takes one of three forms, as proved by Gnedenko [26]. Craighead [14, 15] also conducted empirical tests with the REV technique on 107 large sets of surplus projections. These sets are from 51 separate lines of life insurance and annuity business. However, both the GB2 and REV techniques only model the resultant surpluses, thereby divorcing the models from key input model assumptions, such as the term structure of interest rates. A simple relationship of the interest rate scenarios to the surplus could be a multivariate regression. However, one drawback of using ordinary least squares regression (OLSR) is that the regression is on the mean of the surplus results, whereas with insolvency we are concerned with the tail of the distribution of surplus results (i.e., relatively infrequent extreme values), surplus being negative, which may be far from the mean. The second drawback to multivariate regression is that it is very sensitive to outliers. The values of the associated coefficients are severely distorted when an extreme outlier is used within the regression. Koenker and Basset [44] resolve both of these difficulties when they introduce

Quantile Regression (QR). As the name implies QR allows one to conduct a regression on specific quantiles. These specific quantiles can be chosen so that they are nearer to the observed negative surplus values. Also, QR is a robust regression method that reduces or removes the influence of outliers on the regression coefficients.

We use the 107 different data sets introduced by Craighead [14, 15] to empirically compare the GB2, the REV and QR. Using the traditional Kolmogorov-Smirnov (KS) goodness of fit test we test the fit of the GB2 distribution to the observed empirical surplus distributions. We also use a truncated traditional KS goodness of fit test on the model results of the REV for these same 107 data sets. Generating specific percentile estimates from the empirical surplus distributions, we compare these percentile estimates to those obtained from the associated GB2 distributions, QR regressions and REV results.

We proceed to develop several methods using QR to analyze the sensitivity of different risk drivers to that of specific percentiles of the surplus. We also discuss various ways to test the viability of using subsamples to estimate the information from the QR models.

Just as Robbins et al. [79] use a specific Single Premium Deferred Annuity (SPDA) to compare their results, we summarize our results on three specific SPDA data sets.

The study reveals many limitations to all three methods and the overall results are disappointing. However, the QR sensitivity analysis allows one to locate different points in time when a risk driver affects a line of business.

Key Words:

Extreme Value Statistics, Generalized Beta Type 2 Distribution, Insolvency, Kolmogorov-Smirnov Test, Quantile Regression, Resampling, Ruin Theory, Surplus.

1 Executive Summary

In this paper, I conducted a series of tests to compare and contrast three separate methods to model insurance insolvency. The first method is a traditional risk and ruin modeling paradigm which estimates the best distribution that represents a sample of surplus data, fits that distribution with the data, and draws reserve adequacy conclusions from that distribution. This paradigm does not consider the influence of input scenarios to that of the surplus output nor does it consider the results of extreme value theory. I represented this paradigm with the Generalized Beta Type II family of distributions (GB2).

I proceed in the paper to use Quantile Regression (QR) as a model to see if one can examine the influence of the input scenarios to that of the insolvency output. QR seems to show promise in giving valuation actuaries a new tool that will remove the subjectivity that they have had to confront in the past. QR does not directly address the results of extreme value theory, however it does allow one to determine and examine various extreme quantiles of the insolvency data.

As mentioned above extreme value theory should be considered in the process of examining insolvency results as well. The major result of extreme value theory is that there are only three limiting distributions for extreme data. However, even though it is very good in modeling the extreme events associated with insolvency data, it also cannot link input with output. We will use the Resampled Extreme Value Technique (REV) as the representative-modeling environment in the insolvency study.

All three methodologies show promise on paper. However, by the use of the 107 data sets representing 51 lines of business over three years, I discuss how well each modeling environment holds up under the scrutiny of the reality of the data.

Here are the key conclusions and highlights of the paper:

1. Any use of the GB2 parametric distribution to estimate large computer generated insolvency data is inadequate and should not be used by the valuation actuary. This inadequacy is due to the fact that:

    (a) The GB2 is unimodal. The actual data supports a multi-modal distribution.

    (b) The GB2 is very difficult to estimate and model.

2. The use of the semi-parametric REV model to estimate the tail of the insolvency distribution is also inadequate. This inadequacy is due to the fact that:

   (a) The REV method being a resampling process that requires the averaging of many simulations creates a smooth estimate of the tail. However, the data does not support this smooth estimate.

   (b) The REV cannot replicate the entire support of the insolvency distribution.

3. The QR method cannot be used to estimate the unconditional quantiles that are produced by the actual data.

4. However, the QR method can be used to model the conditional quantiles. By doing so, this allows the actuary and financial engineer to determine various risks that a company is exposed to and at what levels. The limitations in the paper with this type of modeling are:

   (a) The theory for testing the goodness of fit is very new. In fact Koenker's research paper on the goodness of fit was released in December 1999. These goodness of fit tests have been included below and the QR models are adequate except for one line of business.

   (b) Since Section 9 of the paper was so positive in regard to risk determination, I continued to Section 10, to determine if one could use fewer than 10,000 scenarios to help quantify the risk exposure revealed by the QR method. Here knowing that in the limit the coefficients of the QR model will approximate a multi-normal distribution, I created a χ^2 and a Rank test to see if samples less than 10,000 were adequate. These tests, however flawed, reveal that the number of samples required vary by line of business, and I am unable at this time to make any further conclusions.

5. There are various difficulties in the use of Maximum Likelihood Estimators and Kolmogorov-Smirnov goodness of fit tests. These tools should be examined further before being used in extensive data analyses.

6. The most controversial result of the paper is the confrontation of the real data vs. that of parametric fitting. As mentioned in point 2 above, the GB2 family of distributions, which is the richest parametric family currently known in the literature, is inadequate for the task. I believe that this inadequacy causes one to question the entire paradigm of fitting distributions for large data sets. I go on to discuss some of the issues of a new paradigm.

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2 Introduction

There are more things in heaven and earth, Horatio than are dreamt of in your philosophy.
Hamlet Act i sc. 5, l. 166.

In basic ruin analysis, Bowers et al. [5] set up a stochastic process with the following assumptions:

1. Claims count distribution,
2. Claims amount distribution,
3. No interest or asset performance,
4. Constant premiums,
5. Constant expense loads.

Even with these simple assumptions, there is no closed formula for the probability of ruin (that is, when surplus drops below zero), except for one special case. The one exception occurs when this is when the stochastic process is compound Poisson with an exponential claims amount distribution. See De Vylder [19] for a further discussion of traditional ruin/risk theory approaches. In the life insurance industry, regulation and/or professional standards require us to conduct computer simulations on different lines of business to determine when the business performs poorly. We model our business as accurately as possible, allowing for interest and asset performance, changing premium and expense loads. We may or may not make assumptions on the claims count or amount distributions. In addition, we often make many other assumptions such as the term structure of interest rates, future interest rates, asset default probabilities, policyholder psychology, and the relationships of our decrements to the level of interest rates. Computer simulations reveal the behavior of the business relative to these assumptions. We do not know the actual statistical distribution of our business model results. We assume that the computer simulation results are representative (within some degree of confidence) in certain areas of interest, such as the extreme tail. First, we need to determine if our models are valid (again within some degree of confidence). If valid, then we calculate the probability of "ruin," however defined (e.g. reserves are inadequate or all surplus is consumed) within the accuracy of these computer models, or observe the potential risks associated with that product or line of business.

Computer simulations of complex corporate models become very expensive in processing time as the number of scenarios increase. The need to obtain a timely answer often outweighs the need for information from additional scenarios. In fact the industry has created the additional goal of obtaining the most information from a complex corporate model with the smallest set of scenarios. Haken [29] discusses the issue of a macroscopic perspective versus a microscopic perspective concerning information in complex systems. He states

> In order to deal with complex systems, we quite often still have to find adequate variables or relevant quantities to describe the properties of these systems. In all cases, a macroscopic description allows an enormous compression of information so that we are no more concerned with the individual microscopic data but rather with global properties. ... In general we have to guess the nature of the microscopic events which eventually lead to macroscopic data.

This goal leads us to the concept of information theory. Information theory was created by Claude Shannon [84] to manage the transmission of information through a communications network. The use of information theory has impacted many different areas of research, a few of which are communications, statistics and chaos and complexity theory.

In the following we will frequently make references to the concepts of gain, loss, insertion, measurement and maximization of information. However, regrettably we are unable to use these concepts in a rigorous manner. However, we believe that there is a critical need to implement information theory into the issues of insolvency. Hopefully our discussions herein will inspire that research.

We will review the various creative approaches to maximize information while reducing scenario requirements in Appendix E, beginning on page 103.

Most computer business models are limited by the knowledge that we have about the basic assumptions used. We must be careful in how we think about and use these models. At a fundamental level, the models are not correct, and not assumed to be accurate. However, the benefit of using the computer to model actual business products and lines is that we can obtain an understanding of the different risks to which that product or line is exposed. Once we have this understanding, we can consider several methods to reduce the effect of any given risk. Such methods include product redesign, reserve strengthening, deferred expense write-downs, asset hedging strategies, stopping rules (rules that recommend when to get out of a market), derivative positions and reinsurance.

However, once we have gained the basic understanding of the risks and have designed, say, a hedge strategy, we must remember that these models are not accurate, due to oversimplification of the model, lack of knowledge and insight, lack of confidence in the assumptions, or incorrect computer code. We cannot trust the model output as the "truth," but we can trust the knowledge and insight that we have gained from the process of modeling. If done correctly we know both the strengths and weaknesses of the model. For instance, when constructing a hedge to protect against the risks demonstrated by the model, we must not implement a hedge that optimizes against areas of model weakness. Ultimately, the model does not tell us what to do, but the model does make us more comfortable in finally making a business decision.

It is important to keep a clear perspective when using multiple economic scenarios in computer simulations. We can gain significant insight about the risk exposure from the economy using stochastic simulation. By examining multiple possibilities, we can protect ourselves as best as possible. However, we realize that only one path actually emerges. Therefore, the actuary must continually evaluate the economy and make reasoned business decisions to maintain existing business and to acquire new business.

The risk aversion of company management must also govern these business decisions. Insolvency must be considered and avoided. However, the actuary cannot remove all risk of insolvency, because the cost of the associated hedges would become so prohibitive that the company could not afford to conduct business. Accordingly, the actuary should understand where the product or business line places the company at risk and be able to communicate to upper management the specific risk exposure. To see a further discussion of the balancing act between company profit and insolvency risk see Craighead [16].

Next we examine the behavior and restrictions of extreme model results.

2.1 Extreme Value Theory

Valuation actuaries, asset/liability management actuaries, and CFOs of insurance companies confront issues that are vast and complex, including:

1. Calculating the probability and/or impact of bankruptcy either by scenario testing or by determining the company's value at risk.

2. Helping to determine the initial capital allocation for a new line of business.

3. Making sure that reserves are adequate for new and existing lines of business.

4. Understanding how different lines of business are sensitive to the level of interest rates, corporate spreads, volatility of other economic indicators (such as stock indices), and the changes in the levels of these variables.

5. Estimating other risks to which the company is exposed in a timely fashion.

6. Pricing complex policy features to obtain profitability, while maintaining a competitive market position.

7. Aiding in the design and pricing of dynamic hedges to reduce the risk of extreme events.

8. Designing and pricing the securitization of various cashflows to reduce risk based capital requirements.

All of the above issues require timely and accurate valuation of different complex corporate models. When conducting the analysis on models the actuary goes through the following steps:

1. Collect relevant data.

2. Make relevant assumptions.

3. Construct the model.

4. Validate the model for reasonableness.

5. Revise the model

After a corporate model is constructed an actuary uses the results in several ways. Some of these are:

1. Gain insight on the business modeled.

2. Determine risks to the company that the model indicates.

3. Observe the scenarios that give adverse model results.

4. Increase reserves, or create hedges or make product enhancements to reduce the risk exposure or adverse results.

The internal company standards and the external regulatory requirements require the actuary to determine risk levels from corporate models. It is of paramount importance to understand the impact of different economic drivers, product designs or investment/disinvestment strategies have on the behavior of a corporate model. This includes the determination of when (and how often) model results from scenarios fall in 'bad' locations. This knowledge is very important to understand the potential magnitude of the company's risk exposure. While adverse results occur relatively infrequently in scenario testing, the actuary would like to gain more knowledge of these adverse results without paying the cost of projecting enough scenarios to get the number of "hits" in the region of adverse results needed for statistical validity.

These adverse locations are discovered by first placing a valuation on the company's position, scenario by scenario. These valuations are then sorted and put in an increasing or decreasing order. From these ordered results, the location of the adverse results is found at either the highest or lowest valuations. The area of statistics, which analyzes sorted samples, is order statistics. The study of extreme order statistics is called extreme value statistics or the theory of records. The theory of records originally grew out of an actuarial problem. See Gumbel [28] for a statement of this problem and its subsequent discussion.

To understand extreme value statistics at a conceptual level, conduct the following experiment: Take a series of samples from an unknown population. Suppose that these samples can be ranked in some fashion. Order these samples from the smallest value to the largest value. Now observe the median. Now repeat the experiment. Again observe the median. Continue to repeat the experiment while observing the median. By repeating these steps of sampling, ordering and observation, the distribution of the medians will approach a Gaussian (Normal) distribution as the number of experiments increase.

Repeating the above experiment on other percentiles near the median, the distribution of the observed percentiles will also approach a Normal distribution as the number of experiments increase. However, as the percentiles move closer to the extremes, the observed distributions will take on one of three (Non-Gaussian) distributions. The most common of these distributions is the exponential distribution. The other two distributions either have heavier (more area in the) tails or lighter (lesser area in the) tails than in an exponential tail. Most distributions (Zelterman [110] states this to be about 95%) have an exponential tail weight. These exponentially weighted tail distributions have several interesting properties. These properties are discussed in Appendix B.3 on page 92, or in Zelterman [110]. For a further discussion of these three distributions and their associated fitting algorithms see Embrechs et al. [23] or Gumbel [28]. Also in Appendix C.6, on page 101 we discuss the CS ratio. This ratio allows one to estimate from the sample data, which extreme distribution is the best fit of the tail.

In Appendix C.1, page 94, Equation 44 allows the actuary to make confidence interval estimates on different percentiles of the ordered samples. The advantage of this formula is that the actuary does not need to know either the distribution of the underlying population or the distribution of the observed percentile. The actuary only uses other nearby order statistics to determine the confidence interval estimate. Equation 44 can be empirically solved to determine the sample size required to place a certain confidence interval about a specific percentile. However, if the adverse location to be examined is either a very small percentile (such as 0.5%, 1.0%, or 5.0%) or a very large percentile (95.0%, 99%, or 99.5%) with a very high level of confidence (such as 99% or 99.9%), the number of scenarios required can become very prohibitively large.

In summary, to have a high confidence on the behavior of an extreme percentile, many scenarios must be processed on the model. Different methods have been developed for maximizing the information from the models so that statements can be made at desired percentiles and confidence levels based on fewer scenarios. In Appendix E is a survey of these methods. However, in the next section we will examine the known weaknesses of these various modeling environments.

2.2 Scenario Reduction/Information Maximization–Model Weaknesses

In Appendix E are various known and potential modeling environments that may be considered to improve the valuation actuary's knowledge.

Primarily, the modeling environments of Appendix E can be broken into three major areas:

1. Output Only.
2. Input Only.
3. Input and Output.

There are several problems that occur with these environments. These are:

1. Universal Problems:
   (a) Information loss due to model replacement. Trying to increase information while reducing scenario requirements may lead one to replace the results of a complex computer model with a revised, simpler model. The risk associated with model replacement is the potential loss of information from the more complex computer model. In the same way that the measurement of the insertion of information (see Soofi [86]) when replacing a nonparametric model with a parametric model is difficult, the opposite effect is also true. Trading a more complex model for a simpler (possibly weaker) model, we have no measure of the loss of information. We realize that the computer model is not accurate, but it is useful. However, we need to be assured that model replacement does not destroy nor obscure the information of the original. In a perfect world, the replacement model would produce results identical to those of the complex model. However, realizing that this is not possible, one should be able to measure how well the simpler model fits the actual results (within some range of confidence). But ultimately the original time expensive computer model will be more accurate than the simpler model. Taking this view, the priority of subsampling for scenario reduction should take a secondary priority to that of model accuracy. In essence, the results

of the complex computer model contains more "truth" than any resultant model of that data, and should be considered as the standard against any model is compared. Due to the limitations of different goodness of fit tests, we will make model accuracy as our primary focus as we consider the viability of the GB2, REV, and QR methodologies.

(b) Misestimating location, support, skew or magnitude. This is related to the previous problem. The location denotes the center of the distribution, which is frequently estimated by the mean or median. The support is the domain of the random variable. The skew or the shape is whether the distribution has more area to the right or to the left of the location of the distribution. Magnitude is measured by the kurtosis of the distribution. If the distribution is very peaked it is leptokurtic. If flat, it is platykurtic and if mesokurtic it is an in-between distribution. See Hogg and Craig [37] or Press et al. [72]. Whenever using subsampling methods, the subsample may not accurately estimate these basic features of the actual underlying distribution.

The valuation actuary is most concerned with the support. The risk of misestimating the support leads the actuary to the specific problem of over- or under-estimation of specific extreme percentiles. The author has seen this occur at the level of hundreds of millions of dollars.

The misestimation of the location is very serious in derivative pricing because the estimate of the location (mean) is central to calculation of the price. However, means and higher moments are very sensitive to outliers. In fact the proper estimation of the support may wreak havoc with the proper estimation of the mean and vice versa. So maximizing the fit between the sample mean and the population mean may destroy the fit of the support. The need to have the best fit of both the location and the support creates a very unstable optimization problem. This very instability is the cause of many of the subsequent problems.

(c) Sample Size requirements. How many scenarios should be used? Robbins et al. [79] continued to increase the number of scenarios until the running sample standard deviation stabilized. This method only stabilized the estimation of the mean and the standard deviation. However, as the sample standard deviation is highly sensitive to outliers, the sample standard deviation is discontinuous whenever an extreme result is encountered. The tendency to graph the running sample mean and standard deviation on a regular scale leads to the misconception that the sample moments have stabilized. However, in determining running sample moments one has only added one extra sample to the graph. This addition makes a small contribution and possibly gives the impression that the running sample moments have stabilized. The most accurate means to check for stability is to plot the running sample moments on a log scale, which actually discloses whether the sample moments have stabilized. This advice does not lend itself to our objective of scenario reduction since it dramatically increases the number of samples required.

(d) Distortion in the extreme tails. Normally the first through fourth smallest or largest order statistics are dramatically different from the other order statistics. If the sample size is large, the distribution of a central order statistic is a Normal distribution. However as the order statistic moves toward the extremes, the distribution of the order statistic takes on one of the three extreme value statistical distributions. The lack of stability of these extreme order statistics also wreaks havoc on obtaining a stable estimate of any central moments. Again this is an example of the trade-off between modeling the support versus the central moments.

(e) Determination of goodness of fit. The measurement of how well the subsample fits the actual population must also address the determination of the best method to measure this fit. Difficulties due to whether or not the data is binned arise here. The term 'binned' or 'binning' refers to the collection of the data into separate groups or bins to produce summary histograms. Two issues that must be considered are:

  i. Robustness. Is the method robust or are the results of the algorithm sensitive to the data? See Venables and Ripley [101] for a list of references and a further discussion of robust estimators.

  ii. Testing. This is a balancing act between the actuary's trust of the model data versus that of the fitted model. When using a large number of scenarios, graphs of the data may visually appear very smooth. However, if there are many embedded options in the business model the data may have many local extremes. If the resultant data is then fit to a parametric distribution many goodness of fit tests fail. Binning the data, which smooths out the additional extremes, can help alleviate this problem. This is analogous to the 'bandwidth' problem in nonparametric regression. See Venables and Ripley [101].

(f) Confidence intervals. What level of confidence should be used? If a valuation actuary is making opinions on the first percentile and states that he or she is 95% confident in the results, does this create confusion? Said another way, does it make sense for the actuary to state that "there is only a 1% chance of insolvency," while there is a 5% chance that this statement is correct? Should the level of confidence be increased at higher and higher extremes?

2. . Output only problems:

   (a) Not tied to input. In this situation the results have been estimated but there is no idea what the conditions are that actually lead to those conclusions. Sedlak [82] describes this as 'necromancy'.

   (b) Determination of the proper model. This difficulty consists of determining what the best parametric model is or whether to use semiparametric or nonparametric methods.

      - In the past with traditional analysis the tendency was to fit the normal distribution and test that fit. If the results revealed that the normal distribution was a poor fit, then the analysis branched out into examining other parametric distributions.
      - The current state of statistical software tools aids in the use of nonparametric analysis. With these tools, the initial analysis begins with graphs generated with nonparametric methods. After this preliminary analysis, the search for the best parametric or semiparametric models begins.

   (c) Extremes not well modeled. This occurs if the fit algorithm emphasizes the center of the empirical distribution. This also occurs when using resampling methods. When resampling from only the samples of the observed empirical distribution, the resulting resampled distribution will never have extremes greater than the maximum order statistic in the empirical distribution (or less that the minimum order statistic). This creates the problem that the extreme tail is not properly modeled, because the resampled distribution has very little weight in the tail and the support is the same as the empirical distribution.

(d) Distorted probabilities. The probabilities associated with scenarios can be distorted, which can lead to distortion in the final model results. This distortion of the underlying measure occurs in the Cherry Picking, Linear Path and Clustering methods as referenced in Appendix E. The Linear Path Space method accounts for this distortion. However, it is not clear from Ho [34, 35] whether the risk neutral measure that he uses in the original generation of the interest scenarios is not distorted when using the Linear Path equivalence class scenarios.

3. Input only problems:

   (a) Not tied to output. There are no universal methods to choose the best economic scenario generation method or scenario reduction method that can guarantee good results for all possible corporate models. We observed this when using Christiansen's representative scenarios [11].

   (b) Scenario generators. Tenney [96] states that an improperly designed economic scenario generator leads to spurious results. We also observe this below in Appendix A.

   (c) Random number generation. This is where the choice of the random generator influences the results.

4. Input/Output model problems:

   (a) Neural Networks. One of the current most popular Input/Output models is the neural network. However, most neural network algorithms require many data points and a great deal of learning time to find the best fit. Also, the goals of accurately modeling the ruin probabilities and with the fewest scenarios make this approach prohibitive. The author used neural networks to model the

1994 LOB04 line of business (this data is described in Section 3). This was done using a subsample of 1000 from the 1994 reduced scenario sets described in Appendix A.3 and the associated EVAS values as discussed in Section 3 as a learning set. The learning time was three days. After this period, the entire 10,000 reduced scenario data set was used as input to the neural network. We found that the model of the extreme tail was very poor, and the support was severely under-estimated. This result is due to the fact that the support of the subsample and its tails do not have enough information to simulate the behavior of the entire data set.

One major complaint of neural networks is that they are 'black-boxes'. Even though the use of neural nets might be a good method to find the best nonlinear regression from the data, it does not increase the understanding of the drivers of the model. Also, if noise is present in the data, neural nets model poorly.

(b) Proprietary methods. For example, Numerix's fees start around $100,000 for their services. The patented high dimensional LDS generator "Finder" (patent number 5,940,810) by Columbia University, currently costs $12,000.

(c) Proper determination of subregions. Frequently the actuary wants to understand the behavior of the output in certain restricted regions such as the tail. Trying to determine the proper subregion of the input parameter space (which is either the hypercube of random numbers or the economic scenarios) is difficult. The difficulty arises where there may be two or more output values that are very close but the input parameter space coordinates may be very far apart. Also there may be gaps and all the possible input subregions that lead to adverse results may not be observed and hence not modeled. This problem mostly haunts stratified or importance sampling methods. Also, once the subregions are determined additional model runs must be made.

(d) Scenario generators. Again, the assumptions and choice of the economic scenario generators can influence results. See Tenney [96] for further discussion on this issue.

(e) Random number generator. The complex production of the random numbers required in scenario generator.

(f) Sensitivity. How sensitive is the output to small changes of the input.

We now discuss features of the empirical study of the three separate methodologies.

2.3 GB2, REV, and QR: The Three Methodologies

We conduct an empirical analysis of three separate methodologies to a collection of 107 separate data sets, each containing 10,000 separate values. These 107 different data sets contain surplus results from corporate models on 51 separate lines of insurance and annuity business. These corporate models were processed for the years ending in 1992, 1993 and 1994. A more extensive discussion of these data sets is in Section 3. We also have the corresponding 10,000 interest rate scenarios for each year ending 1992, 1993 and 1994 which correspond with the corporate models. See Appendix A.1 for a further description of these scenario files.

Robbins et al. [79] examined several different parametric distributions, with most emphasis placed on the Generalized Beta Type 2 distribution. Their data was the surplus results of a SPDA corporate model obtained from the use of 10,000 scenarios of interest rates. The first thing we will do is to observe the effectiveness of the fit of the GB2 distribution against the above 107 data sets.

Craighead [14, 15] used the REV on the above 107 different data sets. He used the technique to resample the tail and obtain more information about the behavior of the extreme tail of the distribution. This is our second methodology that we compare and contrast to the other methodologies.

Finally we examine the use of Quantile Regression (QR) as our third methodology. We examine the relationship directly between the interest rate scenarios and specific quantiles in each of the 107 data sets.

Our empirical analysis will take the approach of checking the accuracy of the various models resulting from the methodologies to the individual empirical cumulative distribution functions (ECDF) of each of the 107 data sets. These 107 ECDFs will be constructed from Equation 45 on page 94 of Appendix C.2 using the entire 10,000 values in each data set. If we observe model validity for any of these three methodologies we will then consider the issue of using subsampling to reduce the scenario requirements. At the same time this empirical analysis examines how well these methodologies work with different product lines and determine if we can increase our knowledge and understanding of the behavior of these lines of business.

The remainder of this paper will take the following path:

First, in Section 3 we briefly describe the different product lines backing each of these data sets and the specific target variable upon which we conduct all analysis. We also discuss the method that we use to adjust the data sets into a common format that allows us to do a comparative analysis between the three methodologies.

In Section 4 we discuss the complex issues of model adequacy, information content, sample sizes and goodness of fit considerations.

In Section 5 we examine the modeling of the GB2 distribution and examine how effective it is when modeling the 107 data sets.

In Section 6 we use the REV methodology to examine its model effectiveness.

In Section 7 we examine the QR methodology. We demonstrate several ways to display the results as well as develop a means to reveal the qualitative behavior of the coefficients in risk analysis.

In Section 8 because we are unable to use the KS test on the QR methodology, we examine how well the three methodologies model certain percentages of the underlying data sets.

In Section 9 we discuss a report to display various risk drivers and their sensitivity using the QR methodology. We use this report and examine the risk exposure in all of the data series from six separate risk definitions obtained from the associated interest rate scenarios.

In Section 10 we discuss the issue of subsampling and examine its effectiveness with the QR methodology only.

In Section 11 we compare the results of the three single premium deferred annuity (SPDA) line of business data sets against the results of Robbins et al. [79], REV and QR.

In Section 12 we discuss the issue of data dependency tests on the three techniques.

Finally, in Section 13 we end the paper with a list of strengths and weaknesses of each method and a discussion of potential future research.

Appendices follow covering the algorithms associated with each of these topics:

- Interest rate scenarios. We discuss the interest rate scenario files that underlie the different data sets. We also examine how to reduce the required dimension of the scenarios for the QR modeling.

- Methodologies. The theoretical underpinning of the three methodologies is discussed.

- Statistical tests. Confidence interval estimates for specific percentiles are examined. Also, the description of two tests for comparing distributions is included. A specialized goodness of fit test is discussed followed by two specialized subsampling tests are outlined. Finally a ratio to determine extreme percentile behavior is described.

- Statistical graphics. The attributes of boxplots and an associated measure of increased volatility through time are expounded upon.

- A discussion of known and potential modeling environments for insolvency work.

- A discussion of how to determine the confidence intervals of the GB2 parameters.

The Bibliography follows the Appendices. Following the Bibliography, four subsample Figures of a QR analysis are displayed, followed with fifty two separate Tables. The paper then ends with the Index.

3 Data Description

In this section, we describe the data sets that we use in the studies.

See Appendix A for a discussion of the interest rate scenarios that we use in the corporate model projections.

Figure 1 on page 25 shows the basic structure of the modeling environment. The interest rate scenarios, product and asset data are input into the model along with the in-force population and other assumptions and the resultant output is the Equivalent Value of Accumulated Surplus (EVAS).[1] These EVAS values are obtained at the end of the projection period of twenty years and discounted back to the valuation date. These values are somewhat liberal in that if the company became insolvent in an earlier year, but then recovered subsequently, we do not have knowledge of this event contained in the corresponding twenty-year EVAS value.

We have 107 different data files used in [14, 15] to conduct the empirical comparisons. These 107 files represent 51 different lines of business over three separate years. The naming of these data sets is "LOBn" where $n = 1, \cdots, 51$. These files are a mixture of individual versus group products, life and annuity products, fixed and variable annuities, and are tax qualified and tax nonqualified. The majority of the business is deferred annuities in the accumulation phase. However, LOB07, LOB09, LOB15, LOB16, LOB19, LOB31, LOB32, LOB47, LOB50, LOB51 are Nontraditional Life insurance. LOB45 and LOB46 are Traditional Life insurance. LOB08 and LOB41 are annuities in a payout status.

The collection of these 10,000 EVAS values in each data set are for the years 1992 through 1994 and each value corresponds to a specific scenario for that specific year for that line of business. There are several lines of business that are common between the three years. Of course this is not true for all the lines of business because not all data was collected for the 1994 data

[1] Equivalent value of accumulated surplus is somewhat similar in concept to a present value, which is scenario dependent. It is also dependent upon the investment strategy used and is obtained by dividing the surplus at the end of the projection period by a growth factor. This factor represents the multiple by which a block of assets would grow from the valuation date to the end of the period in which we are interested. It is computed by accumulating existing assets or an initial lump-sum investment under the interest scenario in question on an after tax basis with the initial investment and any reinvestments being made using the selected investment strategy. The growth factor is the resulting asset amount at the end of the projection period divided by the initial amount at the valuation date [81].

sets, and there were new lines added and other lines were condensed into larger lines. These inconsistencies in plan groupings do not affect the overall statistical summaries. However, being able to compare results across years is very difficult. This difficulty is also discussed in Section 3.1. In Figures 2, 3 and 4 on pages 26, 27, and 28, the boxplots reveal the overall behavior of each of the lines. Tables 13, 14 and 15 on pages 133, 134 and 135 respectively, are the corresponding tables of summary statistics on each of these lines. Note that the dominance of the LOB04 line in each year creates a compression of the majority of the EVAS values in the graphs of the other lines. One can observe the concentration of the business from the graphs. Observe the LOB04, LOB26, LOB46, LOB47, and LOB49 lines of business. Notice that the severity of outliers of the LOB04 line is high in 1992, reduced in 1993 and dramatically increased in 1994. However, notice in LOB26 that the outliers increase from 1992 to 1993 and appear to remain constant in 1994. Observe that the outliers of LOB46 seem to increase between 1992 and 1993, but the overall support of the two distributions is similar. (Note: The actual EVAS values have been obscured with a linear transformation.)

3.1 Adjusted EVAS

The methodologies created by Robbins et al. [79] and Craighead [14, 15] analyzed the right tail of the distribution. In Robbins et al. the surplus distribution is cast into a total claims distribution by defining $X = A - S$, where A is an upper bound on the surplus. Cox [13] stated that the determination of A was not estimated from the existing subsample but was preset by them. The Generalized Beta Type 2 Distribution has only a positive support and it is necessary to transform the surplus distribution into some random variable with only positive support. Craighead [14, 15] using the negative of the surplus amounts converted the data for right tail analysis. This was done only for convenience because Zelterman [110] did his analysis on only the right tail. Quantile regression is not restricted on the support of the input or output. However, because of the restriction of the positive support for the use of GB2, for consistency we adjust the EVAS values to force all values to be positive. We use these adjusted EVAS data sets across all three methodologies so that we do not have to constantly convert results from the adjusted results to compare between the methodologies.

We use the following formula to adjust the EVAS values S:

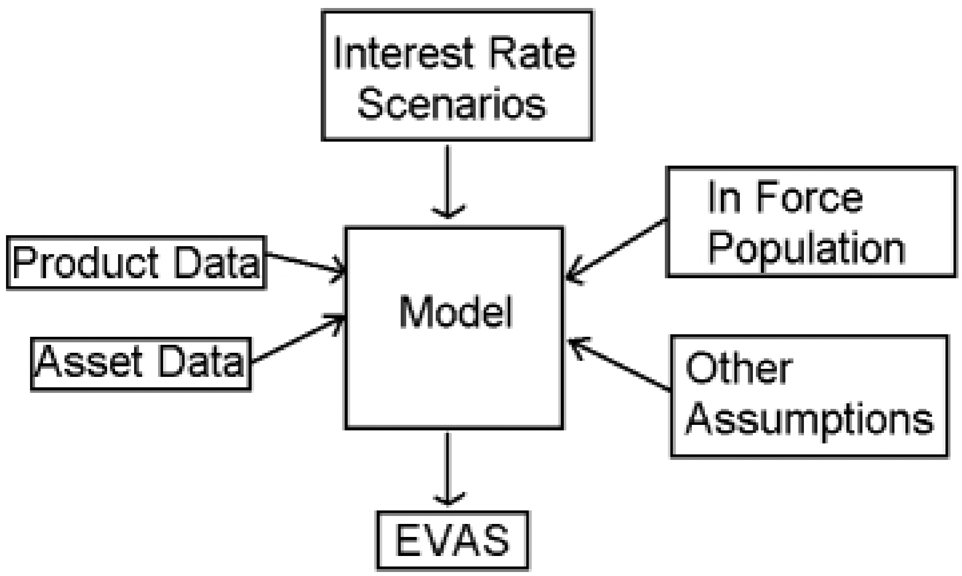

Figure 1: Corporate Model Layout

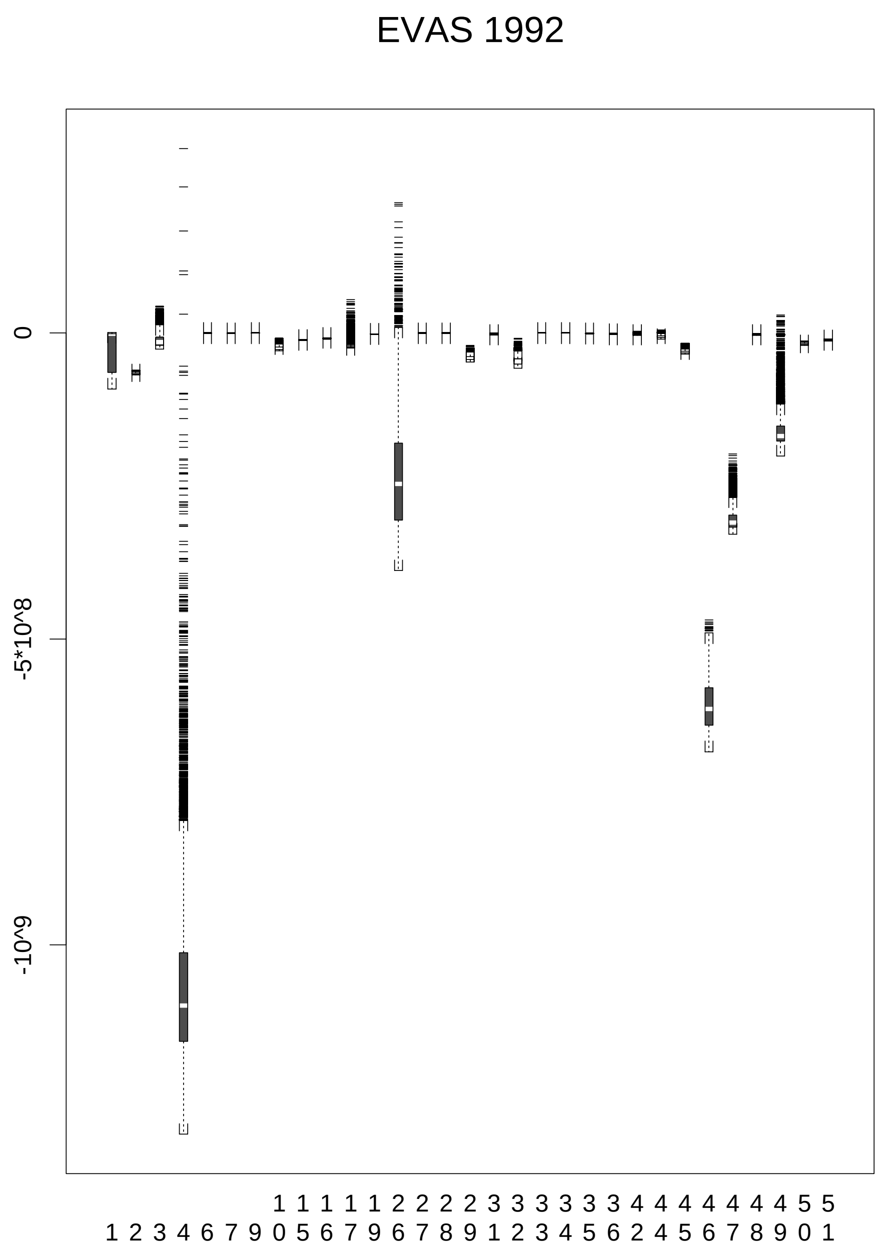
EVAS 1992
0
-5*10^8
-10^9
1 2 3 4 6 7 9 10 15 16 17 19 26 27 28 29 31 32 33 34 35 36 42 44 45 46 47 48 49 50 51

Figure 2:

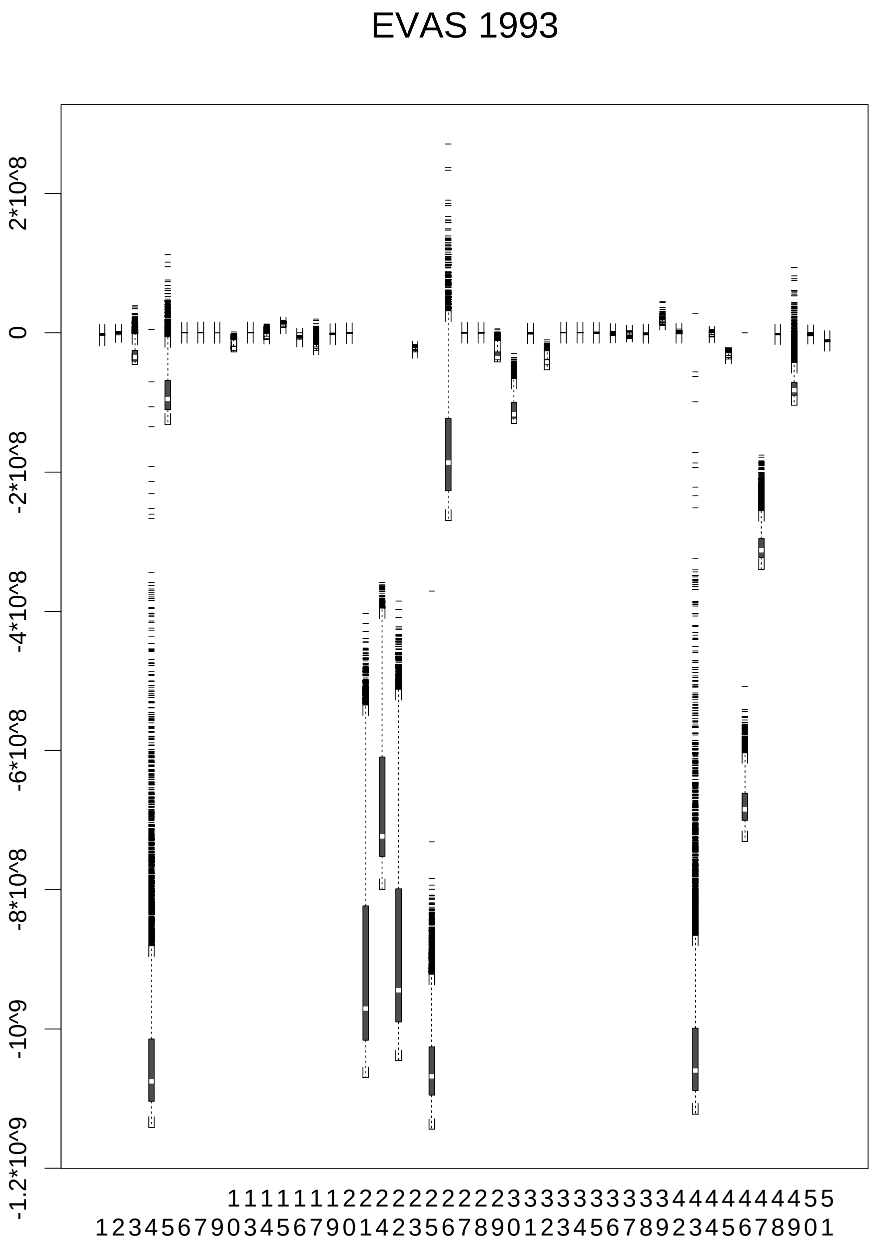
EVAS 1993
2*10^8
0
-2*10^8
-4*10^8
-6*10^8
-8*10^8
-10^9
-1.2*10^9
1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3 4 4 4 4 4 4 4 4 4 5 5
1 2 3 4 5 6 7 9 0 3 4 5 6 7 9 0 1 4 2 3 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 2 3 4 5 6 7 8 9 0 1

Figure 3:

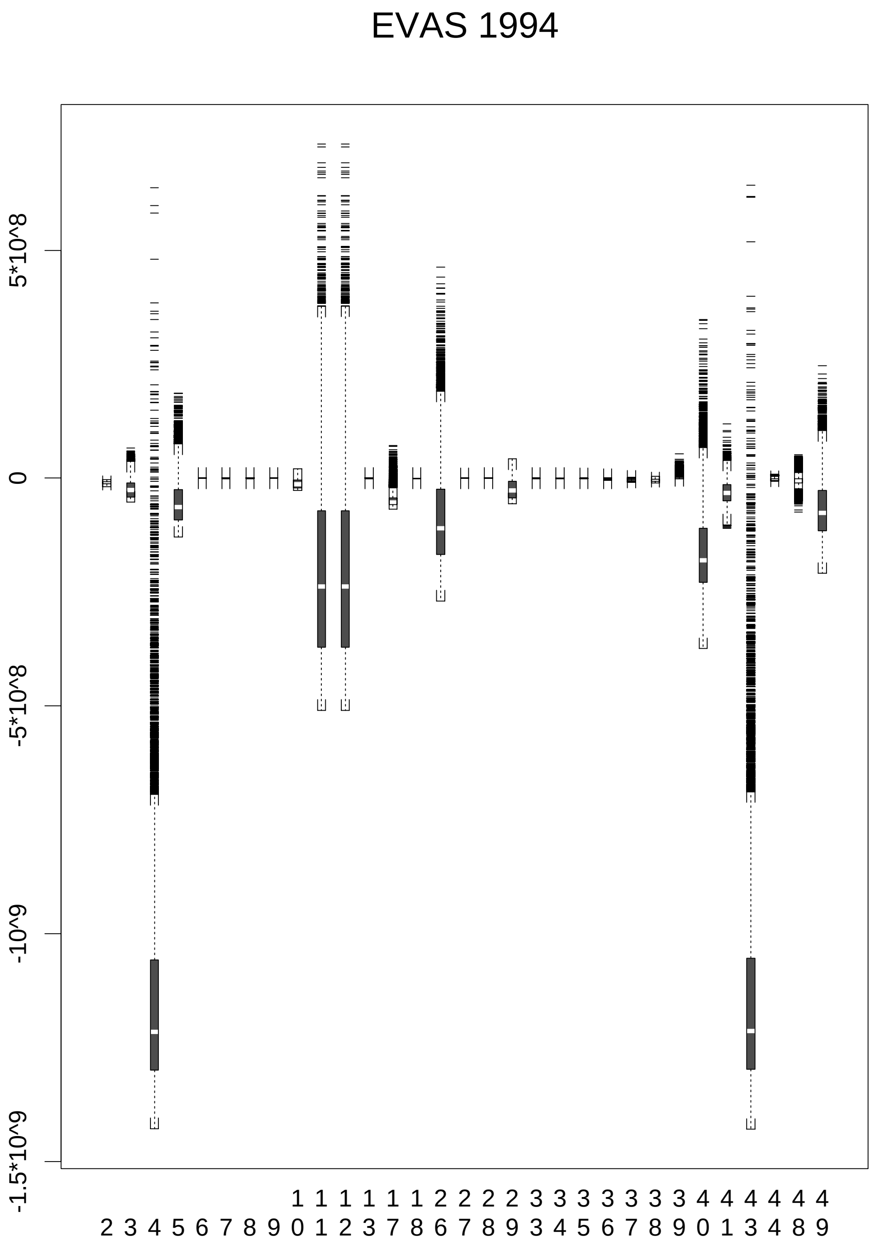
EVAS 1994
5*10^8
0
-5*10^8
-10^9
-1.5*10^9
2 3 4 5 6 7 8 9 10 11 12 13 17 18 26 27 28 29 33 34 35 36 37 38 39 40 41 43 44 48 49

Figure 4:

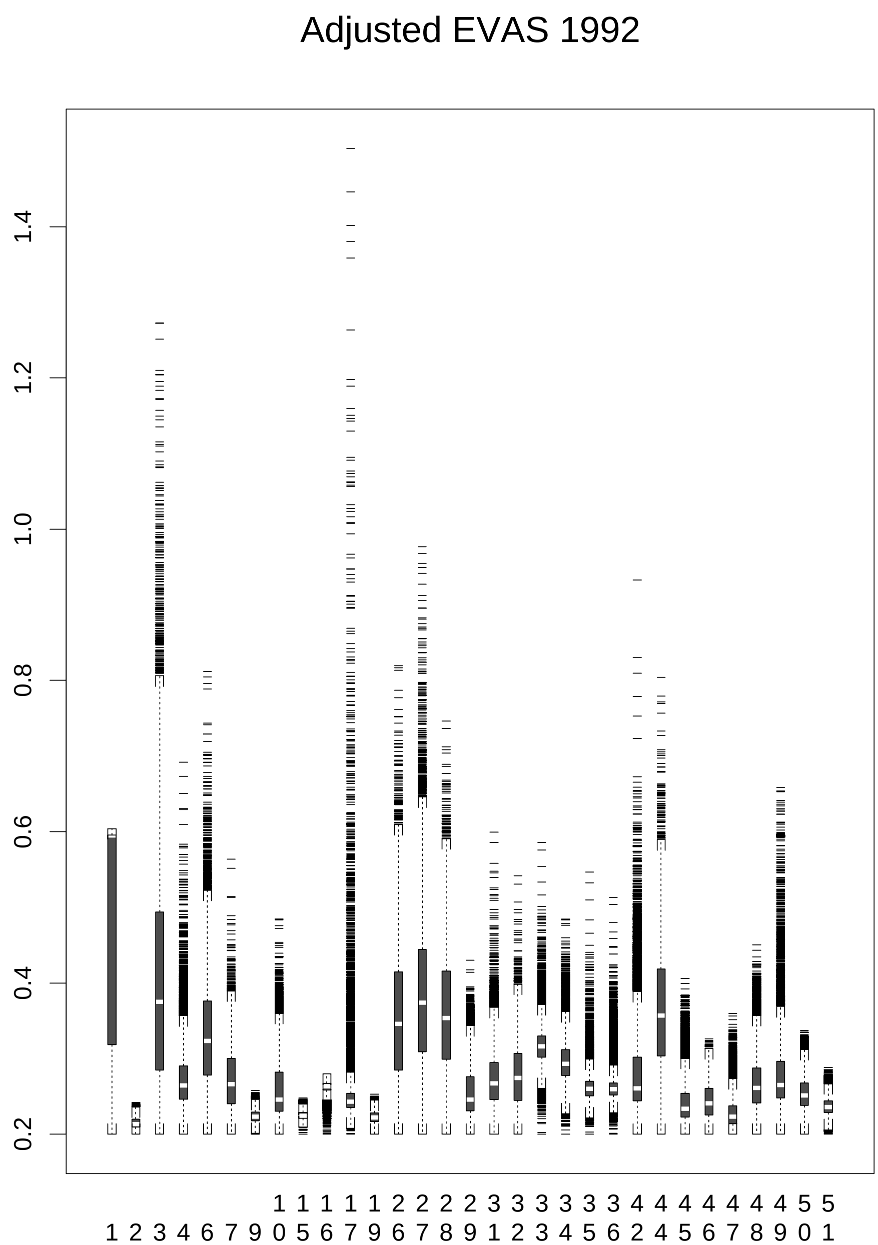
Adjusted EVAS 1992
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1 2 3 4 6 7 9 10 15 16 17 19 26 27 28 29 31 32 33 34 35 36 42 44 45 46 47 48 49 50 51

Figure 5:

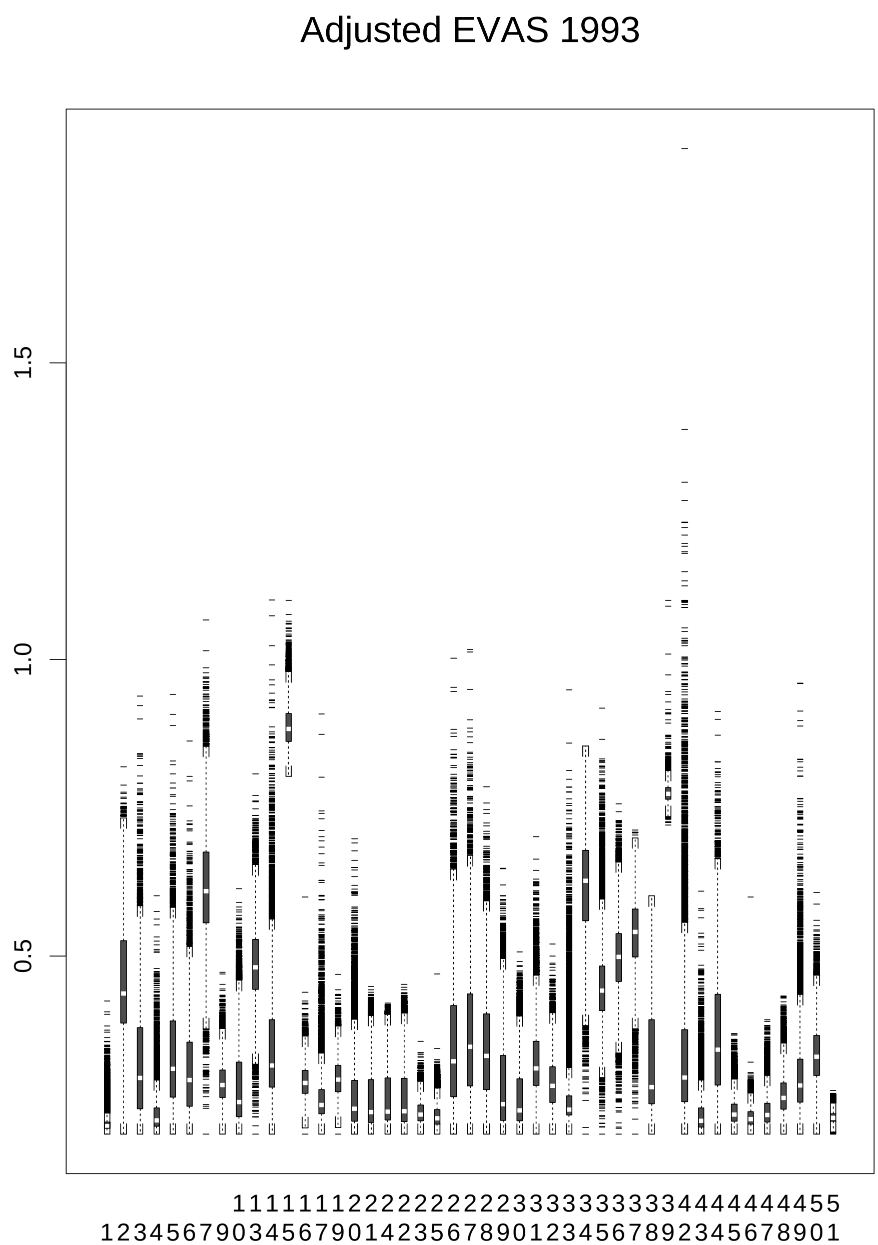
Adjusted EVAS 1993
1.5
1.0
0.5
1 2 3 4 5 6 7 9 10 13 14 15 16 17 19 20 21 24 22 23 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 42 43 44 45 46 47 48 49 50 51

Figure 6:

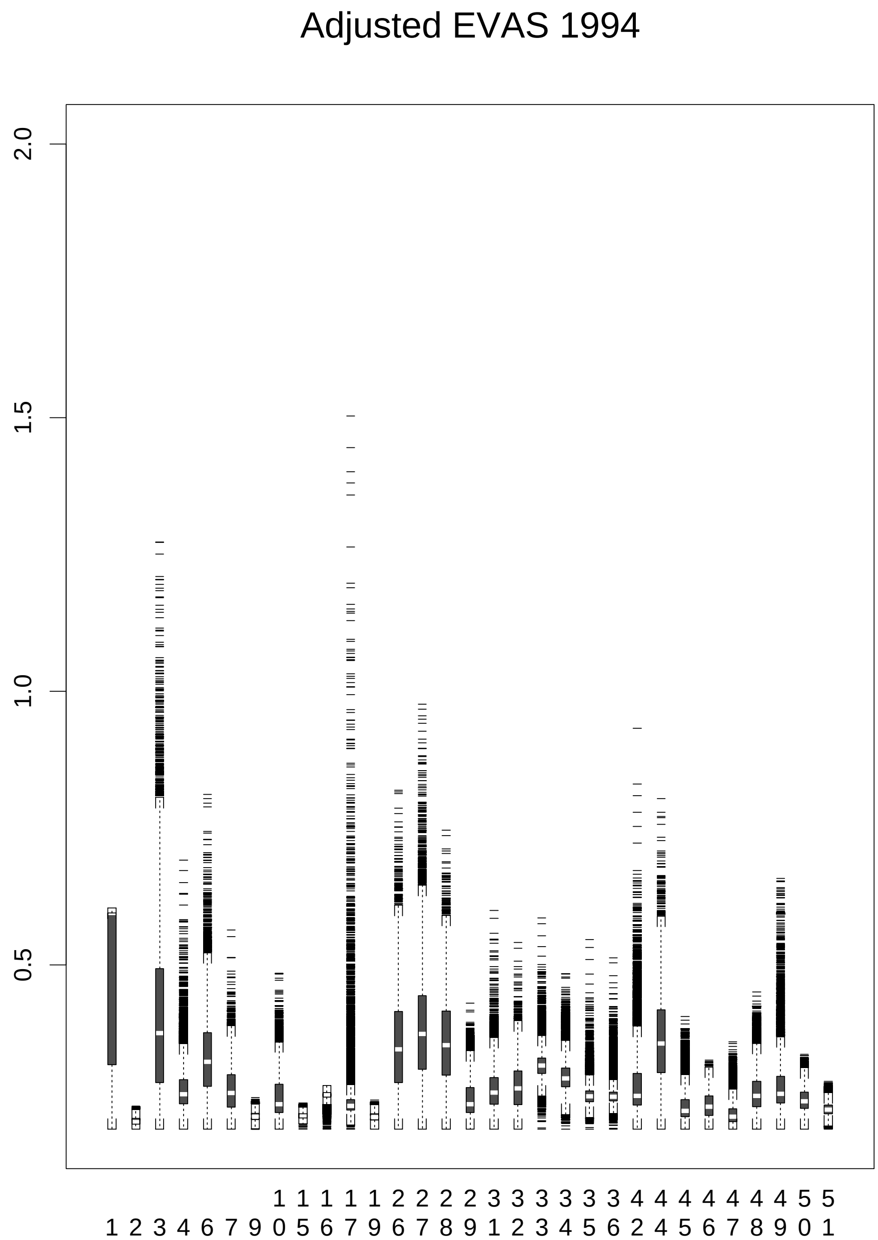
Adjusted EVAS 1994
2.0
1.5
1.0
0.5
1 2 3 4 6 7 9 10 15 16 17 19 26 27 28 29 31 32 33 34 35 36 42 44 45 46 47 48 49 50 51

Figure 7:

$$M(S) = (S - A)/(B - A) \tag{1}$$

The choice of A and B are determined in the following fashion:
If the minimum of S is less than zero then let

$$B = -min(S) \tag{2}$$

whereas if the minimum of S is greater than zero then

$$B = \frac{4max(S)}{5}. \tag{3}$$

For either B use

$$A = \frac{-3B}{2}. \tag{4}$$

For either B we have the following results:

$$M(S) = (S - 3/2B)/(5B/2) \tag{5}$$

and when $S = 0$ then $M(0) = 3/5$.

This adjustment does the following:

1. Common point. The unadjusted EVAS value of 0 maps into the same value for all lines of business, specifically 3/5.

2. Positive Support. This method maps all values to be greater than zero. In fact all values are greater than 1/5.

3. Contraction of Support. This is both a strength and weakness. The strength is that the resultant small support tends to help stabilize the GB2 parametric search. The weakness is that the scale of the adjusted EVAS values is not consistent between lines of business. Before the adjustment the dollar impact is obvious since the actual volume of the business indicated the dispersion of results of the different lines. This adjustment distorts the results in such a way that the lines with small support appear much more risky and large lines less risky. A visual comparison of Figures 5, 6 and 7 on pages 29, 30 and 31 to Figures 2, 3 and 4 on pages 26, 27 and 28 reveals this problem. Also, because B and A are different between years this obscures the analysis between years.

4. Direction of risk reversed. Since the values of EVAS below zero correspond to the adjusted EVAS values above 3/5, the various summary statistics move right when the actual results worsen.

We examined different ways to determine B. First we used B as the negative global minimum over all of the EVAS values for all lines of business for all three years. We did this because we wanted to conduct between year analyses. Also, we wanted to have a common scale of reference for all of the lines of business. However, because the LOB04 line is highly volatile in 1994, this made the results of the lines of business appear to be tiny dust mites near 3/5. These lines created many problems when fitting the GB2 distribution.

Next we used the negative minimum of all EVAS values within a specific year for B. We felt that even if the between year analyses are obscured, we felt the results are comparable within a specific year. However, the severe reduction of small line supports reoccurred.

This led us to finally allow B to be determined by the actual minimum EVAS value within a line of business. In choosing B however, we made the assumption that B was determined from the entire 10,000 population and not from any subsample. In Section 5, this restriction does not jaundice our results because of the multi-modal nature of the data.

It came to the attention of the author much too late in the research process that LOB04 should be eliminated from the analysis or scaled down since LOB04 "washes out" the smaller lines. However, because LOB04 is important to the overall performance of the company and the fact that we wanted to study the effects of all of the data on the three methodologies we retain the above transformation.

Statistics on the adjusted EVAS values for each of the lines of business by year are in Tables 16, 17 and 18 on pages 136, 137 and 138 respectively.

4 Model Adequacy, Information, Sample Sizes and Goodness of Fit Considerations

Since the cardinality of the functions on a set is greater than the cardinality of the set itself, one concludes that the set of models on a data set will have cardinality greater than the cardinality of the data set. This rough cardinality argument places us in difficult straits. So from a large set of potential models, we now have the difficulty of finding an accurate model (just use the data as the model, you will get exact results) versus an adequate model that is not as accurate but will reveal specific data attributes.

Here I would like to introduce an analogy that reveals some of the core difficulties that we are addressing.

Imagine that you want to play solitaire. However, the only decks that you have access are frequently exposed to small children. In this situation you must determine if the deck is complete. What algorithm should you use to determine if the deck is complete?

Say that you can make no assumptions about the character of the deck, for example, the deck of cards to be tested for completeness may be the combination of two separate decks with the exact same decorative backs. In these situations the checking algorithms that you can consider are more complex and time consuming. For example, you could sort the deck into suits and check if each suit is complete. Or you could sort into face values and see if each face value contains all suits. The advantage to these methods is that you will know exactly which cards are extra or missing when you have completed the task. The disadvantage is that the sorting takes a great deal of time.

However, if you can make the assumption that you only have one deck of cards that have this decorative backing the testing can be simplified. Then by just counting the cards, you will be confident that you have a complete deck if you obtain 52. Any value less than 52 you can conclude that the deck is incomplete. With any value greater than 52, your original assumption about only one deck with the same decorative backing is incorrect (or you didn't remove the Jokers). In these situations you would either abandon your original goal of playing solitaire or begin a more extensive sorting algorithm to determine the extra and/or missing cards. However if you counted 52, you could play. The advantage of this algorithm is that it is much quicker than any sorting and categorization algorithm referred to above. The disadvantage is that your hypothesis test indicates that the deck is complete with some element of uncertainty. Moreover, you have no knowledge of the missing or extra cards. In fact if by chance the number of extra cards and the number of missing cards matched, you would obtain 52 cards, and as you play, you might or might not determine the inaccuracy.

The above analogy points out:

1. The importance of the original purpose influences whether complete information is critical or not.

2. The means to have complete information with surety is by some expensive sorting and categorization process.

3. The hypothesis test is usually related to a specific simplification of the overall understanding of the data being analyzed. Since the card player knows that a deck of cards is made up of 4 suits with 13 different values, then card counting will be sufficient.

4. The hypothesis test is very quick.

5. The hypothesis test may not be totally accurate.

6. In the event that the hypothesis test fails, either abandon the original purpose, or revert to a more expensive algorithm to obtain complete information.

7. The subsequent use of the deck (model) may or may not reveal the inaccuracy.

In our analysis of the three methodologies, we have a more difficult situation than with the card analogy. This arises from:

1. We do not know the complete make up of our data sets.

2. We do not know whether the methodologies can adequately reveal the information contained within the data.

3. We do not know if a subsample can adequately reveal the critical information of the entire data set.

4. We are not clear on how to measure the effectiveness of maximizing information while reducing the scenario requirements.

5. We are unclear on what information is critical to our understanding of the underlying computer model.

6. We realize that as sample sizes increase, the information contained therein will also increase. Regrettably due to this increased complexity, our understanding of the data may not increase. The data will approach the true distribution of the underlying population, but this distribution may be so complex that no known family of distributions will adequately fit the observed data.

7. We know that many options are embedded within the corporate models and these options may cause the ECDF to be very complex, but we don't know their effect much less their interaction under any given scenario. We also realize that any smoothing of the data may obscure the effects of these embedded options.

8. We realize that the data sets contain some unmeasurable error. All of the assumptions that led to the corporate models each have accuracy issues. How much does the lack of information due to these errors, cause the actuary to take the results with the proverbial 'grain of salt'? How does one measure the informational loss here against the informational gain by increasing the number of scenarios processed? Does this difficulty create the need of a categorization of different types of information?

9. We realize that model smoothness may not match the observed data sets. We also realize that we have not specifically designed our goal of maximizing information in context with this smoothing issue. This is very similar to the difficulties in graduation theory, where the smoothing parameter is used to adjust the graduation model between the accurate rough data and a simpler smooth graduation of the data. See London [52] for a further discussion of this problem. This is also observed in nonparametric density fitting where the bandwidth parameter corresponds to the smoothing parameter in graduation theory. See Venables and Ripley [101] for a further discussion and references to this problem. A difficulty with parametric and semiparametric methods is that this smoothing process takes place when fitting the data. This leads to the problem where smoothing versus accuracy in a model is tested but not controlled by the analyst.

10. We know that the traditional KS goodness of fit test is structured to determine the worse case deviation of a model's distribution from that of the ECDF based on the actual data. As the data set size increases the ability of passing this test becomes more difficult because the ECDF becomes finer and finer and approaches the actual CDF underlying the data set. By using this test, it does point out that a specific CDF may not be adequate. The only difficulty with the test is that it supplies only the information that the CDF is inadequate and does not advise you in replacement of the CDF.

11. We know that the traditional KS test is structured to proceed on only unbinned data. The traditional test does not allow one to use some form of smoothing with the KS test. We also know that the traditional KS test does not allow any forms of binning, smoothing, using key value restrictions, or reducing the sample sizes on the data. If one does any of these methods without modifying the related probability Equation 48 on page 96 when using some specific nonnegative weight function $\psi(x)$, then the results of hypothesis tests used to comparing model distributions to the actual data are questionable.

Our examination of how well our methodologies can model each of the complete data sets is of course more complex than trying to determine whether a deck of cards is complete or incomplete. First and foremost, we are trying to determine if the separate methodologies' results produce a reasonable match to the actual results. Possibly by making some assumption about the quality of the data, we could obtain a simple and quick test to state that our efforts are valid within some level of confidence. Possibly we could take random samples of the data and test if the resultant models from these subsamples are accurate. However, these tests could be very dependent upon the data, and might have to vary from data set to data set. Also, we would have to devise a test to determine if the models based on the subsamples are accurate.

Due to the large number of data sets and the complexity of the methodologies, we will place our emphasis on accuracy as measured by the entire data set. Because of the complexities we outlined above, we will not examine any subsampling issues when examining this accuracy issue. Here we will use the traditional KS test, which will compare the ECDF of the data to that of a specific model's distribution. The severity of this method in testing is discussed above and Appendix C.2. However, this goodness of fit test has had a long acceptable history of use and the necessary theory to conduct hypothesis testing is very complete.

Here we see that reality begins to reveal that general modeling philosophy is as limited as Horatio's philosophy in Hamlet. However, the difficulty that most of us confront is that we make conclusions and derive results with a paradigm that is inadequate and such inadequacy is not known until revealed by reality. By conducting studies on a large number of large sample data sets, reality will frequently point out the inadequacies and inaccuracies in various methodologies. Once these inadequacies are observed, we must develop new paradigms that give us new economies of thought and modeling.

5 Modeling of GB2

Please refer to Appendix B.1 for a theoretical discussion of the GB2 distribution.

This was the most technically difficult aspect of the research. The following is a description of the process that we use.

The actual difficulty arose with respect to being able to properly control the optimization process. We took five different approaches to this process before finally determining the most efficient. The first approach was to consider the solver in Microsoft's Excel for the optimization. Using maximum likelihood estimation (denoted MLE) we determined that there are many potential solutions of the GB2 distribution. Using this method, we also determined that because each of the GB2 parameters had to be positive, that frequently the optimizer sent the next experiment to be processed into a region where the parameters are negative. This frequently occurs with most nonlinear optimizers because the software must be limited in some fashion to handle boundaries. Some methods to alleviate this problem are to create reflecting boundaries or absorbing boundaries. Another method is to determine some type of transformation that removes the boundaries and allows the optimizer free rein in its parametric choices. However, Excel's solver is not this flexible. The other insight that we gained is that if the scale of the data used in the estimates is very large, we do not get a stable estimate of the parameters. Frequently this occurred when the parameters became too large, and the computer algorithms had overflow errors. Defining the adjusted EVAS values in the fashion that caused the support to be small helps alleviate this difficulty. Also, this led to running multiple optimizations on each line for a given sample size of adjusted EVAS values. This required different starting values for the parameters and hence it became critical to be able to control the optimization process. Since the solver in Excel is not macro driven, the requirements to produce multiple runs by human intervention for the 107 data sets is excessive. The sheer number of possible studies would then be at risk for potential operator error. We decided that we needed an optimizer that ran reliably with little intervention.

We next tried a Levenberg-Marquardt process (see Press et al. [72]) within Microsoft QuickBasic to try to determine the parameters of the negative log likelihood. We abandoned this method because we were unable to use a data set's entire 10,000 adjusted EVAS values without having to make major revisions to the Levenberg-Marquardt code.

We next developed a random simplex algorithm along the lines of the floppy triangle in S+. See Hendrix [32] or Babcock and Craighead [3] for a description. We abandoned this when the algorithm did not produce a very diverse list of MLE values. We determined at a later time that a simplex method requires only 1 more experiment than the number of variables in the simplex where we used 500 experiments. We believe that this is the cause of the poor performance.

We next used the *ms* function in the S+ language to minimize the negative log likelihood which is equivalent maximizing the log likelihood in MLE methods. We then randomly chose the parameters between 0 and 1 using an uniform distribution. However we found that different lines of business pushed the τ parameter above 15 and created a discontinuous cumulative distribution function (CDF). See Appendix B.1 for a further discussion of this difficulty. We revised the algorithm to discard the results anytime that τ exceeded 15. We also at this time tried to maximize the KS test goodness of fit probability instead of minimizing the negative log likelihood. At this point a further complication arose when the algorithm overflowed and the *ms* function did not process all the data sets without crashing. Since the *ms* function is an internal S+ function we were unable to revise the algorithm to prevent these difficulties.

Finally we turned to another simplex algorithm which Press et al. [72] discuss which they call the *amoeba* method. We also changed our venue for the analysis. We returned to the Excel environment and used Microsoft's Visual Basic for Applications (VBA) to code the *amoeba* algorithm. We also proceeded to code the other related methodologies within this environment as well. The difficulty where negative parameters caused overflows within the GB2 CDF evaluation again arose. We initially modified certain constants within the *amoeba* algorithm to reduce the chance of encountering these negative parameters. However, not finding success, we revised the *amoeba* method so that whenever a negative set of parameters was obtained, the algorithm would exit and a new random simplex of experiments is generated. With this approach we are able to run multiple optimizations on each line of business. Note: In these optimizations we minimize the negative log likelihood.

Other methods that were untried are:

1. Use of formulas for the gradient and the Hessian matrix to determine the MLE. We did not resort to these types of MLE solution because

we decided to solve the log likelihood directly. However, if we had, we would have still had a problem that would require a difficult non-linear solution with boundary conditions. However, Klugman et al. [51] do discuss the means to obtain the formulas for the gradient, but not the Hessian.

2. Levenberg-Marquardt algorithm. (See Press et al. [74]). This method also produces a parameter covariance matrix, which allows one to determine the significance of the parameter estimate. (See Appendix F for the difficulties in estimating this.) Earlier, I abandoned this method, because of a memory restriction with an existing software package.

3. Pre-estimate. The GB2 distribution contains many separate families of distributions. One could make initial estimates of the parameters from a simpler family. This was one technique used by Robbins et al. [79].

4. Generalized Methods of Moments (GMM). Here, we create a target function that relates all of the known moments to the parameters. By minimizing the target function, we will obtain the GMM parameter fit of the model to the data.

5. Simulated Methods of Moments. This is similar to GMM, however, here we just create simulations of many different sets of the parameters and obtain the best parameterization by minimizing some target function.

Due to the complexity of the optimization process, we only process twenty-five separate optimizations with each line of business. After sorting the negative log likelihood from lowest to highest, we capture the best GB2 parameters for each line. Tables 19, 20 and 21 on pages 139, 140 and 141 respectively contain these results. Observe that in each table that none of the models fit the actual data when measured by the KS test. This is because p consistently falls below the acceptable 5% level.

We also examined methods to maximize the goodness of fit probability directly, but again we did not have much success. However, in the minimization of the negative log likelihood, we found five lines of business whose GB2 model met the KS test. However, these models were not the best MLE models! The results on these five lines are displayed in Table 22 on page 142. Notice in Table 22 that LOB50 in 1993 has four separate solutions that pass the KS test. LOB42 has three separate solutions that pass the KS test. So out of 2,675 separate optimizations only 11 separate optimizations passed the KS goodness of fit! From this we believe that all the information in the 10,000 trials cannot generally be reduced to 4 parameters.

One major attribute of the GB2 distribution is if it has a mode, there is only one. However because of all of the options that are embedded in the different lines of business, the data is multi-modal. This limitation is another reason that the GB2 methodology should not be considered with insolvency analysis. Realizing that multi-modal distributions are better approximated by using mixture models; we modeled the adjusted EVAS values for the 1992 LOB04 line of business using gamma mixture models as in Robbins et al. [79]. We examined from two to twelve separate gamma distributions in these mixture models. However, when analyzing the KS probability of fit on all of these separate models, none passed at the 95% confidence level. One observation is that the KS distance D reduces for each additional gamma distribution added, but the reduction is so slow that it is estimated that we need hundreds of gamma distributions in the mixture to lower the distance enough to pass the KS test.

As previously mentioned Robbins et al. [79] produced 10,000 surplus values from a SPDA model. They then determined that 500 is a good subsample size by examining when the first and second central moments stabilized. With this subsample they then fit the GB2 and several other distributions. By using graphs they visually compared the results of the different models to each other and the empirical distribution of the 10,000 EVAS values. They then used the KS test to examine how well the GB2 and other distributions passed the KS test on the 500 subsampled data. The other time they used the KS test is to test the fit on a set of key values. There is no reference in their report to the fact whether they conduct the KS test on their models with their entire 10,000-element data set.

Using the above methodology, it took over 1 week to process 25 separate *amoeba* MLE estimates for the entire 107 data sets. In fact 250 or 500 separate *amoeba* MLE estimates would have been better to obtain the best fit.

The purpose of the KS test when comparing a data set to a known distribution is to determine if the sample data could be samples from that distribution. In the KS test, as the sample size increases the resultant ECDF will converge to the actual (though possibly unknown distribution). What we have found by using the entire data set is that the actual distribution that the ECDF is converging to is not the GB2 distribution. These tests point out that the GB2 distribution is just not rich enough to be used when considering the actual complexity of the data. Now, if the actuary wants to give up some of the information contained in the data and use the parametric distribution, they may use subsamples with the realization that the parametric distribution will have limited information. Dr. Zhang [112] agrees with us in this conclusion. Now the KS test performed so poorly with the GB2 methodology when compared to the 107 separate data sets of 10,000 adjusted EVAS.

If we wanted to subsample this problem, we would have to:

1. Choose various sample sizes $s_1, s_2, \ldots, s_m$.

2. Determine the number of repeated samples of these separate sample sizes, probably 25, preferably higher.

3. Conduct the *amoeba* MLE estimate on each of those samples. Again we probably would have solved the MLE 25 times, but could have used more.

4. Repeat this process on each of the 107 data sets.

As one can see there would have been roughly $(107)(25)(25)(\sum_1^m s_i)$ separate MLE solutions. Our current method of MLE estimation makes organizing and processing of this number of tests too prohibitive. So, we do not conduct any subsampling analysis with the assumption that if the model does not fit the whole, it does not reasonably fit the part.

An associated problem would be to place some type of confidence intervals upon each of the GB2 parameters. This difficulty is discussed in Appendix F. We also did not attempt to obtain these confidence intervals.

6 Modeling of REV

Please refer to Appendix B.3 for a theoretical discussion of the REV methodology.

The implementation of the REV algorithm by Craighead [14, 15] was made available to us at the same time as the 107 EVAS data sets.

Upon determining that the KS test revealed that the GB2 distribution fit very poorly to the actual 10,000 EVAS values on a line by line basis, we realized that the same problem may affect the REV methodology. REV is designed as a subsample method only so we take the first 1,000 adjusted EVAS values from the 10,000 adjusted EVAS values for each line of business and for each year. We sampled from the 10% right tail and then compared both the mean and the median resampled order statistics to the actual values of the right truncated 10% tail of the 10,000 adjusted EVAS values. Note that the truncated KS Test as discussed in Appendix C.2.1 is used to compare the results. The results of the tests (with the corresponding CS ratio) are in Tables 23, 24 and 25 on pages 143, 144 and 145 respectively. Observe that the truncated KS probability p for both the mean and median resampled order statistics are all less than 0.05. This revealed that the REV method also fails when compared with the complexity of the actual 10,000 values. In previous papers Craighead [14, 15] used a binning process before using the KS test. This led him to questionable conclusions, since he did not adjust the associated KS probability series, (which corresponds to Equation 48 on page 96). Again because of these negative REV results, we will not conduct an in-depth subsampling analysis.

7 Modeling of QR

Please refer to Appendix B.2 for a theoretical discussion of the QR methodology.

Since QR analysis links the input interest rates scenarios with the resultant EVAS values, please refer to Appendix A to see a description of how the interest rates are generated and an empirical analysis of certain key rates in the scenarios. Also observe the consideration in Appendix A.3 for dimensional reduction on the scenarios.

Dr. Stephen Portnoy made us aware of a new QR algorithm had been developed by himself and Koenker [67]. This was obtained from the Internet site referenced in [48]. The implementation of this algorithm gave us the ability to conduct extensive QR analysis on the entire 107 data sets of 10,000 adjusted EVAS values.

We need to look at several ways to display and report QR results. These initial results and reports will be restricted to the QR results of the LOB04 line of business for the 1992 adjusted EVAS values. We will initially limit the design matrix for the QR analysis to that of the ninety-day rates for 1992. We also compare these results to a standard multivariate regression.

Observe the formula

$$R_q = B_{0,q} + B_{1,q}X_1 + B_{2,q}X_2 + \cdots + B_{19,q}X_{19} + B_{20,q}X_{20} + U_q. \qquad (6)$$

R_q is the response (specifically at the qth quantile), and the $X_i = Y_i^{90}$ are the levels of the ninety-day rate at the beginning of each year i. The $B_{i,q}$ are the related coefficients for the specific quantile q and U_q is the error. The assumption that $Quant(U_q) = 0$ leads to the result:

$$Quant(R_q) = B_{0,q} + B_{1,q}X_1 + B_{2,q}X_2 + \cdots + B_{19,q}X_{19} + B_{20,q}X_{20}. \qquad (7)$$

In Figure 8, one can see graphs of the intercept and coefficients $B_{i,q}$ for Year 1 through Year 8 by quantiles $q \in \{0.05, 0.10, 0.15, \cdots, 0.95\}$ on the x-axes. Note since the value of Y_0^{90} is constant across all scenarios, X_0 is not considered, but the impact of the starting level of Y_0^{90} is contained in the intercept $B_{0,q}$, however $B_{0,q}$ does contain other results that arise from the QR algorithm. The $B_{i,q}$ are shown as solid lines and their corresponding ninety-fifth percent confidence intervals are shown as dashed lines. These graphs correspond to those in Koenker and Machado [49]. Obviously the benefit of these graphs lies in the fact that one can observe the behavior of

Coef	Coef.	Std. Error	t value	$Pr(> \|\|t\|\|)$
Intercept	0.11723658	0.001313442	89.259079	0.000000e+000
Year 1	0.58620344	0.020093049	29.174439	0.000000e+000
Year 2	0.51941350	0.019206742	27.043291	0.000000e+000
Year 3	0.13123854	0.018878247	6.951839	3.831158e-012
Year 4	0.04179273	0.018489273	2.260377	2.381931e-002
Year 5	0.06049989	0.018465348	3.276401	1.054981e-003
Year 6	0.09423138	0.018287695	5.152721	2.616428e-007
Year 7	0.09500054	0.017983444	5.282667	1.299994e-007
Year 8	0.06231484	0.017689709	3.522660	4.291487e-004
Year 9	0.10616833	0.017661876	6.011158	1.906530e-009
Year 10	0.16744882	0.017042942	9.825113	0.000000e+000
Year 11	0.21872430	0.016961486	12.895350	0.000000e+000
Year 12	0.18612674	0.016487646	11.288861	0.000000e+000
Year 13	0.14755104	0.017159182	8.598955	0.000000e+000
Year 14	0.15774396	0.016167999	9.756554	0.000000e+000
Year 15	0.06374222	0.016006956	3.982158	6.877701e-005
Year 16	0.02022683	0.015944597	1.268570	2.046242e-001
Year 17	0.02780050	0.015983540	1.739321	8.200919e-002
Year 18	0.00503466	0.015479114	0.325255	7.449949e-001
Year 19	-0.02711263	0.015320868	-1.769654	7.681534e-002
Year 20	-0.01923336	0.013442517	-1.430785	1.525231e-001

Table 1: 10,000 Multivariate Regression Adjusted 1992 LOB04

the coefficients across many quantiles. Since the analysis is on the adjusted EVAS values the actual risk analysis is reversed. For example, the 5'th percentile reveals the upside potential for the line and the 95'th percentile the downside risk. Also, if the fifth percentile is lowered it indicates that the line has become more profitable. Similarly if the ninety-fifth percentile increases the risk to the line increases. This implies that if a specific coefficient $B_{i,q}$ is positive and the level of the X_i increases then the value of R_q moves up. For example, when examining Year 1 coefficients by quantile, one sees that the associated coefficients are positive and roughly decreasing. If the level of X_1 is high, the positive $B_{1,q}$'s cause the R_q to increase and cause the EVAS to decline. Note the losses are consistent across all of the quantiles. However, observing Year 5, the graph of the $B_{5,q}$ from the fifth percentile to the eightieth percentile is negative and then turns positive. This implies that as the level of the ninety-day rate X_5 increases the actual percentiles move left and hence improves the business performance. However from the eighty-fifth percentile R_q moves up and hence cause poorer business performance.

For the sake of comparison, an ordinary least squares regression (OLSR) on the adjusted EVAS values for LOB04 is in Table 1. Also $R^2 = 0.7499532$. Observe that the probabilities associated with all of the t values make all of

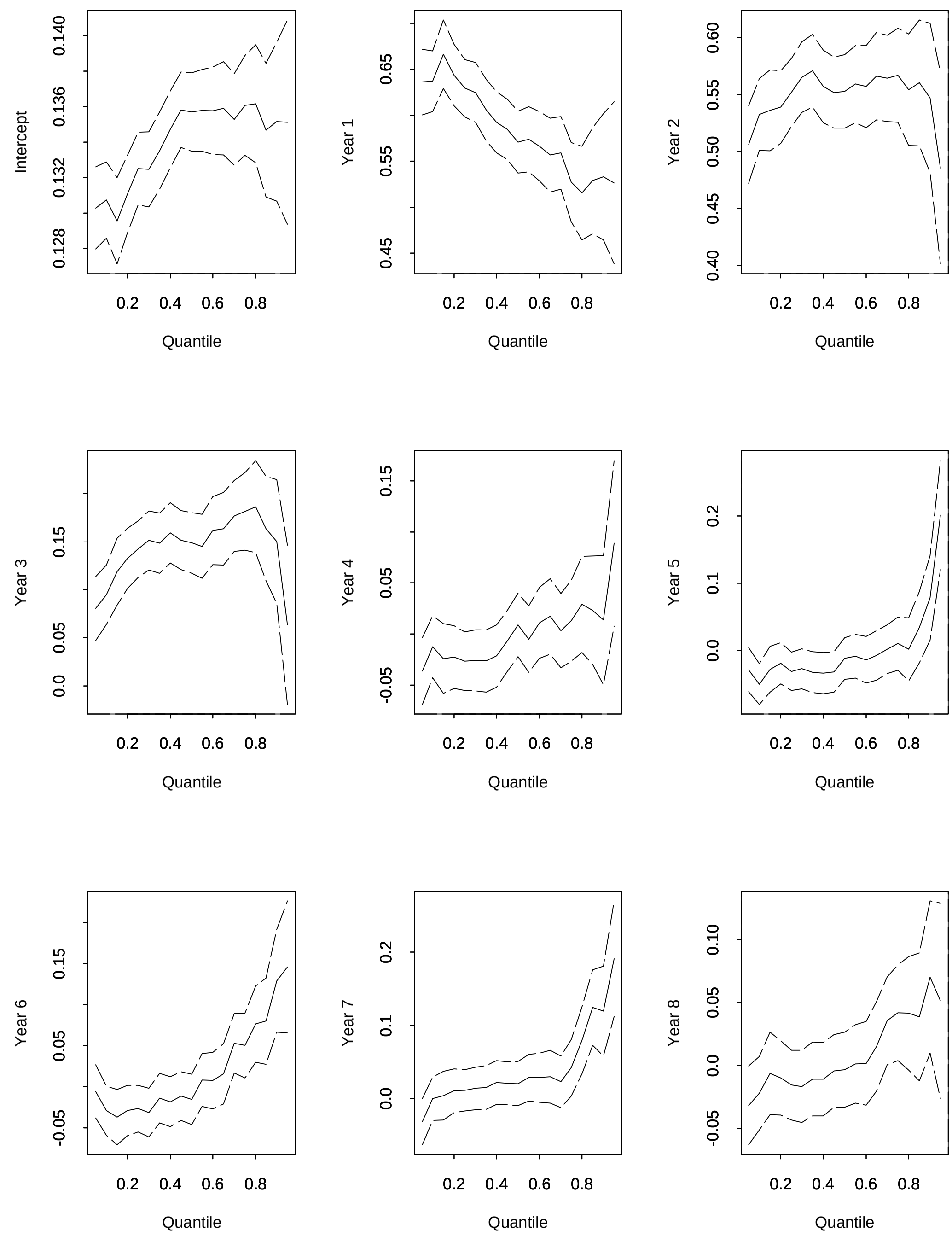

Figure 8: 1992 Adjusted LOB04 QR Behavior

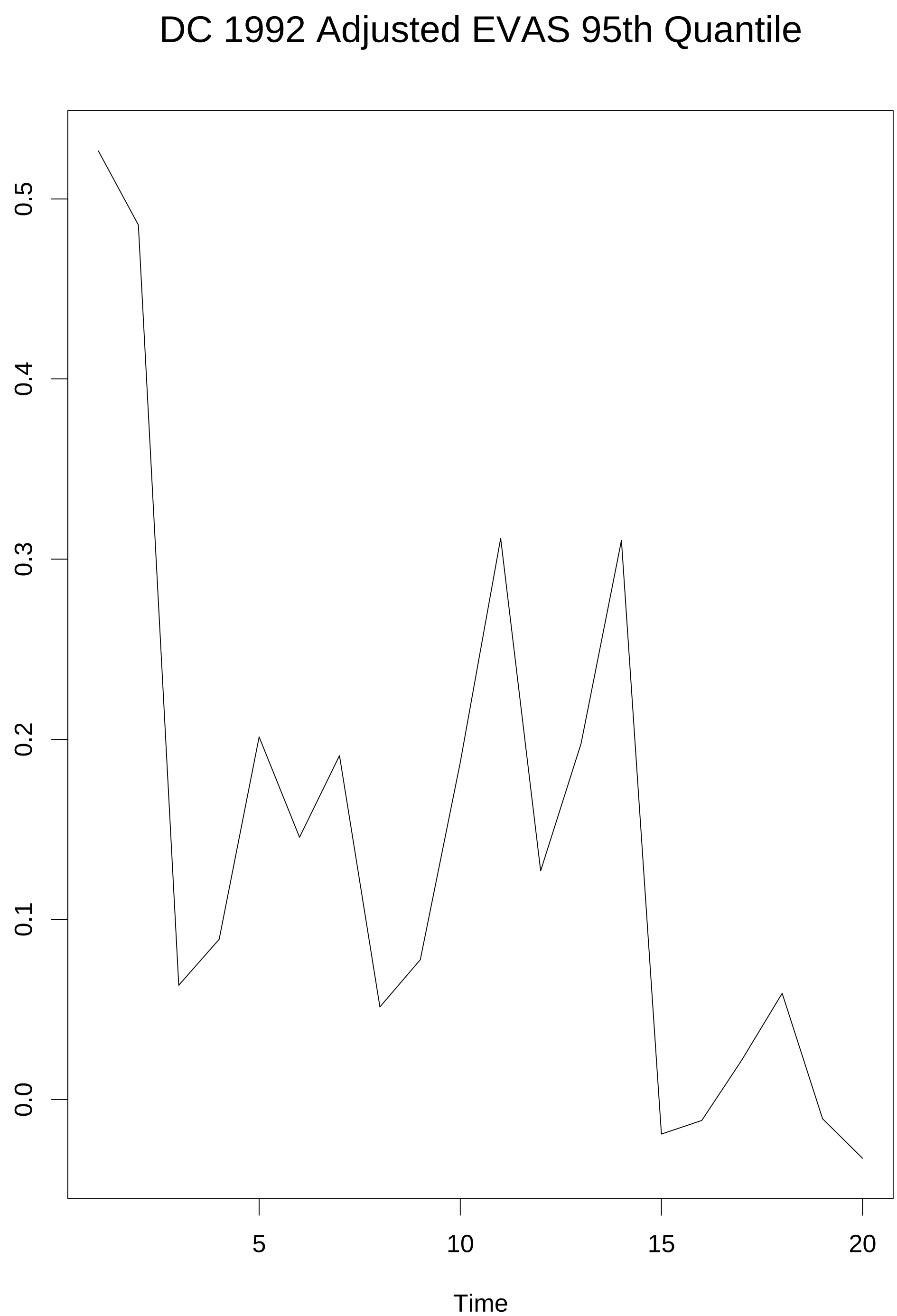

Figure 9: 1992 Adjusted LOB04 QR

Coef	$\tau = .05$ (se)	$\tau = .25$ (se)	$\tau = .50$ (se)	$\tau = .75$ (se)	$\tau = .95$ (se)
Intercept	0.1303 (0.001)	0.1325 (0.001)	0.1357 (0.001)	0.1361 (0.001)	0.1351 (0.003)
Year 1	0.6361 (0.018)	0.6294 (0.016)	0.5709 (0.017)	0.5275 (0.022)	0.5265 (0.045)
Year 2	0.5062 (0.017)	0.5521 (0.015)	0.5531 (0.016)	0.5671 (0.021)	0.4855 (0.043)
Year 3	0.0805 (0.017)	0.1424 (0.015)	0.1488 (0.016)	0.1818 (0.021)	0.0635 (0.042)
Year 4	-0.0363 (0.017)	-0.0267 (0.015)	0.0089 (0.016)	0.0129 (0.020)	0.0889 (0.041)
Year 5	-0.0288 (0.017)	-0.0313 (0.015)	-0.0122 (0.016)	0.0099 (0.020)	0.2014 (0.041)
Year 6	-0.0056 (0.016)	-0.0265 (0.015)	-0.0152 (0.016)	0.0502 (0.020)	0.1457 (0.041)
Year 7	-0.0315 (0.016)	0.0113 (0.014)	0.0205 (0.015)	0.0423 (0.020)	0.1910 (0.040)
Year 8	-0.0318 (0.016)	-0.0156 (0.014)	-0.0033 (0.015)	0.0419 (0.019)	0.0514 (0.040)
Year 9	0.0984 (0.016)	0.0969 (0.014)	0.0918 (0.015)	0.1129 (0.019)	0.0776 (0.040)
Year 10	0.1212 (0.015)	0.0958 (0.014)	0.1278 (0.015)	0.1651 (0.019)	0.1871 (0.038)
Year 11	0.1436 (0.015)	0.1790 (0.013)	0.2005 (0.015)	0.2080 (0.019)	0.3116 (0.038)
Year 12	0.1890 (0.015)	0.2024 (0.013)	0.2209 (0.014)	0.2036 (0.018)	0.1270 (0.037)
Year 13	0.1007 (0.015)	0.1220 (0.014)	0.1470 (0.015)	0.1359 (0.019)	0.1975 (0.038)
Year 14	0.1069 (0.015)	0.1252 (0.013)	0.1391 (0.014)	0.1577 (0.018)	0.3103 (0.036)
Year 15	0.0757 (0.014)	0.0786 (0.013)	0.0719 (0.014)	0.0569 (0.018)	-0.0191 (0.036)
Year 16	0.0518 (0.014)	0.0486 (0.013)	0.0241 (0.014)	0.0176 (0.017)	-0.0116 (0.036)
Year 17	0.0187 (0.014)	-0.0013 (0.013)	0.0256 (0.014)	0.0508 (0.017)	0.0219 (0.036)
Year 18	0.0087 (0.014)	0.0094 (0.012)	0.0124 (0.013)	0.0068 (0.017)	0.0589 (0.035)
Year 19	-0.0259 (0.014)	-0.0129 (0.012)	-0.0214 (0.013)	-0.0174 (0.017)	-0.0105 (0.034)
Year 20	0.0021 (0.012)	-0.0159 (0.011)	-0.0274 (0.012)	-0.0361 (0.015)	-0.0327 (0.030)

Table 2: 10,000 QR Coefficients on Adjusted 1992 LOB04

the coefficients very significant in the regression.

Another method recommended by Portnoy [68] on the display of QR results is in Table 2. This is just another way to display the results in Figure 8. Notice that only the coefficients and the standard errors for the quantiles $\{0.05, 0.25, 0.50, 0.75, 0.95\}$ are displayed. Note that Table 2 displays all years from 1 to 20 as well as the intercept. Examining the 95'th percentile we see that the R_{95} is most affected by the positive $B_{i,q}$ in years 1, 2, 11, and 14 and negative $B_{i,q}$ in years 20 and 15. However the coefficients above year 14 have absolute value close to zero, so the overall influence in the latter years is small. Later we see in other tables that the $B_{20,q}$ coefficients have a fair amount of influence on various lines. This is due to the fact that the existing business model is terminated at the end of year twenty. This effect is caused by the fact that the reserves in those lines of business are substantial in the twentieth year. Observe how the coefficients and the standard errors are very similar between different years for the various quantile regressions when compared to the OLSR in Table 1. The benefit of the OLSR is that one can see that each variable in the design matrix is significant, but with $R^2 \approx 0.75$ the OLSR explains only 75% of the variability of the results. We can see that there is a benefit of using the quantile approach. We will discuss in Section 9 a goodness of fit statistic similar to R^2 in OLSR. The 95% quantile regression does reflect additional risks that are not revealed from the OLSR. The effect of the positive $B_{i,q}$ in OLSR is in years 1, 2, 11 and 10 which is slightly different than with the 95% quantile. Also the magnitude of the $B_{i,q}$ are higher in year 1 and 2 in the OLSR than with the 95% quantile regression. This demonstrates that the use of an OLSR would point to different years as well as over- or under-emphasize an extreme quantile regression. However, the value of using OLSR does reveal whether the design matrix is adequate for the regression models.

Our interest for a specific quantile is in the sensitivity of the responses to the coefficients through time. By examining the time series of coefficients as in Figure, we can see this effect. The intercept has been excluded. This is because the magnitude of the intercept may at times obscure the other coefficients of the predictors. Notice from the figure that the responses are most sensitive to years 1, 2, 11 and 14. Also, notice that the coefficients go to zero after year 14. One can consider the addition of confidence intervals to the graphs also. We will enhance this type of analysis in Section 9.

We now proceed to compare the three different methodologies by percentile estimation.

8 Percentile Comparisons

The use of the KS test has led us to conclude the overall support and even the truncated support of the actual 10,000 adjusted EVAS values are not well estimated by either the GB2 or the REV process. Even though the KS test reveals that neither model does well across all of the data, the methodologies might be a good point estimate of specific extreme percentiles. However in order to compare the three actual methodologies we examine specific percentiles with the plan to use any of the three methodologies to approximate the actual percentiles as observed in the 10,000 adjusted EVAS values.

8.1 Comparison of Actual, GB2, REV, QR Percentiles

Appendix C.1 describes how by the use of order statistics, that one is able to estimate the confidence interval of a specific percentile in a data set. In this section we describe the results of using this estimation technique to look at the ninety-fifth and ninety-ninth percentiles of the adjusted EVAS values. Because the GB2 distribution is a complex four-factor distribution, the ability to obtain some type of confidence interval estimate ratchets up the algorithmic complexity, which we are unable to address. See Appendix F for a discussion of this. Therefore, we will not estimate the 95'th or 99'th percent confidence intervals on the GB2 results below. However we are able to make these confidence interval estimates using the REV method.

Let the predicted values from a specific QR be denoted as $\hat{R}_q$. The residuals are $R_q - \hat{R}_q$, which correspond to the errors U_q of the QR. The only assumption on the distribution of $R_q - \hat{R}_q$ in QR analysis is that $Quant(R_q - \hat{R}_q) = 0$. Because quantiles are determined by sorting then

$$Quant(R_q - \hat{R}_q) \neq Quant(R_q) - Quant(\hat{R}_q). \tag{8}$$

This creates a difficulty when trying to see how well the QR model actually matches the respective quantile in R_q. The method that we use to estimate the 95'th and the 99'th percentiles from a QR is to see if the 95'th or 99'th percentiles of the residuals $(R_q - \hat{R}_q) \approx 0$. From these values, we take the corresponding $\hat{R}_q$ as the 95'th or the 99'th percentile estimate. We do not have an effective means to create a confidence interval for QR model results. This is a topic for further research.

With the methodologies by which we could obtain confidence intervals, we placed a higher confidence interval requirement upon each of these percentiles. This is because of our concern with the issue of setting a high confidence level when making statements about extreme percentiles. We chose 95'th percent confidence intervals for the ninety-fifth percentiles for the order statistics and REV and 99'th percent confidence intervals when examining the ninety-ninth percentiles.

We do not know if having the confidence intervals set equal to the corresponding percentile is adequate to provide reliable ruin probabilities. The relationship of these confidence intervals with insolvency studies is a topic of future research.

In Tables 26, 27 and 28 on pages 146, 147 and 148 respectively, we can see the comparison of the 95'th percentile.

Summarizing Table 26, 27 and 28 on in Table 3 one sees that GB2 ninety-fifth percentile falls within the order statistic confidence intervals 48 out of 107 times. With REV we see that the median resampled ninety-fifth percentile falls in the order statistic confidence intervals 76 out of 107. We observe that the QR percentile estimates are very poor, having only 6 values fall within the respective intervals. This clearly demonstrates that the QR method will not model the unconditional quantiles very well at least at the higher percentiles.

Since the REV method can estimate a confidence interval about the median, if it is larger than the order statistics confidence interval, it is more conservative. We see that in total that is true in 90 out 107 examples.

In Tables 29, 30 and 31 on pages 149, 150 and 151 we can see the comparison of the 99%.

Summarizing Table 29, 30 and 31 in Table 4 one sees that using GB2 the ninety-ninth percentile falls within the order statistic confidence intervals 17 out of 107 times. Where with REV we see that the median resampled ninety-ninth percentile falls in the confidence interval 38 out of 107. Again we see that the QR results again are very poor with only 7 values falling within the confidence interval.

The results for the ninety-ninth percentile are similar with the REV confidence intervals.

We see that the REV methodology at the 95% and 99% have better point estimates than any of the other methods. REV percentile estimates outperform the GB2 estimates because the GB2 distribution is estimated to fit the entire support of the 10,000 adjusted EVAS values, whereas the REV

Summary of Tables 26, 27 and 28

Methodology	Year	Number of Point Estimates in Order Confidence Interval	Number of Order Confidence Interval Estimates in REV Confidence Interval
GB2	1992	12:31	NA
REV	1992	22:31	26:31
QR	1992	1:31	NA
GB2	1993	22:45	NA
REV	1993	34:45	38:45
QR	1993	4:45	NA
GB2	1994	14:31	NA
REV	1994	20:31	26:31
QR	1994	1:31	NA
Summary			
GB2		48:107	NA
REV		76:107	90:107
QR		6:107	NA

Table 3: Scorecard of Methodology Estimate of 95%

Summary of Tables 29, 30 and 31

Methodology	Year	Number of Point Estimates in Order Confidence Interval	Number of Order Confidence Interval Estimates in REV Confidence Interval
GB2	1992	7:31	NA
REV	1992	12:31	28:31
QR	1992	4:31	31:31
GB2	1993	6:45	NA
REV	1993	22:45	44:45
QR	1993	3:45	45:45
GB2	1994	4:31	NA
REV	1994	4:31	31:31
QR	1994	0:31	31:31
Summary			
GB2		17:107	NA
REV		38:107	103:107
QR		7:107	NA

Table 4: Scorecard of Methodology Estimate of 99%

is designed to concentrate its estimation in the tails. Also there appears to be some influence from the volatility of the underlying interest rates. In the 1993 time period, the interest rate volatility is down and the estimates are tighter than in 1992, which are tighter than in 1994.

We are very disappointed with the formula output for the QR methods. We see that the regression does not model the unconditional 95% and 99% very well. However, QR is designed to model the conditional and not the unconditional quantiles, Portnoy [68]. However, we will observe in the next section an application of QR, which uses the conditional quantiles and shows great promise in stochastic corporate model analysis.

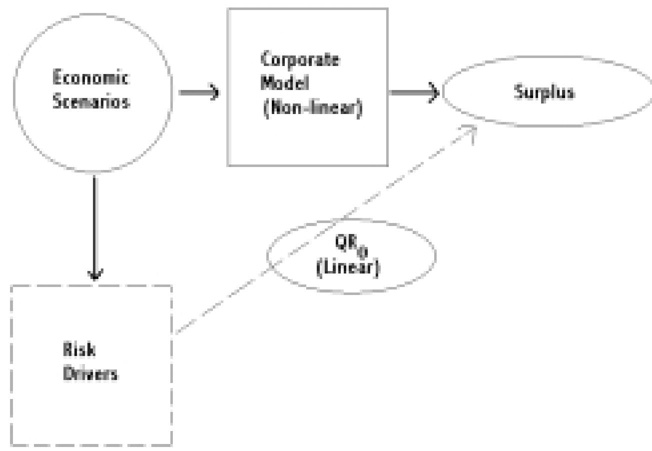

Figure 10: Concept of Risk Drivers

9 QR methodology and Risk Driver Sensitivity

In the prior sections we have been concerned with modeling results from the distribution of the EVAS. However, from this point, we examine the Input/Output relationship between the scenarios and the adjusted EVAS and disregard most of the distribution's properties. The actuary has a need to approximate the relationship between the input economic scenarios and the surplus output results without additional computer processing. Also, if one is able to target the location of adverse results when developing this relationship, all the better.

In Figure 10 we have a non-linear computer corporate model which takes economic scenarios as input and produces certain model output, which represents the surplus of the corporate model. Next, we define a risk driver to be a function of the economic scenarios through time, which distills the most informative characteristics of the economic scenarios that have an impact on the model output. The QR_θ function is an linear approximation at a specific quantile θ of an relationship between the risk drivers and the surplus output of the original non-linear computer model. For example, extracting the time series of 90-day Treasury Bill from each scenario is an example of a risk driver. Another example, is the time series of the spread of the 10-year Treasury Note less the 90-day Treasury Bill rate.

We will use the following six design matrices as representative risk drivers and examine how they affect the adjusted EVAS values of the 107 data sets.

1. The levels of the ninety-day Treasury bill rates from time 1 to 20. These are denoted as $L^{90} = \{Y_1^{90}, Y_2^{90}, \cdots, Y_{20}^{90}\}$.

2. The levels of the ten-year Treasury bond rates from time 1 to 20. Denote these as $L^{10} = \{Y_1^{10}, Y_2^{10}, \cdots, Y_{20}^{10}\}$.

3. The change in ninety-day Treasury bill rates. Denote this as $D^{90} = \{Y_1^{90} - Y_0^{90}, Y_2^{90} - Y_1^{90}, \cdots, Y_{20}^{90} - Y_{19}^{90}\}$.

4. The change in ten-year Treasury bond rates. Denote this as $D^{10} = \{Y_1^{10} - Y_0^{10}, Y_2^{10} - Y_1^{10}, \cdots, Y_{20}^{10} - Y_{19}^{10}\}$.

5. The spread between the ten-year Treasury bond rates and the ninety-day bill rates from time 1 to 20. This is denoted by $P = \{Y_1^{10} - Y_1^{90}, Y_2^{10} - Y_2^{90}, \cdots, Y_{20}^{10} - Y_{20}^{90}\}$.

6. The change in spread between the ten-year Treasury bond rates and the ninety-day bill rates. This is denoted by $\Delta P = \{(Y_1^{10} - Y_1^{90}) - (Y_0^{10} - Y_0^{90}), (Y_2^{10} - Y_2^{90}) - (Y_1^{10} - Y_1^{90}), \cdots, (Y_{20}^{10} - Y_{20}^{90}) - (Y_{19}^{10} - Y_{19}^{90})\}$.

At this point we develop a statistic that reveals the relevant information. We do this by developing a means to measure how a specific coefficient can be used to locate the epoch, which has the greatest influence on the quantile regression. This approach takes on a qualitative nature in that we do not try to predict the actual percentile as in Section 8, but we use it to report the location of potential risk or profit. The actual value of the individual coefficient $B_{i,q}$ in Formula 7 is not as critical to our understanding, as its relative magnitude is when compared to all of the other coefficients for a specific risk driver. Borrowing from Principal Components Analysis (PCA), one determines the influence of a specific eigenvalue by ranking each of the eigenvalues divided by the sum of the eigenvalues. See Dillon [20], Johnson and Wichern [40], and Mardia et al. [61] for a further discussion of PCA.

Modifying PCA to properly account for the negative coefficients, we measure the impact $S_{i,q}$ of a specific coefficient $B_{i,q}$ at the qth quantile

$$S_{i,q} = B_{i,q} / \sum_{i=1}^{n} |B_{i,q}|. \tag{9}$$

$S_{i,q}$ is expressed as a percent where n is the total number of coefficients. Also note that the impact does not include the intercept $B_{0,q}$ in the formula. This is because the intercept does not contribute to the variability of the R_q and also obscures the influence of the other coefficients of the predictors by its magnitude.

However, before using the impact statistic $S_{i,q}$ we must also consider whether the underlying QR model is significant. Otherwise, we could be misled by the results. Koenker and Machado [49] have developed a goodness of fit statistic for QR, which they refer to as the R^1 statistic that corresponds to the OLSR R^2 statistic.[2] They also discuss another statistic called a Wald estimator, which can be used in a fashion similar to the F-Ratio test in OLSR to determine model adequacy. See Appendix C.3 for a further discussion of how the Wald Estimator is determined. Below, we will use the R^1 and the Wald estimator as a goodness of fit measures. Also, we will use a simplified version of the Wald Estimator as a test (similar to the use of the Student t in OLSR) whether a specific variable is significant to the model. Please refer to [49] for a further discussion of the use and interpretation of R^1, the Wald Estimator, and other statistics.

In Tables 32, 33, and 34 on page 152 through page 154 are tables of the R^1 statistics of Koenker and Machado [49], for the 107 separate adjusted

[2] The design matrix of a regression demonstrates its completeness in how well the inner product of the coefficients against the design matrix replicates the responses R_q. In OLSR the effectiveness (or goodness of fit) is measured by the R^2 statistic. As one adds relevant variables to the design matrix then R^2 will move closer to 1, thus indicating that the design matrix contains sufficient variables. However, if R^2 is close to zero, this implies that variability within the residuals is not well explained with the OLSR model. This implies that additional variables should added to the design matrix or one should try other types of regression. Note: By the use of the Student t test, one can determine if a variable is significant to the model even when R^2 is small. However, low values of R^2 still point to model ineffectiveness. However, an OLSR model can still be ineffective with high R^2 due to other problems with the residuals. For instance, if the residuals are serially correlated or if the variance of the residuals is not constant then other problems ensue with the model effectiveness. See Venables and Ripley [101] for a further discussion and for other references relative to the use of OLSR.

EVAS data sets at the 95% quantile.. These R^1 values are for each of the six risk drivers. These show that for example, that all of the delta models (D^{90}, D^{10}, and ΔP) contain identical information of the main effects (L^{90}, L^{10}, and P). This is because the main effects models are telescoping sums of the delta models. However, the main models reveal when the business models are sensitive to levels of rates and spreads. Whereas, the delta models reveal when the business models are sensitive to changes in these rates and spreads. Note that LOB01 and LOB47 in 1992, LOB32 and LOB47 in 1993, and LOB08 and LOB41 in 1994 have values of R^1 that drop below 0.1. Note that LOB01 in 1992 have the poorest values for R^1.

In Tables 35, 36, and 37, on page 155 through page 157, one can observe the values of the Wald Estimators. (Note: These are only done on the three main effect models.) Here the first column will signify whether all of the coefficients other than the intercept are significant. The regression is significant at a 95% level if the value exceeds $\chi^2(\nu = 1) = 3.84$. Note: All of the regressions are highly significant at the 95% level.

The Wald estimator, which can be also used in a fashion similar to the Student t statistic in OLSR to indicate whether a specific variable in the design matrix is significant. Below, we will use the Wald estimator as a test for variable significance. Here, we calculate the Wald Estimator to assume that all of the variables other than the one in question are zero. This Wald Estimator will then be $\chi^2(\nu = 1)$. If the Wald Estimator is greater than 3.84 we have a variable that is significant at 95% confidence. This test shows that a specific variable is highly significant to the regression as if it stands alone. However, some type of stepwise regression should be conducted to actually determine the most significant variables for a regression. However, we will simplify the determination of model by assuming the interaction between separate variables is independent.

In Table 5 we display the QR results that correspond to 95.0% of the 90-day rate level risk driver L^{90} mentioned in above. This quantile regression has a R^1 value of .59097, and the Wald Estimator is 303,980, which indicates a significant regression model. Note in column one that the epoch corresponds to the coefficient subscript. Also note that Epoch 0 is the intercept. The other coefficients correspond to the value of the specific risk driver at different years. In the second column, we have the actual QR coefficients. The third column is the impact statistic from Formula 9 where we see the impact of $B_{i,q}$ contributes to the performance of $Quant(R_q)$ as seen in Equation 7. In the fourth column, the Wald estimator is derived and indicates the significance

Time	Coeff.	Scaled Coeff.	Wald Estimator	Significant Scaled Coeff.
0	0.1351200	0.000000	2840.2000	0.000000
1	0.5265400	0.168770	78.9970	0.168770
2	0.4855200	0.155630	150.5600	0.155630
3	0.0634770	0.020347	1.6159	0.000000
4	0.0888490	0.028479	4.2235	0.028479
5	0.2013800	0.064551	10.9040	0.064551
6	0.1457100	0.046706	4.6622	0.046706
7	0.1910000	0.061223	6.3278	0.061223
8	0.0514230	0.016483	0.6452	0.000000
9	0.0776180	0.024879	1.3946	0.000000
10	0.1870800	0.059966	10.5170	0.059966
11	0.3115700	0.099868	21.1460	0.099868
12	0.1270100	0.040710	4.7210	0.040710
13	0.1975500	0.063320	5.1211	0.063320
14	0.3103300	0.099472	14.0660	0.099472
15	-0.0191380	-0.006134	0.0524	0.000000
16	-0.0115500	-0.003702	0.0197	0.000000
17	0.0218980	0.007019	0.0919	0.000000
18	0.0589330	0.018890	0.6675	0.000000
19	-0.0105470	-0.003381	0.0560	0.000000
20	-0.0326600	-0.010469	0.3677	0.000000

Table 5: 90-day Rate Sensitivity for LOB04 1992 Adjusted EVAS

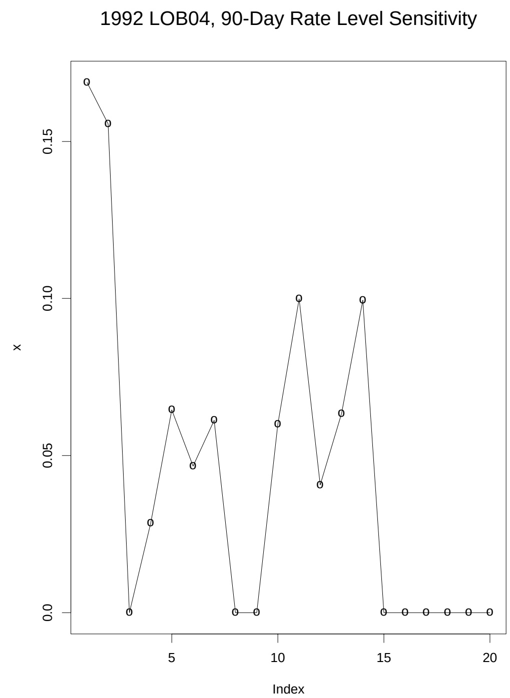

Figure 11: 1992 LOB04 QR Risk Driver Sensitivity

of the specific coefficient in the second column. Since the Wald Estimator is $\chi_1^2(\nu = 1)$ the significant value for a χ_1^2 distribution at 95% is 3.84. However, we will use 4.00 to indicate significance, which is more conservative.

Finally, in column five, the S_i is copied if the Wald estimator is significant. Note that Figure 11 is a graph of this column.

Note that LOB04 in 1992 is highly sensitive in Years 1, 2, 11, and 14, with the impact statistic being 16.9%, 15.6%, 10.0%, and 9.9%. Since our analysis is on the 95% quantile of the adjusted EVAS, the value Equation 7 will move further right if level of the 90-day rate is high. This implies LOB04 is at risk in high rate levels in the above years.

We use the following convention to indicate risk and its impact: "sign year (impact)." For example, the entry in Table 38, on page 158 for LOB42 in the second lowest % is "-4(11.4)." The sign indicates the direction of the risk. If positive, it indicates that the response $Quant(R_q)$ in Equation 7 on page 45 degrades if the level of X_i where $i = year$ is high. If negative the R_q degrades if the level of X_i is low. Here "4" indicates the year that the R_q is most sensitive.

At this point we need to discuss the effect of the X_i on the R_q. We assume that we are examining R_q on actual EVAS values and not the reversed adjusted EVAS values. If the X_i does not take on negative values, the risk analysis on the R_q is more complex. In this situation, we must examine the $B_{i,q}$ with both the lowest negative coefficients and the highest positive coefficients. Here if the level of X_i is high and the corresponding $B_{i,q}$ is negative then R_q degrades, otherwise if the level of X_i is low, R_q improves. If the level of X_i is high and the corresponding $B_{i,q}$ is positive then R_q improves, where R_q degrades if the level of X_i is low.

However, if X_i does take on negative values, the risk analysis is somewhat simplified. At this point, we need only to examine the large $|B_{i,q}|$ when using the risk direction mentioned above. Here if X_i is positive and $|B_{i,q}|$ is large and $B_{i,q}$ is negative, R_q declines. If X_i is negative and $B_{i,q}$ is positive, R_q also declines. By using the risk direction, we do not need to think about the relationship between X_i and its corresponding $B_{i,q}$.

Following is the QR analysis for all of the 10,000 adjusted EVAS values for all of the lines of business and for all years. At this point we examine the ninety-fifth percentile, which as we mentioned before is the risky fifth quantile for the actual EVAS values. However, we use the sign of the risk to denote the direction of risk as if we are working with the actual EVAS values. By this means we can actually see the risk exposure. Note: if fields in the tables are blank, there were no significant variables.

The ninety-day rate analysis is in Tables 38, 39, and 40 on pages 158, 159, and 160 respectively. Since the level of the ninety-day rates cannot be negative, we must examine the largest positive and smallest negative coefficients separately. Also, we must examine the significance of the specific coefficients because the lowest negative coefficients (as in the 1992 LOB04 line of business) or the highest positive coefficients (as in the 1992 LOB33 line of business) may be near zero. We observe for instance that the LOB07 line of business has a major risk in the first year's interest rate with its significance level at 31.9%. Note that if the level of the rate is very high the performance of that line degrades. However if the level of the rate is very low as in the LOB09 line of business the benefit associated with the negative corresponding coefficient reduces the influence on the line performance. So this line is sensitive to low rates in that the positive attributes on R_q is reduced. These tables reveal the lines that are sensitive to high versus low interest rate environments. For example, the LOB44 line of business in 1992, we see that the line is exposed to high interest environments in year 2, 3 and 7. This is indicative of the fact that a high environment increases the probability of lapse in this line and in the seventh year a high environment increases lapses due to the fact that the surrender charge period has run off.

Note that the risk exposure changes between years because of the reduction of the interest rate volatility and the change in the underlying business model. For example, look at the performance of the LOB04 line of business through time, we see the high interest rate environment sensitivity is in the early years (1 and 2) in 1992 and later years (12 and 10) in 1993 and returns to early years (3 and 4) in 1994. Recall from Section 3, how the support of the LOB04 line is wide in 1992 then narrows in 1993 and then dramatically rewidens in 1994. At this point we believe the 1993 sensitivity in the later epochs occurs purely because the interest rate volatility is so narrow in 1993.

Similarly the sensitivity of the business to the level of the ten-year rates are in Tables 41, 42, and 43 on pages 161, 162, and 163 respectively.

The change of the ninety-day and ten-year rates in Tables 44, 45, and 46 on pages 164, 165, and 166 respectively indicate how a changing interest rate environment affects the various lines of business. Because the change in interest rates can both be negative and positive, we only use the absolute magnitude of the coefficients in the determination of the reports. Notice for example, how the LOB04, LOB07, LOB10, LOB26, LOB27, LOB28, LOB31, LOB32, LOB44, LOB45, LOB46 and LOB48 lines of business are most sensitive to ninety-day interest rate changes in an upward direction in the first three years for 1992. Note how the LOB33, LOB35 and LOB36 lines are sensitive to changes in the same years in a downward direction.

Finally in Tables 47, 48, and 49 on pages 167, 168, and 169 respectively, we examine how the spread between the ten-year rate and the ninety-day rate affects the lines of business. For example, we can see that the LOB47 line of business in 1992 is at risk in an inverted yield curve environment in the first year. Where other lines strong positive yield curves influence the performance of the business in later years. In the Delta Spread analysis we can examine the effect of spread compression of the Treasury yield curve on the business. For instance in the 1992 LOB33 we see that we are at risk for spread compression in the first three years.

In LOB01 in 1992, we observe that there are no significant variables in the L^{90} and L^{10} impact statistics. However, we see that in the D^{90}, D^{10}, P, and ΔP there are significant variables, even though the R^1 values are all below 1.50%. We can see in QR just as in OLSR, one can have extremely low R^2

values, but significant variables. Even though these significant variables do a poor job of explaining the variability of the R_q, yet the significant variables are important in understanding the modeling of R_q.

The pricing actuary can use this quantitative approach to determine design flaws when examining low quantiles and positive upside design features in high quantiles. The valuation actuary can use this type of report to locate various risks and their location in time in existing lines of business. By aggregating the results, the valuation actuary can also estimate the overall company exposure and determine if the company should increase reserves or enter some form of a derivative hedge. This methodology effectively allows one to understand the various embedded options from the business. Also, a financial engineer could use these methods to understand the mechanics of the hedge backing the modeled derivative.

This methodology examines the effect from all scenarios on the specific R_q. In the past the actuary may have used different deterministic scenarios to determine the direction of the markets that created risk exposure to the business. However, the deterministic scenarios do not indicate the impact or aid in the determination of the exact location in the projection period that the business is at the highest risk.

The above methodology also allows the actuary to conduct risk analysis on several different risk drivers. In fact in the past the actuary did not consider some of the above analyses without extensive additional computer runs. This increased ability may initially raise more questions for the actuary to analyze, but this type of risk analysis appears to be an excellent tool to conduct these analyses.

We do not want to comment on all of the information that is revealed with these QR reports, but we did want to display the information to give the reader an appreciation of the potential of this methodology for determining and analyzing assumptions which are critical risk drivers.

Note: We also conducted the above analysis on the original EVAS values and obtained identical results. This was done to see if the adjusted EVAS linear transformation affected the QR risk driver analysis.

10 Subsampling

The prior section all of the risk QR analysis was conducted using 10,000 scenarios. This yielded such excellent results we will pursue in this section a rough analysis to see if we can determine the necessary number of scenarios to conduct our risk QR analysis.

However, there is the difficulty of determining how many scenarios are needed to obtain this information. A graphical demonstration on addressing the subsample size is shown in Graphs 19, 20, 21, and 22 on pages 123, 124, 125, and 126. Notice that the coefficients stabilize as the sample size increases. In each of these graphs we have graphed the QR coefficients based on 10,000 samples time series versus that of the subsamples. We can see visually that the coefficients begin to stabilize at 1,500 for this line of business. The first three coefficients have somewhat stabilized between 600 and 700 scenarios. The first four appear to have stabilized at 800 scenarios. However, even if the number of scenarios is small at least the actuary may gain some insight into the major risk periods. However, there is a great deal of noise in the QR regressions.

Below we will conduct two very rough studies to see if the number of required scenarios to conduct the risk QR takes on a stable value. In this analysis, we will not use the R^1 nor the Wald Estimator to determine the regression significance. We will also not use the Wald Estimator to determine the significant variables. Another weakness of this analysis is that we will not conduct a hypothesis test on when the coefficients $B_{i,q}$ can be approximated by a multinormal distribution. Even with these weaknesses we can make several observations.

In the following analysis, we will examine single subsamples of size $\{1000, 1500, 2000, 2500, 3000, 4000, 5000, 6000, 7000, 8000, 9000\}$.

In Tables 50, 51 and 52 on pages 170, 171 and 172, notice that the smallest sample size that passes the χ^2 test. This test is discussed in Appendix C.4, page 99. The QR analysis is on the 95% quantile on the adjusted EVAS values. Note that the subsample size is fairly large. There are only three lines in all three years with a subsample size of 1,000 and only two lines with 2,500 subsamples.

In our analysis of QR in Section 7 the specific years of highest risk are determined by finding the coefficients that are large in absolute value. Possibly, if we take a qualitative approach, we might be able to redeem the disappointing results of the χ^2 subsample test in Tables 50, 51 and 52. Here our interest is aroused if the subsample coefficients are of the same sign as the main set's coefficients, and if the coefficients are substantially away from zero.

We discuss this Rank subsample approach in Appendix C.5, page 100. Tables 50, 51, and 52 shows that the smallest sample size that passes the Rank test with $\epsilon = 0.001, 0.002, 0.003, 0.004, 0.005, 0.006$ and 0.007.

We found that the χ^2 test reveals the lower sample sizes 79 times out of 107 data sets. On the remaining 28 data sets, the Rank test improves over the χ^2 test. However, we believe that these sample sizes are still too prohibitive for most lines of business. Possibly, by using a quantile that is closer to the center of the distribution, we might see an improvement in the sample sizes. Also, due to the fact that the funnel of doubt of the various risk drivers widen in the later epochs, this variability may impact the scenario requirement. Further research needs to be conducted here, especially with the implementation of the R^1, Wald Estimator, and a hypothesis test on the coefficients $B_{i,q}$.

11 Examination of Single Premium Deferred Annuities

Since Robbins et al. [79] used a SPDA contract to conduct their analysis, we will describe in depth the results of the SPDA line of business in the data sets. The SPDA line of business is LOB44. The EVAS files of this line were obtained from the in-force population. These contain multiple contracts with varying initial premium and several different crediting strategies. In Tables 13, 14 and 15 on pages 133, 134 and 135, one can observe that the minimum and the standard deviation of the EVAS values for LOB44 reveal that the business is sensitive to up-interest rate environments. Recall that the starting ninety-day interest rates for 1992 through 1994 are 5.766%, 4.447% and 6.315% respectively. The minimum EVAS value is low in 1992, rises in 1993 and drops further than that of 1992 in 1994. The EVAS variance starts wide in 1992, narrows in 1993 and widens further than that of 1992 in 1994.

In Tables 19, 20, 21 and 22 on pages 139, 140, 141 and 142, we observe that the GB2 model is not adequate for all years and across all random studies for the SPDA line.

Robbins et al. [79] have $\alpha = 0.692$, $\gamma = 3.221$, $\lambda^{-1} = 17,539$ and $\tau = 3.454$. We observe that the γ and the τ parameters in all three years are of the same order of magnitude as that of Robbins et al. [79]. The dramatic difference between our λ values with Robbins et al. is due to the fact that the support of our adjusted EVAS values is very small. It appears that λ^{-1} has some correlation to that of the size of the distribution's support. We are unable to compare the relevant α values.

In Tables 23, 24, and 25 on pages 143, 144, and 145, reveal that the REV method does not model the extreme tail of the SPDA lines well.

Next we will examine the associated sensitivity QR Tables for the SPDA line. Here we will use the sign to indicate the risk direction.

We see that the SPDA is sensitive to the level of the ninety-day rates in years 2, 3, and 7 in 1992, years 1, 7, and 6 in 1993 and years 6, 5, and 1 in 1994.

SPDA is sensitive to the level of the ten-year rates in years 2, 3, and 1 in 1992, years 1, 7, and 2 in 1993 and years 1, 5, and 6 in 1994.

SPDA is sensitive to the change in the ninety-day rates in years 1, 2, and 3 for all three years.

SPDA is sensitive to the change in the ten-year rates in years 1, 2, and 3 in 1992, years 1, 2, and 4 in 1993 and years 1, 2, and 3 in 1994.

SPDA is sensitive to the level of the ten-year to the ninety-day rates spread in years 11, 13, and 9 in 1992, years -13, -11, and 1 in 1993 and years -4, 5, and 1 in 1994.

SPDA is sensitive to the change in the ten-year to ninety-day rates spread 2, 1, and 3 in 1992, years 1, 2, and 3 in both 1993 and 1994.

We believe that the above risk QR analysis reveals:

1. The SPDA line is very sensitive to early years in the projection. This is due to the fact that the asset performance of the investment of the initial premium is very sensitive to the level of the credited rate when to compared to that of the competitor's rate which is a function of the new money rates.

2. The SPDA line is sensitive to when the surrender charge period ends, which is seven years.

3. The SPDA line is sensitive to the initial rates and the associated interest rate volatility

We are unclear to the reason why the business is sensitive to yield curve inversion in years 13, 11, and 4.

We found that the R^1 hovered about 0.70 for all of the design matrices except for the spread and delta spread design matrices which was at 0.27. This indicates that 70% of the variability of the 95'th quantile of the adjusted EVAS is explained by the levels and the changes of interest rates, only 27% of the variability of the adjusted EVAS is explained by the spread and spread compression.

In the percentile comparison Tables 26, 27 and 28 on pages 146, 147 and 148, the REV estimates for the ninety-fifth percentile are consistent with the actual order statistic confidence intervals for all years. None of the other methodologies had reasonable results.

In the percentile comparison Tables 29, 30 and 31 on pages 149, 150 and 151, the REV estimates for the ninety-ninth percentile are consistent with the actual order statistic confidence intervals for only 1992. None of the other methodologies had reasonable results.

We found that the ninety-fifth percentile of the adjusted EVAS values is below 0.6 for year 1992 and 1993 but above 0.6 in 1994. The ninety-ninth percentile is below 0.6 for 1992 but above in 1993 and well above in 1994.

x	ECDF	GB2	REV
0.2	0.0001	0.0024	
0.3	0.1445	0.1256	
0.4	0.4441	0.4520	
0.5	0.6695	0.7157	
0.6	0.8328	0.8579	
0.7	0.9276	0.9271	.931
0.8	0.9732	0.9609	.972
0.9	0.9918	0.9780	.989
1.0	0.9982	0.9871	.995

Table 6: Comparative F(x) Results between the GB2 and REV Methodologies

This indicates that the probability of ruin is below one percent in 1992 and above one percent but less than five percent in 1993 and the probability of ruin is above five percent in 1994.

A comparison between the GB2 and the REV methodologies and the ECDF of the 1994 SPDA adjusted EVAS values is in Table 6. Recall that the REV only replicates the tail and not the entire support. Notice that the REV methodology is close to the ECDF, but it does not pass the KS goodness of test in Table 25 on page 145.

12 Data Dependency

The issue of data dependency affects not only the results of how well a model fits the observed data but also the algorithms that are used to obtain the parameters for the model. The stability of the solution of the associated parameters can be dramatically affected if an extreme data value is encountered within the sample used to create the model. For example when using the mean to measure the location of a distribution, extreme sample values will cause the estimator to move dramatically. Extreme values do not affect the results if the median is used to measure the location of the distribution. This is because the ranking of the samples determines the location of the median. The difficulty in insolvency studies is that extreme samples are the very values that one wants to examine and model. However, as these extremes are encountered, they also impact the model smoothness and accuracy of the fit. What one hopes when dealing with overall model accuracy is that the model's parameters do not shift dramatically when an extreme sample is encountered in the fitting algorithm.

The literature surrounding the concept of designing robust estimation processes has been extensive. See Venables and Ripley [101] for a list of references and a further discussion of robust estimators.

Finding a stable estimate of the GB2 parameters as discussed in Section 5 is fraught with frustration and difficulty. The need to conduct extensive subsample testing is required to prove this data dependency. We did not attempt this due to the time consuming nature of locating stable parameters estimates. However, from examining the solutions from the different random simplexes we are inclined to conclude that either the algorithm or the actual GB2 distribution is very data dependent.

In an unpublished study conducted several years ago we found that the REV algorithm showed a weak dependency upon the values sampled.

Koenker and Basset [44] construct the entire concept of quantile regression around the concept of robust estimation. This at least deals with the problem of outliers creating misleading models, but however, there is a form of data dependency that occurs within the QR environment. This is the inherit assumption that the conditional quantile of the error term of the regression is data independent. This can be stated as follows:

$$Quant(\mathbf{0}|x) = Quant(\mathbf{0}) \tag{10}$$

with probability $= 1$, which says that the quantile of the error term u_t at $\mathbf{0}$

is independent of the value of x. See Appendix B.2 for the use of notation. Buchinsky [7] discusses a method to test for this independence. We do not address this issue in our QR analysis.

The entire issue of algorithmic data dependency in insolvency testing requires more extensive research.

13 Conclusions and Future Research

We will next develop a list of strengths and weaknesses of each of the methodologies and finish with a list of further research topics and concluding remarks.

13.1 Strengths and Weaknesses

The strengths of the GB2 methodology are:

1. Models the entire support.
2. Follows traditional ruin theory.
3. Contains many families of familiar distributions

The weaknesses are:

1. Complex.
2. Not tied to the input scenarios.
3. Does not model the tail well.
4. Has a single mode.
5. Has only a positive support.
6. Difficult to obtain a stable solution.
7. Difficult to model when the τ parameter exceeds 15.
8. Little or no insight gained from the values of the parameters.
9. Difficult to construct confidence intervals. This is due to fact that the confidence interval estimate is dependent upon a complex interaction of the four parameters.

The strengths of the REV methodology are:

1. Models extreme tail.

2. Simpler to model. Just sort and resample the normalized distances

3. Less information is injected into the process because it is semiparametric.

4. Easier to construct confidence intervals. One just has to use a higher percentile for each order statistic instead of just the median.

The weaknesses are:

1. Not tied to the input scenarios.

2. Does not model the entire support.

3. Not a traditional risk or ruin approach.

4. Requires a large amount of data (sample size = 1000).

5. Overly conservative in reserves.

6. Does not accurately model lighter or heavier than exponential tails.

The strengths of the QR methodology are:

1. Fast. We were able to conduct 642 (6x107) regressions under 2 hours.

2. Ties the input scenarios to the output.

3. Sign and magnitude of coefficients give insight into risk exposures.

4. Targets specific regions of the output's behavior.

The weaknesses are:

1. Complex.

2. Poor residual distribution.

3. Does not model actual quantiles, just the conditional quantiles.

13.2 The Need of a New Paradigm?

We observe that GB2 represents the traditional risk/ruin theory of fitting distributions, that REV represents the use of extreme value theory and that QR represents a simplified linear model with emphasis on various quantiles. We see with large sample sizes that the traditional method of risk/ruin theory using the richest known distribution fails miserably. We also see that there are difficulties with the extreme values methodology when considering large sample sizes as well.

The development of a new paradigm should consider at least the following:

1. Input/Output linkage.
2. Being able to target various areas of the output distribution.
3. Use Extreme Value Theory to improve the understanding of the extreme exposures.
4. Expanded Goodness of fit tests for models.
5. Speed and ease of use.
6. Tests to determine the scenario adequacy issue.

Appendix E give a survey of known or potential modeling environments that may allow for this new paradigm.

13.3 Future Research

In the process of examining the three methodologies we have found the following questions and areas that require further development and research:

1. An anonymous referee observed that since each of the data sets used herein contained 10,000 points, there may be justification to censor these data sets to reduce the variation of the observations. Future research in censored quantile regression in Powell [70] and chapter 15 of Maddala and Rao [58] merged with the new QR algorithm of Portnoy and Koenker [69] may allow one to more accurately model various quantiles of the corporate models without having to make adjustments to the data sets.

2. Can we use the concept of the entropy of a distribution and/or use information theory to revise our current paradigm for the modeling of insolvency?

3. What is the relationship between the level of confidence and the extreme percentiles when used in insolvency analysis?

4. What nonnegative weight functions ψ should be used to improve on goodness of fit tests, such as the KS test? The improvements must allow the analyst to control model smoothness (or information insertion) over the accuracy of results. Robbins et al. [79] and Craighead [15] have already created the following three possibilities:

   (a) See how well a model matches on just a subsample of the entire data set.

   (b) Choose only the key values where accuracy is the most important and test how well a model matches on those values.

   (c) Use a binning method. This method requires the use of a parameter to determine the number of bins. Once binned, compare the model results to the ECDF of the binned data.

   The research required is to determine the respective probability equation (that corresponds to Equation 48 on page 96) for each of these approaches.

5. Should other goodness of fit tests, other than the KS test, be used to analyze the data? Examples such as the Cramér-Von Mises Statistic, the Kulper Statistic, the Watson Statistic, and the Anderson-Darling Statistic show promise.

6. Should there be some revision to the MLE, that also allows the analyst control over the smoothness versus accuracy issue?

7. How does one create confidence intervals on the output of a QR analysis?

8. Is there a method to reduce sample sizes required with QR tests? Possibly either by using a quantile that is closer to the center away from the extremes or R^1 tests we might see an improvement in the sample sizes.

9. Can the use of extreme value theory be used to improve the performance of either the GB2 or the QR process? Can we use the CS ratio and determine the tail type of the surplus output and improve the model?

10. Can the use of Low Discrepancy Sequences be merged with any of the methodologies and improve the performance of these methodologies?

11. Can mixture models be used to improve any of the different methodologies?

12. In QR analysis there is a finite set of separate regressions that can represent all the quantiles. Can this fact be used to better model the input and output relationship?

13. Can QR analysis be used to help model other complex surplus models such as target surplus?

14. Can QR analysis be used in modeling complex exotic derivatives? Can it help in the design of the dynamic asset strategy?

15. Can a test be developed that measures the level of data dependency within the separate methodologies? Can this be combined with the balancing act between smoothness and accuracy?

13.4 Concluding Remarks

We have taken 107 separate data sets representing the surplus performance of 51 lines of business separately over a three-year period. Each of these data sets contains 10,000 distinct values that were obtained from an extensive and expensive series of corporate studies. Our position is to believe that the 10,000 EVAS values contained in each of these files is the best representation of the actual information. We proceed to examine these data sets and examine a process by which we can adjust these EVAS values to be able to use the three modeling methodologies. We examine how well our three methodologies model the data for each line of business for each year. Comparing the results of the first two methodologies directly against the empirical cumulative distribution function with a KS test does this analysis. Here we found that these methodologies did not compare well at all. We next examined the methodology of quantile regression and found that it gave us several methods to compare the effect of the input scenarios against the computer model output results. Here we developed several reports that can indicate the interrelationships between the input and output. We next examine how well specific percentiles are estimated by all three methodologies. Here we find that the REV seems to outperform the other methods, however with many limitations. Next, we develop QR reports that reveal risks at specific times for the different lines of business. Then we examine the performance of the specific SPDA line of business under each methodology and across all years. After a discussion of whether the methodologies are data dependent, we create a list of strengths and weaknesses of each methodology, and discuss the need for a new paradigm for large sample analysis. Our final conclusions are that the data itself is the still the best representation of the contained information, and that one can use the QR methodology to determine the location and impact of specific risk drivers.

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Patricia, you are my better three-fourths. Thanks for your patience and willingness to help oversee the children, pets and house while I just sat and wrote and wrote and wrote. In no way this research could have been accomplished without you. I truly adore you for your love and sacrifice.

Finally, I dedicate this paper to my late father-in-law, John Robert Bird, Jr. We sorely miss you and fondly recall your influence on all of our lives.

A Interest Rate Scenarios

A.1 Formulas Used in Interest Rate Scenario Generation

The generation of the yield curves used in the interest rate scenarios is not arbitrage free. This requires setting up a diffusion process of state variables and making sure that the various par bond prices are consistent with the resultant bond pricing partial differential equation. Instead, we used a two-factor model with a lognormal diffusion process on the short-rate (ninety-day) and a lognormal diffusion process on the long-rate (ten-year). This model does not have mean reversion and has fixed boundaries above and below. These fixed boundaries are not reflecting. See Tenney [94, 95, 96] for a further discussion of the behavior and requirements of good interest rate generators.

Below we use the notation Ym_t, where m denotes the maturity of the interest rate on the yield curve and t denotes the time epoch. The only exception of this notation is that we use Y_t^{90} to denote the value of the ninety-day rate instead of $Y.25_t$. Note $m = \{1, \ldots, 20\}$

First obtain the initial yield curve. Set Y_0^5 to be Constant Maturity Treasuries (CMT) five-year interest rate for the last day of the year and calculate the ninety-day rate to be

$$Y_0^{90} = Y_0^5 e^{\mu_{90}}. \tag{11}$$

μ_{90} and σ_{90} and σ_{10} are based on a historical lognormal analysis of the short and long rates. We assume below that μ_{10} is zero.

With maturity m, we use the log regression formula

$$N(m) = 1.349ln(2m+1) + 1.051ln(m+1) \tag{12}$$

to assure a "nice" positive or inverted yield curve. This formula precludes the possibility of humped yield curves.

Define the spread slope constant

$$C = (Y_0^5 - Y_0^{90})/N(5). \tag{13}$$

Letting m range from one to twenty, we obtain the entire initial yield curve from

$$Ym_0 = Y_0^{90} + CN(m). \tag{14}$$

For time $t > 0$, the subsequent yield curves are based on a lognormal diffusion processes of the ten-year rate and the ninety-day rate as follows. The ten-year rate is projected as follows:

$$Y_{t+1}^{10} = Y_t^{10} e^{\sigma_{10} Z_{10}}. \tag{15}$$

The ninety-day rate is projected as:

$$Y_{t+1}^{90} = Y_t^{90} e^{\mu_{90} + \sigma_{90} Z_{90}} \tag{16}$$

Z_{90} and Z_{10} are uncorrelated standard normal samples.

These values are then bracketed. The ninety-day brackets are 0.5% and 20% and the brackets of the ten-rates are 1% and 25%.

However, in the belief that inverted yield curves are only observed in a rising interest rate environment, if the yield curve is inverted and the rates are falling (measured by the fact that $Y_{t+1}^{90} > Y_{t+1}^{10}$ and $Y_{t+1}^{10} < Y_t^{10}$) then the Y_{t+1}^{90} is adjusted to be:

$$Y_{t+1}^{90} = Y_{t+1}^{10} e^{\mu_{90}} \tag{17}$$

This new value of Y_{t+1}^{90} is then bracketed as before.

Now define the spread slope constant

$$C = (Y_{t+1}^{10} - Y_{t+1}^{90})/N(10) \tag{18}$$

and obtain the entire yield curve by interpolating by the following formula:

$$Ym_{t+1} = Y_{t+1}^{90} + CN(m) \tag{19}$$

A.2 Empirical Analysis of Interest Rate Scenarios

The graphics in this section use boxplots. See Appendix D.1 and Appendix D.2 for a description of these types of graphs as well as a brief discussion of the concept of the Funnel of Doubt (FOD).

There are 10,000 different scenarios each for 1992, 1993, and 1994. Each interest rate scenario is a 21 by 21 matrix of interest rates. A single row in the matrix represents a yield curve where the first entry represents the ninety-day T-bill rate in bond equivalent yield (denoted BEY) format. The second entry is the one-year constant maturity treasury (denoted CMT) bond in BEY format. The third entry is the two-year CMT in BEY format. Continuing in this fashion, finally the twenty-first entry is the twenty-year CMT in BEY format.

We only examine the ninety-day rate and the ten-year rate that correspond to these scenario files.

Figures 12, 13 and 14 on pages 81, 82, and 81 are displays of the FOD of the ninety-day rates. See Tables 7, 8 and 9 on pages 127, 128 and 129 for the corresponding tables of statistics for these rates. Observe the widening of the dispersion of as time passes (time is on the x-axis of the graphs and denoted as *Epoch* in the tables). Notice that the median appears to remain constant through all the epochs. Note from the Tables that the mean is increasing. Also, notice that the outliers fall above the upper whiskers. This implies that distribution of interest rates is right skewed, and not a Normal distribution. This is of course consistent with the fact that the interest rates are generated from a lognormal process, which is formulated in Appendix A.1. Notice also from Tables 7, 8 and 9 that the starting rates for the interest rates are 5.766%, 4.447%, and 6.315% are for 1992, 1993 and 1994 respectively. Since the formula of the variance of a lognormal distribution contains an expression including the mean of the distribution, this implies that the higher the mean or the starting position of the rates the higher the volatility of the interest rates. Notice that the interquartile distance narrowed between 1992 to 1993 and then rewidened from 1993 to 1994. Also, observe that there is a 25% maximum limit placed on these ninety-day rates. Note that there are no downside outliers on these rates.

Figures 15, 16 and 17 on pages 84, 85 and 86 are displays of the FOD of the ten-year rates. See Tables 10, 11 and 12 on pages 130, 131 and 132 for the corresponding tables of statistics for these rates. The volatility behavior for the ten-year rates corresponds with the ninety-day rate volatility observations. This of course is due to the fact that the starting values of the ten-year rates are at 6.7%, 5.83% and 8.094%. Note the severity of the volatility in the 1994 set that the maximum of 20% collapses all the outliers down to just the whiskers in the last three epochs.

As we saw in Section 3, the overall increase of the volatility of the interest rates gives rise to a dramatic increase in the volatility of the surplus values.

A.3 Reduction of Interest Rate Scenarios for QR

Each a scenario is represented by a matrix which contains 441 different values. There are 441 values because of 21 epochs (time 0 through time 20) times 21

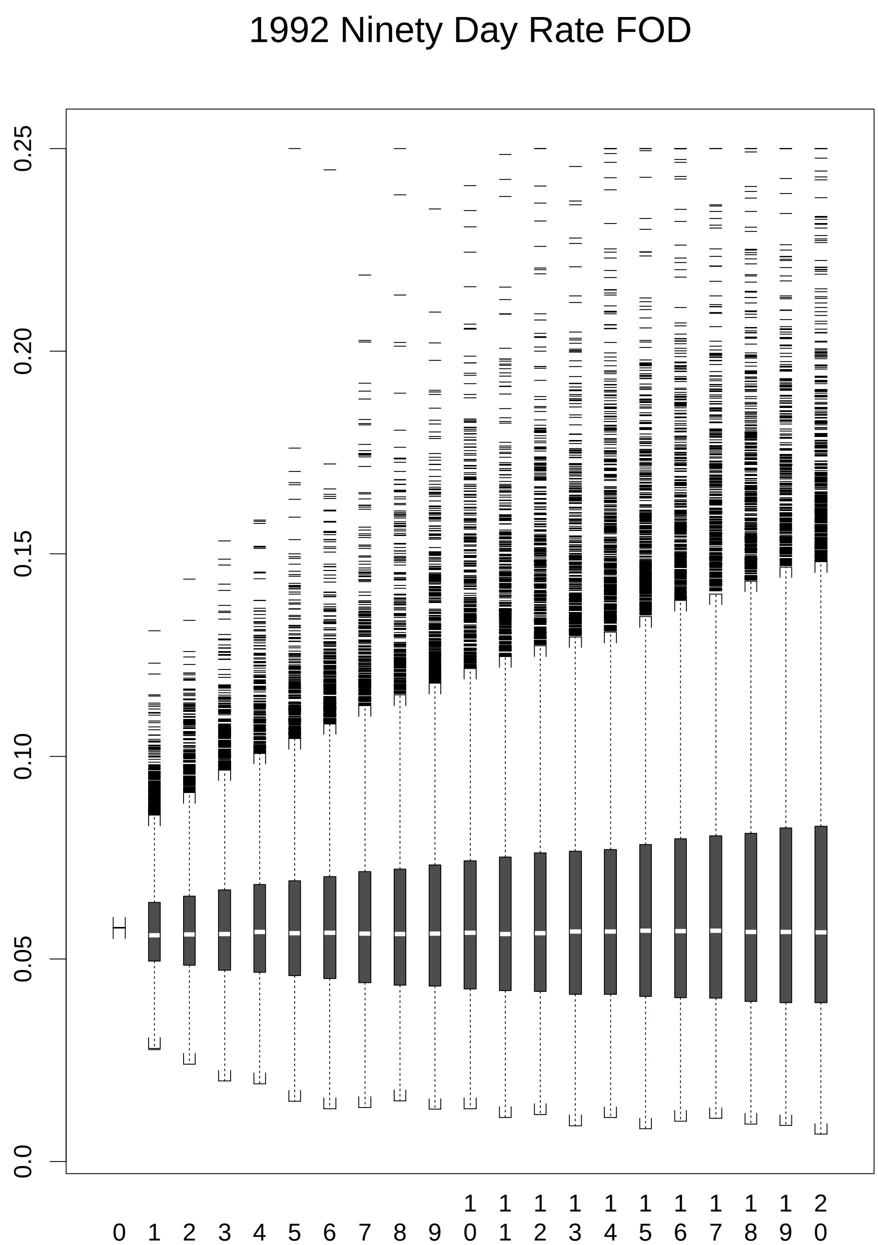

1992 Ninety Day Rate FOD
0.0
0.05
0.10
0.15
0.20
0.25
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Figure 12:

1993 Ninety Day Rate FOD

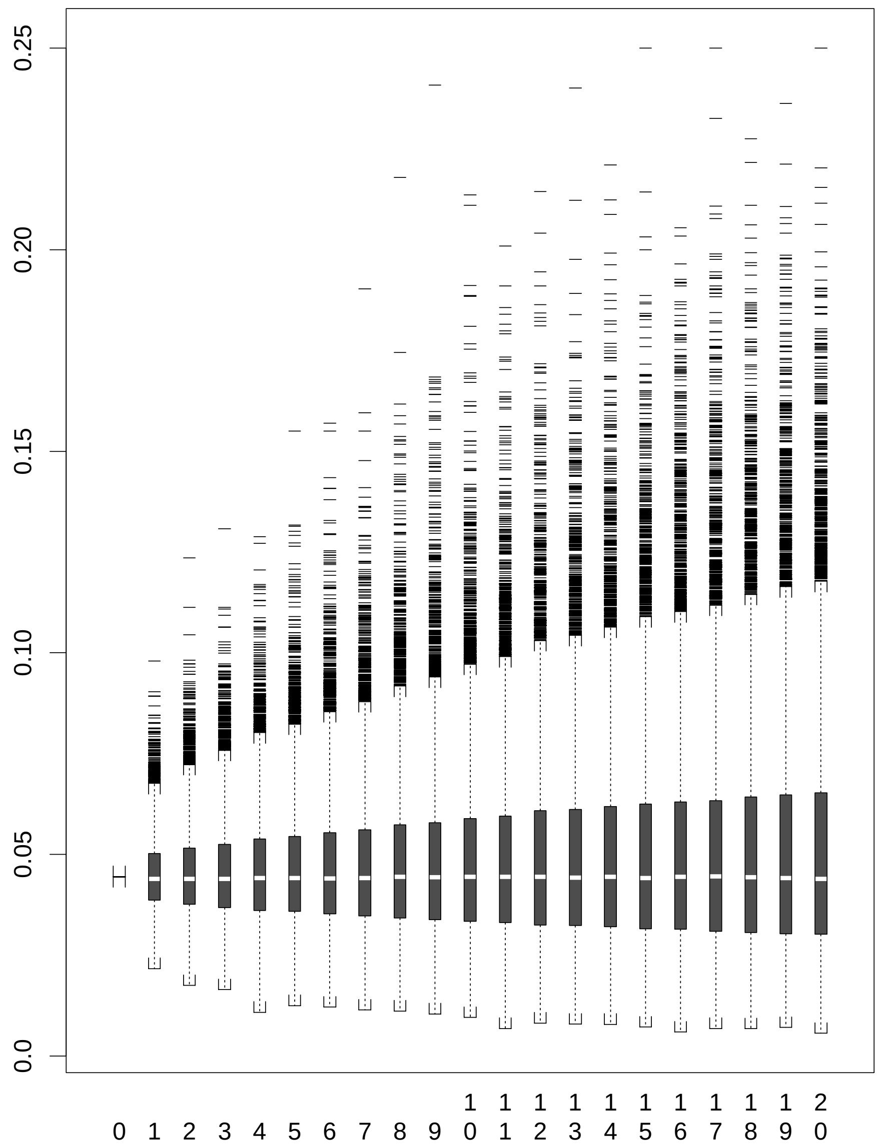


Figure 13:

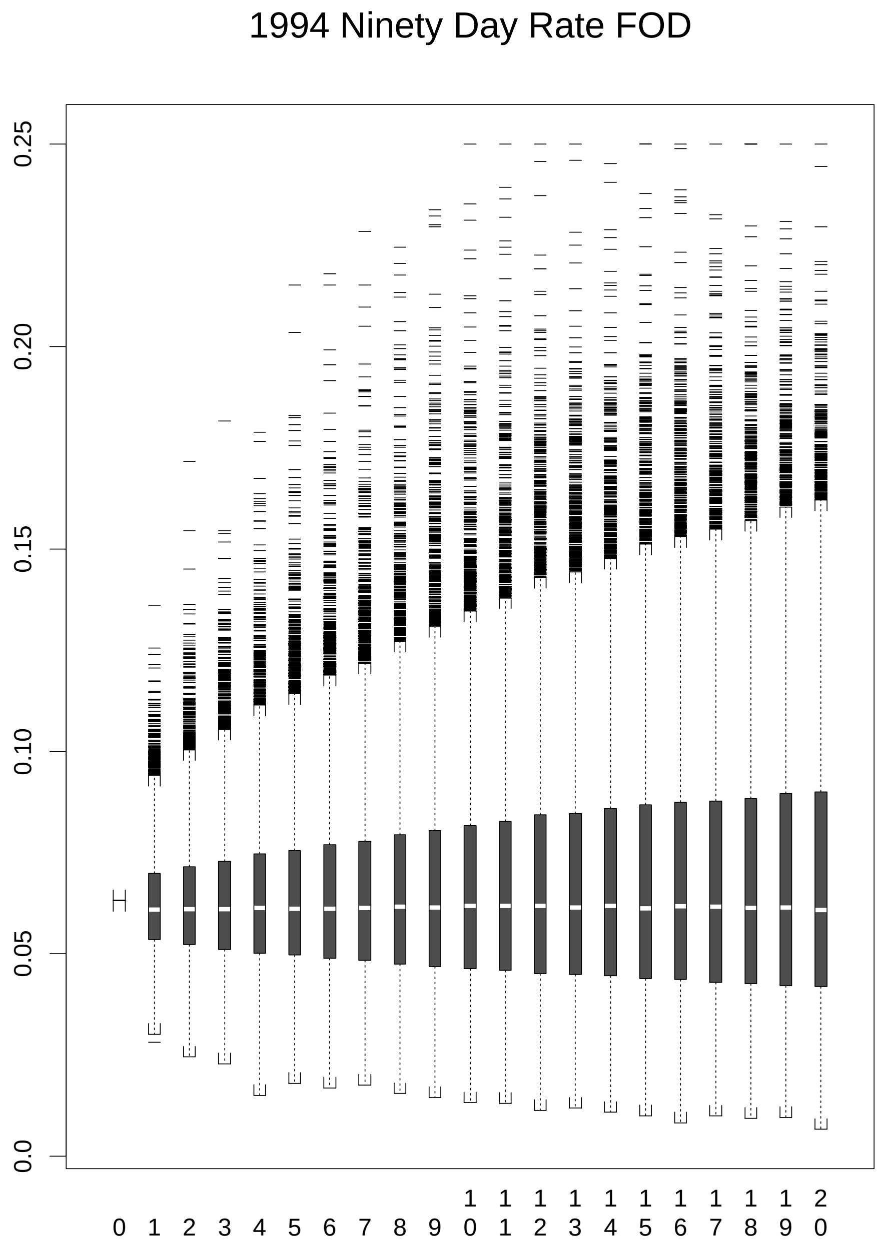

1994 Ninety Day Rate FOD
0.0
0.05
0.10
0.15
0.20
0.25
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Figure 14:

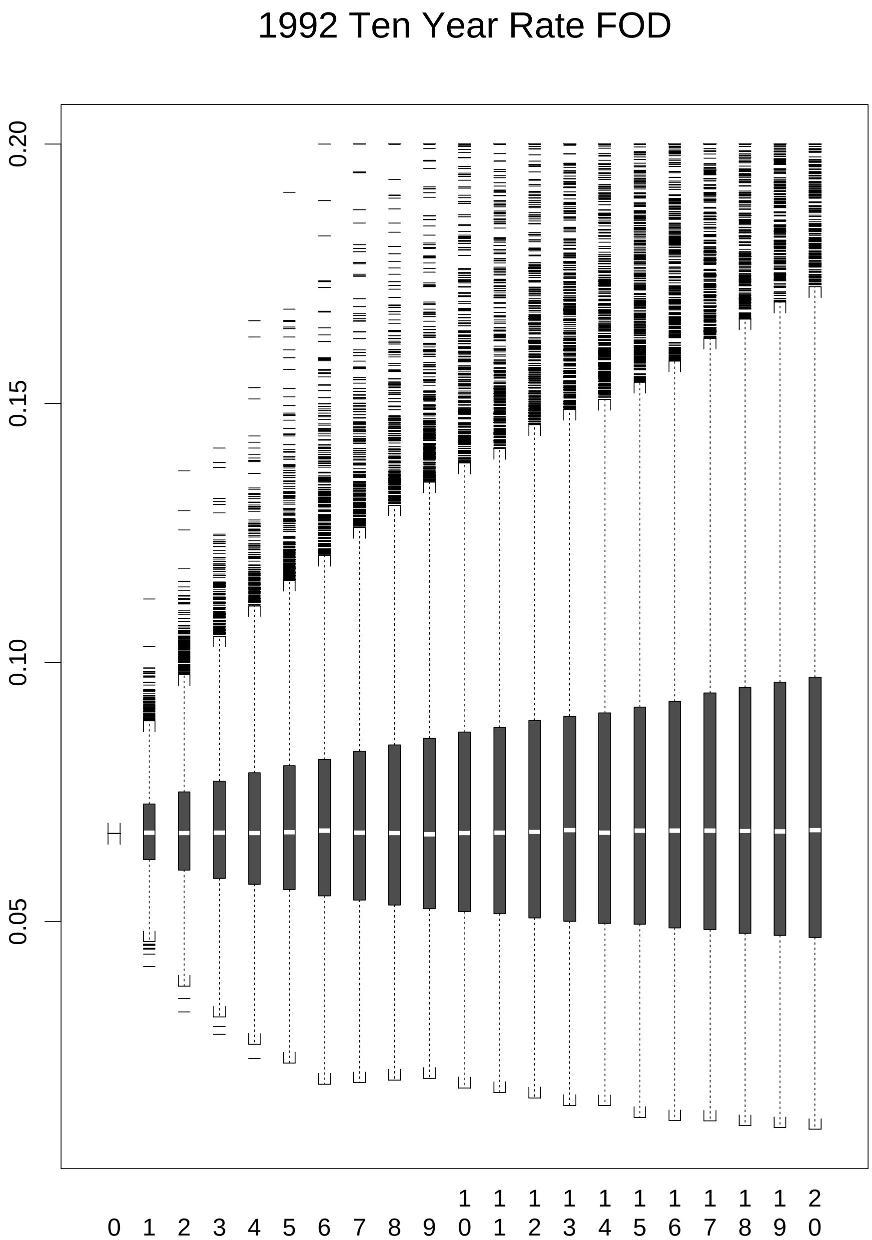

1992 Ten Year Rate FOD
0.20
0.15
0.10
0.05
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20

Figure 15:

1993 Ten Year Rate FOD

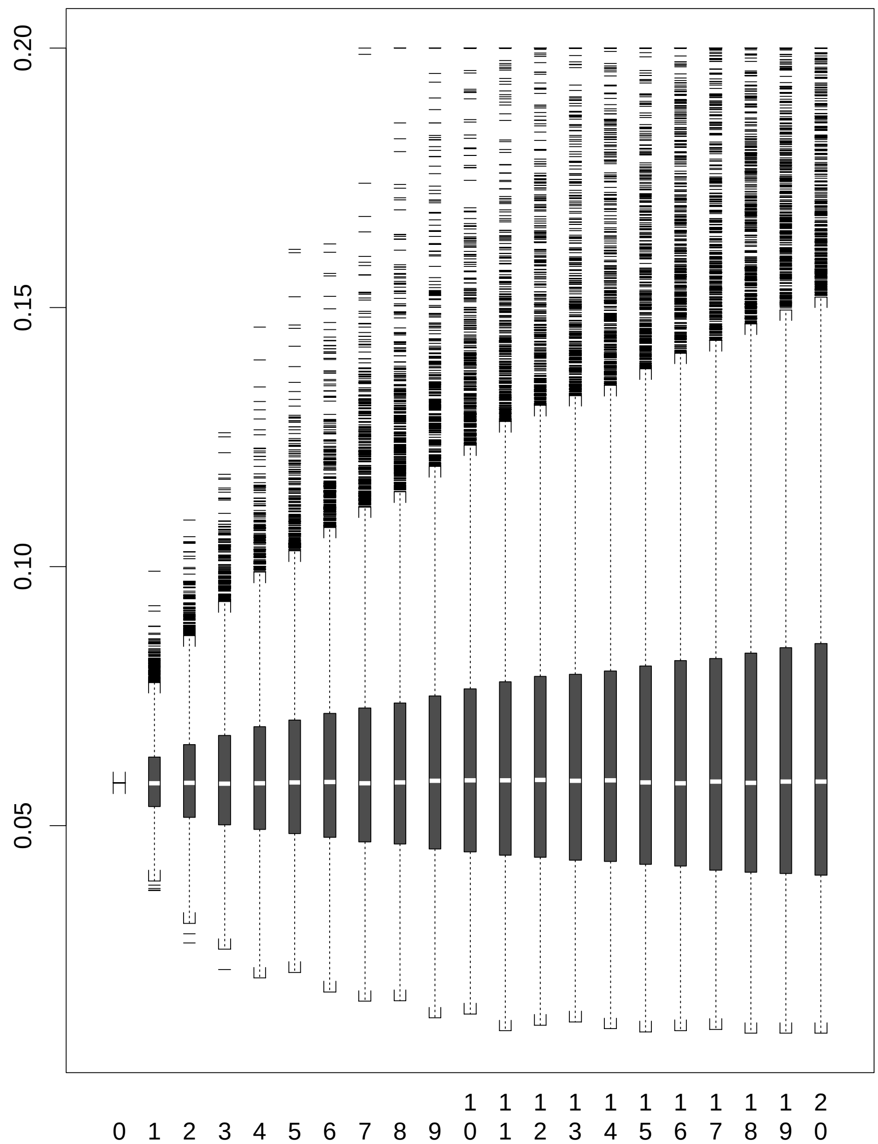


Figure 16:

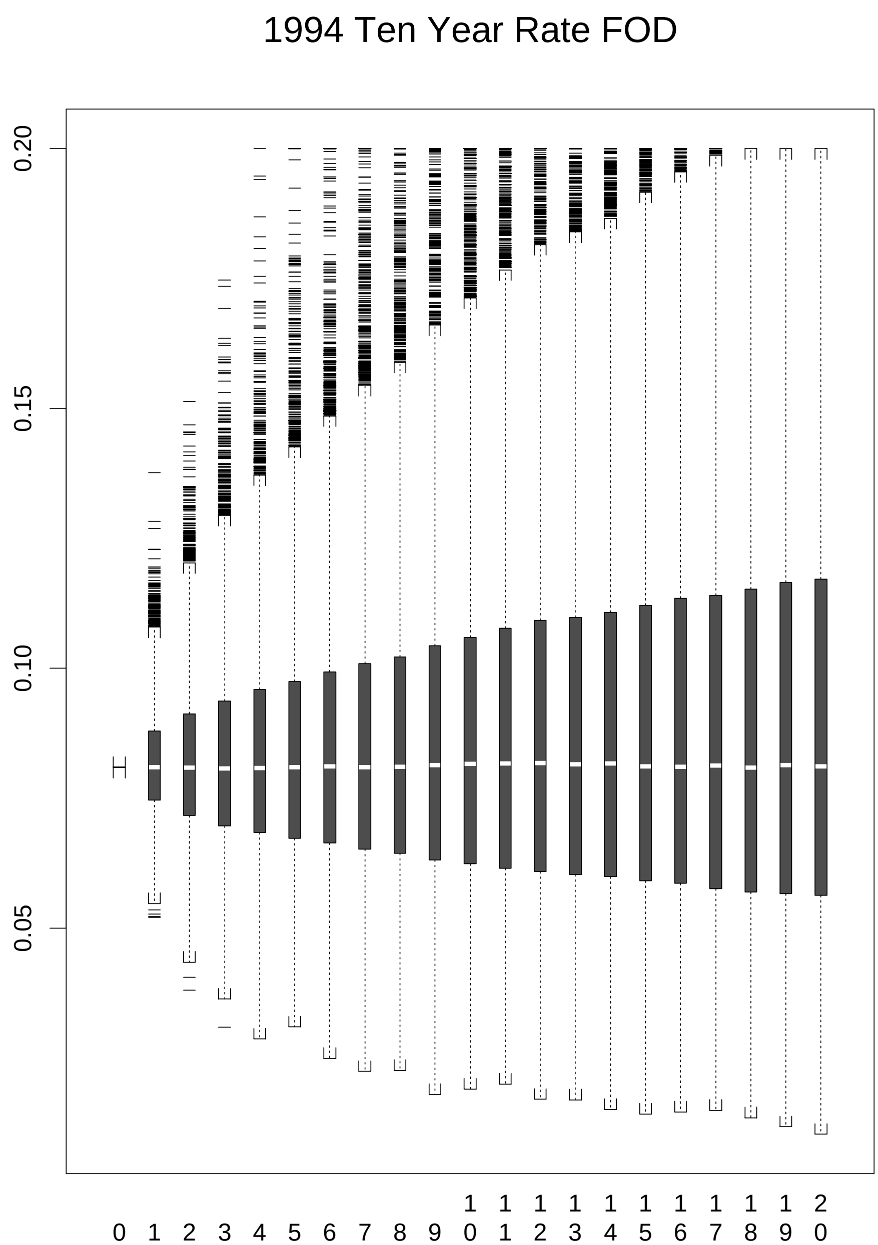

1994 Ten Year Rate FOD
0.20
0.15
0.10
0.05
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20

Figure 17:

maturities (90-day rate, 1-year rate,$\cdots$,20-year rate). A subsequent quantile regression analysis with a design matrix with 441 columns is very difficult to conduct on 10,000 scenarios. The entire scenario generation process is based on the diffusions of the ninety-day and the ten-year rates results from the initial five-year rate. Because of this relationship, most information for a specific scenario can be contained in a vector of interest rates at the ninety-day and ten-year rates through time. The only information that is lost is when the maximum rate restrictions (recall this is 25% for the ninety-day rate and 20% for the ten-year rate) are applied to other maturities than the specified maturity vector. This information loss is not a major influence since the restrictions occur in the later epochs of the scenario. On the average also, the heavy discounting of the cashflows in these later years may help to minimize the influence these high rates have on the overall EVAS values. These influences cause little to no actual information loss when using a single vector of rates to represent the scenario's overall attributes.

B Methodologies

B.1 Generalized Beta Type 2 Distribution

Robbins et al. [79] use the Generalized Beta Type 2 distribution to examine methods to reduce scenario requirements. Also see Cummins et al. [17] and McDonald and Richards [54] for possible other applications of the use of this distribution in insurance models. Also see Klugman, Panjer, and Willmot [51] who also examine the GB2 distribution, which they call the transformed beta distribution.

The GB2 distribution has four parameters: $\alpha > 0, \gamma > 0, \lambda > 0$, and $\tau > 0$.

Its probability density function is

$$f(x) = \frac{\lambda^{\tau\alpha}\tau x^{\tau\alpha-1}}{B(\alpha,\gamma)(1+(\lambda x)^{\tau})^{\alpha+\gamma}} \tag{20}$$

Its log likelihood function is

$$l = nln(\frac{\lambda\tau}{B(\alpha,\gamma)}) + (\tau\alpha - 1)\sum_{i=1}^{n} ln(\lambda x_i) - (\alpha+\gamma)\sum_{i=1}^{n} ln(1+(\lambda x_i)^{\tau}). \tag{21}$$

Its cumulative density function (CDF) is

$$F(x) = \frac{z^{\alpha}}{\alpha B(\alpha,\gamma)} {}_2F_1(\alpha, 1-\gamma, \alpha+1; z) = I_z(\alpha,\gamma), \tag{22}$$

where $z = \frac{(\lambda x)^{\tau}}{1+(\lambda x)^{\tau}}$ and ${}_2F_1$ is the generalized hypergeometric series and I_z is the incomplete Beta series. Here

$$\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx \tag{23}$$

and the beta function is defined as

$$B(\alpha,\gamma) = \frac{\Gamma(\alpha)\Gamma(\gamma)}{\Gamma(\alpha+\gamma)}. \tag{24}$$

From Abramowitz and Stegun [1] and Press et al. [72], the formula for the incomplete beta function is

$$I_z(\alpha,\gamma) = \frac{1}{B(\alpha,\gamma)} \int_0^{z} t^{\alpha-1}(1-t)^{\gamma-1} dt \tag{25}$$

where $0 \leq z \leq 1$. The corresponding formula as given by Robbins et al. [79] in their Appendix 4 is incorrect.

Using code for the incomplete Beta function from Press et al. [72], The author was able to create the necessary code to evaluate the GB2 distribution except when τ is very large. When τ is large the relationship $z = \frac{(\lambda x)^\tau}{1+(\lambda x)^\tau}$ creates an unnatural discontinuity when using a computer to evaluate the expression. When using single precision computations, if the value of z moves from .9999999 to 1.000000, the resultant CDF appears to have a discontinuity. This also occurs when using double precision. In the search for the best parameters to satisfy the GB2 distribution, I had to constrain τ to be less than 15.

B.2 Quantile Regression

In multivariate linear regression a column vector of T responses $\{Y_t\}$ are related to a design matrix X of predictors in the following way:

$$\begin{aligned} Y_t &= \beta_0 + \beta_1 X_{t1} + \beta_2 X_{t2} + \cdots + \beta_k X_{tk} + u_t. && (26) \\ E[Y_t] &= \beta_0 + \beta_1 X_{t1} + \beta_2 X_{t2} + \cdots + \beta_k X_{tk} && (27) \end{aligned}$$

Let $t = 1, \cdots, T$ and denote the 'errors' as $\{u_t\}$. These 'errors' are where the predicted value from the formula in X_{ti} does not exactly correspond to the observation Y_t. The $\{u_t\}$ are considered to be independent and identically distributed with a distribution F_u and $E[u_t] = 0$.

Another way to look at the problem is a comparison between a model

$$\hat{Y}_t = \beta_0 + \beta_1 X_{t1} + \beta_2 X_{t2} + \cdots + \beta_k X_{tk}, \tag{28}$$

which tries to predict the responses Y_t from that of some linear combination of the elements of the design matrix. The residuals $u_t = Y_t - \hat{Y}_t$ then are how well or how poorly the model fits the actual responses. In OLSR the expectation of the residuals are considered to be zero. Also since the expectation operator is linear then $E[u_t] = E[Y_t] - E[\hat{Y}]$.

In multivariate linear regression, the β_i are determined by minimizing the following sum

$$\sum_1^T (Y_t - (\beta_0 + \beta_1 X_{t1} + \beta_2 X_{t2} + \cdots + \beta_k X_{tk}))^2. \tag{29}$$

The determination of the $\{\beta_i\}$ is referred to as an OLSR or as a l_2 regression estimator. See Portnoy and Koenker [67] for a further discussion of l_2-estimation. After the $\{\beta_i\}$ are determined, the equation relates the sample mean of the Y_t to the predictors.

However, one major difficulty of using OLSR is that the values of the $\{\beta_i\}$ can be very sensitive to outliers in the responses $\{Y_t\}$. The area of robust statistics has arisen to deal with this outlier sensitivity. See Venables and Ripley [101] for a series of references on robust statistics.

Koenker and Basset [44] develop quantile regression (QR), where the regression is related to specific quantiles instead of the mean. We will now describe the process.

Let $\mathbf{x_t} = \{1, X_{t1}, X_{t2}, \cdots, X_{tk}\}$, and $\beta_\theta = \{\beta_0, \beta_1, \cdots, \beta_k\}$ and consider the following:

$$Y_t = \beta_0 + \beta_1 X_{t1} + \beta_2 X_{t2} + \cdots + \beta_k X_{tk} + u_{\theta_t} \tag{30}$$

or in matrix format:

$$Y_t = \beta_\theta \mathbf{x_t}' + u_{\theta_t}. \tag{31}$$

$$Quant_\theta(Y_t | \{X_{t1}, X_{t2}, \cdots, X_{tk}\}) = \beta_0 + \beta_1 X_{t1} + \beta_2 X_{t2} + \cdots + \beta_k X_{tk} \tag{32}$$

or in matrix format:

$$Quant_\theta(Y_t | \mathbf{x_t}) = \beta_\theta \mathbf{x_t}'. \tag{33}$$

$Quant_\theta(Y_t | \{X_{t1}, X_{t2}, \cdots, X_{tk}\})$ denotes the conditional quantile of Y_t, which is conditional on $\{X_{t1}, X_{t2}, \cdots, X_{tk}\}$, the regression vector. The distribution $F_{u\theta}$ of u_{θ_t}, the error term is not specified. Formula 32 implies that $Quant_\theta(u_{\theta_t} | \{x_{t1}, x_{t2}, \cdots, x_{tk}\}) = 0$ for a specific component vector $\{x_{t1}, x_{t2}, \cdots, x_{tk}\}$.

Let's look at this from the perspective of the residual or error term. Assume that there is a model

$$\hat{Y}_t = \beta_0 + \beta_1 X_{t1} + \beta_2 X_{t2} + \cdots + \beta_k X_{tk}, \tag{34}$$

which tries to predict certain behavior of the responses Y_t that is some linear combination of the elements of the design matrix. The residuals $u_t = Y_t - \hat{Y}_t$

then are a measurement of how well the model relates to the actual responses. The difference between QR and OLSR is that instead of the fact that $E[u_t] = 0$ one assumes that $Quant_\theta(u_t) = 0$. This leads to the relationship

$$Quant_\theta(u_t) = Quant_\theta(Y_t - \hat{Y}_t) = 0. \tag{35}$$

The determination of the $\{\beta_i\}$ that allows this relationship to hold will produce the necessary model. However, because the determination of a quantile requires sorting, the quantile operator is not linear. Hence

$$Quant_\theta(Y_t - \hat{Y}_t) \neq Quant_\theta(Y_t) - Quant_\theta(\hat{Y}_t). \tag{36}$$

Koenker and Basset [44] made the following observation: Let Y be a random variable with distribution F. Let $\{y_t : t = 1, \cdots, T\}$ be a random sample on Y. The θth sample quantile for $0 < \theta < 1$ is defined to be any solution of the following minimization problem:

$$min_{b \in \mathbb{R}} \left[\sum_{t \in \{t : y_t \geq b\}} \theta |y_t - b| + \sum_{t \in \{t : y_t < b\}} (1 - \theta)|y_t - b| \right]. \tag{37}$$

From the above Koenker and Basset are able to generalize the l_1 regression estimator from the median to all quantiles $0 < \theta < 1$, by finding the $\{\beta_i\}$ that minimizes the following:

$$\sum_{t=1}^{T} \rho_\theta(Y_t - (\beta_0 + \beta_1 X_{t1} + \beta_2 X_{t2} + \cdots + \beta_k X_{tk})), \tag{38}$$

where $0 < \theta < 1$ and

$$\rho_\theta(u) = \begin{cases} \theta u & \text{when } u \geq 0, \\ (1 - \theta)u & \text{when } u < 0. \end{cases} \tag{39}$$

Buchinsky [7] discusses that under certain regularity conditions the consistency and asymptotic normality of $\hat{\beta}_\theta = (\hat{\beta}_0, \hat{\beta}_1, \cdots, \hat{\beta}_k)$ (which is the estimator of $\beta_\theta = (\beta_0, \beta_1, \cdots, \beta_k)$) that

$$\sqrt{n}((\hat{\beta}_0, \hat{\beta}_1, \cdots, \hat{\beta}_k) - (\beta_0, \beta_1, \cdots, \beta_k)) \xrightarrow{L} N((0, 0, \cdots, 0), \Lambda_\theta) \tag{40}$$

$$\sqrt{n}(\hat{\beta}_\theta - \beta_\theta) \xrightarrow{L} N(\mathbf{0}, \Lambda_\theta) \tag{41}$$

where

$$\Lambda_\theta = \theta(1-\theta)(E[f_{u\theta}(\mathbf{0}|\mathbf{x_t})\mathbf{x_t}\mathbf{x_t'}])^{-1}E[\mathbf{x_t}\mathbf{x_t'}](E[f_{u\theta}(\mathbf{0}|\mathbf{x_i})\mathbf{x_i}\mathbf{x_i'}])^{-1}. \qquad (42)$$

If the density of the error term u_θ at $\mathbf{0}$ is independent of x_t then formula 42 simplifies to

$$\Lambda_\theta = \frac{\theta(1-\theta)E[\mathbf{x_t}\mathbf{x_t'}]^{-1}}{f_{u\theta}^2(0)}, \qquad (43)$$

which corresponds to the result in Koenker and Basset [44].

For any y, if $f_{u\theta}(y|x)$ is independent of x, then the only difference between the quantile regression parameters β_θ for all quantiles θ is in the intercept β_0.

The major advantage of using quantile regression is the ability to relate the behavior of a specific quantile of the responses to the design matrix X of predictors. The partial derivative of the conditional quantile of y_t with respect to a specific regressor k is $\partial Quant_\theta(y_t|\mathbf{x_t})/\partial X_{tk}$. Interpret this derivative as the change in the θth conditional quantile due to the change in the kth element of x. Note: when the kth element of x changes this does not imply that when the y_t in a specific quantile θ changes that it will remain in the θth quantile.

A further presentation of the above material is contained in Buchinsky [7].

B.3 Resampled Extreme Value Technique

This technique approximates by bootstrapping the joint distribution of the k order statistics. Let $\{Y_i\}$ for $i = 1, \cdots, n$ be n samples of a distribution with an exponentially weighted right tail. To model this tail of the distribution, sort the $\{Y_i\}$ into the increasing order statistics $X_{1n} \geq X_{2n} \geq \cdots \geq X_{nn}$. To model the left tail, sort $\{-Y_i\}$ into its increasing order statistics.

Define the vector of normalized spaces $\mathbf{d} = \{d_i\}$ of the $k+1$ order statistics as $d_i = i(X_{in} - X_{i+1n})$, $i = 1, \cdots, k$.

Zelterman [110] discusses that the $\{d_i\}$ are approximately identical independent exponential distributed random variables when n is much larger than k. Let $\{d_1^*, \cdots, d_k^*\}$ be a bootstrap resample from $\mathbf{d}$ of size k drawn with replacement[3]. The bootstrap resample $X_{1n}^* \geq X_{2n}^* \geq \cdots \geq X_{nn}^*$ of the k largest order statistics is defined by

[3]Choose a random natural number between 1 and k, assuming a discrete uniform distribution.

$X^*_{jn} = X_{k+1n} + \sum_{i=j}^{k} i^{-1} d^*_i \ (j = 1, \cdots, k)$.

In the bootstrap, all that is needed is the fact that the normalized spaces are approximately independent and identically distributed. We do not use the fact that they are also approximately exponentially distributed. Zelterman [110] goes on to show the theoretical underpinning of the above technique. Note also that the different X^*_{1n} are not limited by the original X_{1n}. The collective $\{X_{in}\}$ simulates a sample from the joint distribution.

The resampling size for the 5% table is 500, where in the 1% table it is 1000. Upon resampling we collect the resampled X^*_{in} for specific i. This technique essentially cuts the tail off and attaches additional tails with distances based on the original tail. Theoretically, when using this technique on heavier tailed distributions it underestimates the quantiles, where in lighter tails, it overestimates them. However, Craighead [15] observes that even though this algorithm would at times underestimate and overestimate these distributions, the semiparametric nature of the algorithm is effective on these distributions as well.

C Statistical Tests

C.1 Percentile Estimation using Order Statistics

Let $\{X_n\}$ be a random sample of size n from some common unknown distribution. Let $\{Y_n\}$ be the ascending order statistics on the $\{X_n\}$. Let ξ_p represent the pth percentile. Hogg and Craig [37] derive the following formula:

$$Pr(Y_k < \xi_p) = \sum_{w=k}^{n} \frac{n!}{w!(n-w)!} p^w (1-p)^{n-w}. \tag{44}$$

Using the above formula we can determine various confidence intervals on ξ_p.

For instance for a sample size of 10,000 for the two sided 95% confidence interval of ξ_{95} is between $Y_{9456} < \xi_{95} < Y_{9541}$. Where the two sided 99% confidence interval of $\xi_9 9$ is $Y_{9872} < \xi_{99} < Y_{9924}$.

See Hogg and Craig [37] for further examples and uses.

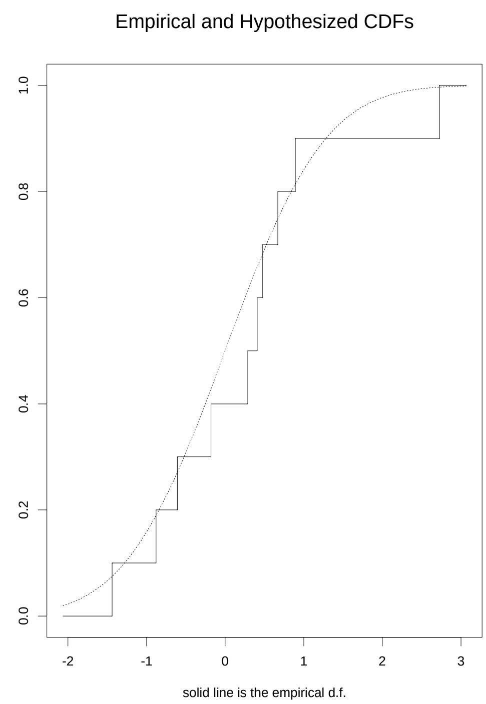
C.2 Kolmogorov-Smirnov testing

To be able to set up a hypothesis test to see if two distributions are identical, one classical method is the Kolmogorov-Smirnov test. (Others are the Cramér-Von Mises Statistic, the Kulper Statistic, the Watson Statistic, and the Anderson-Darling Statistic. See Stephens [88] and Durbin [21] for further discussions and references on these tests.) The Kolmogorov-Smirnov or KS test is applicable to an univariate distribution (say $F(x)$) whose samples have not been binned. One can easily determine the empirical cumulative distribution function $F_n(x)$, which is an unbiased estimator of $F(x)$. Let $\{x_i\}$ represent n samples from $F(x)$. Order the $\{x_i\}$ in increasing order. Denote these order statistics as $\{X_{in}\}$. Let

$$F_n(x) = card(\{X_{in} | X_{in} \leq x\})/n. \tag{45}$$

where $card(A)$ denotes the number of elements in a set A. So $F_n(x)$ is constant between consecutive X_{in}, and jumps by the same amount $1/n$ at each new X_{in} encountered. See Figure 18 on page 95.

Note that the cumulative distributions agree at the smallest and the largest allowable values of X_{in}. Here the cumulative distributions take on the values of zero and one respectively. The difference between the two cumulative distributions is measured by absolute value of the area, or the integrated



Empirical and Hypothesized CDFs
0.0
0.2
0.4
0.6
0.8
1.0
-2
-1
0
1
2
3
solid line is the empirical d.f.

Figure 18:

mean square distance. However the KS test measures the maximum value of the absolute difference between the CDFs. So to compare $F_n(x)$ to that $F(x)$, the KS D statistic is

$$D = \max_{-\infty<x<\infty} |F_n(x) - F(x)|. \tag{46}$$

If one needs to compare two separate empirical distributions $F_{n1}(x)$ and $F_{n2}(x)$ the KS D statistic is

$$D = \max_{-\infty<x<\infty} |F_{n1}(x) - F_{n2}(x)|. \tag{47}$$

The null hypothesis where samples are drawn from the same distribution can be tested because the distribution of D can be calculated.

In the single empirical cumulative versus parametric cumulative distribution test let $Y = \sqrt{n}D_n$. In the dual empirical cumulative distribution test, let $Y = \sqrt{\frac{n_1 n_2}{n_1+n_2}}D$.

Now as the sample size $n \to \infty$, the probability of acceptance of the null hypothesis can be calculated from

$$Pr(D > observed) = 2\sum_{j=1}^{\infty}(-1)^{j+1}e^{-2j^2Y^2}. \tag{48}$$

(Note: the relationship $Y = \sqrt{n}D_n$ assumes that the parametric distribution being tested had be fully determined. If the data is used to estimate the parameters, then Stephens [88] states that $Y = (\sqrt{n}+0.12+.11/\sqrt{n})D_n$. It is unclear whether Robbins et al. [79] made this adjustment. In our studies we used $Y = \sqrt{n}D_n$, however the second formula would only make our test fail more frequently, because it would inflate the value of Y, which in turn would make the chance of passing the test smaller. Since we observed so many failures of the KS-test, this adjustment would have just made our results worse.)

The above series is a monotone function, and when $Y = 0$ the series equals one, and when $Y = \infty$ the series equals zero. To accept the null hypothesis at a 95% confidence level, the value of the series must exceed 0.05. Normally a sample size of $n \geq 20$ is large enough.

However, note that as n increases there must be a corresponding decrease in the value of D or else it becomes more and more difficult to accept the null hypothesis. The sample size of 10,000 forces the D to be quite small to compare the empirical cumulative distribution to that of GB2, which is discussed in Section 5. Also note that the above versions of the KS test are conducted on unbinned samples only. Darling [18] and Durbin [21] discuss the use of a nonnegative weight function $\psi(x)$ to generalize Kolmogorov-Smirnov statistics. Here

$$D = \max_{-\infty<x<\infty} |\psi(x)(F_n(x) - F(x))| . \tag{49}$$

By choosing different $\psi(x)$ one is able to give greater weight to discrepancies at different x locations. Also, one can set $\psi(x)$ to zero for some values of x. However by using a Poisson process, one goes through a very complex mathematical argument to modify Equation 48, which is beyond the scope of this paper. See Durbin [21] for a further discussion of the derivation of the revision of Equation 48. Durbin also devises a generalized KS test for binned data, by the specialized use of a ψ function. Craighead [15] binned his data before calculating the D statistic. However, he did not used the revised Equation 48 discussed in Durbin [21] and hence his results would mislead one to believe that REV produced highly successful results. Robbins [79] used several subsetting methods on their KS test such as limited subsampling and key value fits. It is unclear whether they modified Equation 48 to reflect the implied weight functions that they used to restrict the traditional KS test.

See Press et al. [72] or London [53] for the algorithms for the traditional KS test.

C.2.1 Truncated KS test

Truncated distributions in general take on the form

$$F(x|y < X \leq z) = \frac{F(x) - F(y)}{F(z) - F(y)} \tag{50}$$

where $F(x)$ is the cumulative distribution. When examining a truncated left tail distribution, $y = -\infty$, and $F(y) = 0$, whereas in a truncated right tail distribution, $z = \infty$, and $F(z) = 1$. So a left tail 10% truncated tail, one examines

$$F(x|X \leq z) = \frac{F(x)}{.1} \tag{51}$$

and a right 10% tail examines:

$$F(x|y < X) = \frac{F(x) - .9}{.1} \tag{52}$$

After making the above adjustments, one can use the standard KS test on the resultant truncated cumulative distributions.

Note: Because of this adjustment, the value of D must be very small to handle being grossed up by both the square root of the sample size and the truncation adjustment. This implies that the KS test passes less frequently when conducted on truncated distributions than on full distributions assuming that the sample size does not change.

C.3 Wald Estimator

Suppose that you have estimators $\hat{\theta}_j$ of parameters θ_j where $j = 1, 2, \ldots, p$ such that

$$\sqrt{n}(\hat{\theta} - \theta) \longrightarrow N_p(\mathbf{0}, \Sigma) \tag{53}$$

by vector convergence.

Also, suppose that you have covariance estimators such that

$$\widehat{\Sigma} \longrightarrow \Sigma \tag{54}$$

in probability.

Suppose that you want to test say, $H_0 : \theta_j = 0$, for $j = 1, \ldots, q$ with $q < p$, then

$$(\widehat{\theta(q)} - \theta(q))' S.\widehat{inv}(q)(\widehat{\theta(q)} - \theta(q)) \tag{55}$$

is approximately $\chi^2(p - q)$ where $\theta(q)$ is the first q coordinates of θ, $\theta(\hat{q})$ is similarly defined, and $S.\widehat{inv}(q)$ is the inverse of the upper left $q \times q$ submatrix of $\widehat{\Sigma}$.

The Wald test is set up with the QR software by estimating the coefficient covariance matrix from gradients used in the interior point method, when determining the QR coefficients. This covariance matrix is then used as the $\widehat{\Sigma}$ above.

The above description of the Wald Estimator is courtesy of Dr. Stephen Portnoy. Another presentation of this is in Press et al. [75].

C.4 Chi Squared subsample test for QR method

In ordinary least squares regression, if the errors are independent and identical normal distributions, the coefficients of the regression are from a multivariate normal distribution. See [2].

We make the following assumptions on our quantile regressions to create a statistical test to examine how well a subsample quantile regression matches an entire data set quantile regression.

1. The coefficients from the QR regression is from a multivariate normal distribution as the sample size increases. This corresponds to Equation 43, on page 92. Here a hypothesis test should be conducted to determine when one is able to make this assumption. However in Section 10 this is not done.
2. Let τ be the specific quantile on which the regression is conducted.
3. Let N be the number of scenarios.
4. Let m be the number of predictors in the design matrix.
5. Portnoy [68] said that the relationship of N to τ is as follows:
$$1/\sqrt{N} \ll \tau \ll 1 - 1/\sqrt{N}. \tag{56}$$
6. Let n be the subsample size. Note: $n < N$.
7. Let $\{B_{0N,q}, B_{1N,q}, \cdots, B_{mN,q}\}$ be the coefficients from the entire set of scenarios.
8. Let $\{S_{0N,q}, S_{1N,q}, \cdots, S_{mN,q}\}$ be the standard errors of the coefficients from the entire set of scenarios.
9. Let $\{B_{0n,q}, B_{1n,q}, \cdots, B_{mn,q}\}$ be the coefficients from the subsampled set of scenarios.
10. For each $i = 0, \cdots, m$ $\ (B_{iN,q} - B_{in,q})/S_{iN,q}$ is from a standard Normal distribution.

11. Given the assumption that the above item, the following formula

$$\sum_{0}^{m}((B_{iN,q} - B_{in,q})/S_{iN,q})^2 \sim \chi^2_{m-1} \tag{57}$$

We have 21 coefficients in the design matrix for all of our QR analyses and so we are examining χ^2_{20}. If the above statistic exceeds the critical value of 31.41 we can conclude that the set $\{B_{0n,q}, \cdots, B_{mn,q}\}$ of subsample coefficients are not the same as the $\{S_{0N,q}, S_{1N,q}, \cdots, S_{mN,q}\}$ from the entire sample with a 95% confidence. This is a right tail test only.

C.5 Rank subsample test for the QR method

This is a nonparametric method to obtain the relative information contained in the QR coefficients. The test is structured based on the following assumptions:

1. Let N be the number of scenarios.
2. Let m be the number of predictors in the design matrix.
3. Let n be the subsample size. Note: $n < N$.
4. Let $\{B_{0N,q}, B_{1N,q}, \cdots, B_{mN,q}\}$ be the coefficients from the entire set of scenarios.
5. Let $\{B_{0n,q}, B_{1n,q}, \cdots, B_{mn,q}\}$ be the coefficients from the subsampled set of scenarios.
6. The signum function

$$S(x) = \begin{cases} 1 & \text{when } x > 0, \\ 0 & \text{when } x = 0, \\ -1 & \text{when } x < 0. \end{cases} \tag{58}$$

7. The indicator function

$$I_A(x) = \begin{cases} 1 & \text{when } x \in A, \\ 0 & \text{when } x \notin A. \end{cases} \tag{59}$$

8. Let ϵ be the indicator of how close a variable is near zero.

9. Let the set $A = \{x : -\epsilon < x < \epsilon\}$ be an open ϵ neighborhood around zero.

10. Define the union function for a specific ϵ

$$\begin{aligned} U(x,y) &= (1 - I_A(x)) + (1 - I_A(y)) - (1 - I_A(x))(1 - I_A(y)) \\ &= \begin{cases} 1 & \text{when } |x| \geq \epsilon \text{ and } |y| \geq \epsilon, \\ 1 & \text{when } |x| < \epsilon \text{ and } |y| \geq \epsilon, \\ 1 & \text{when } |x| \geq \epsilon \text{ and } |y| < \epsilon, \\ 0 & \text{when } |x| < \epsilon \text{ and } |y| < \epsilon. \end{cases} \end{aligned}$$

.

11. For each subsample and a specific ϵ construct the following sum

$$R(n) = \sum_{i=0}^{m} U(B_{iN,q}, B_{in,q})|(S(B_{iN,q}) - S(B_{in,q})| \qquad (60)$$

$R(n)$ is large if separate coefficients are of opposite sign and the coefficients are significantly different than zero. $R(n)$ is small when the separate coefficients are of the same sign and/or very near zero.

C.6 CS Ratio Measurement of Extreme Tails

This statistical test determines when the tail of a distribution is heavier, equal to or lighter than an exponential distribution. This ratio is introduced in Castillo et al. [10]. We refer to this test as the CS ratio. Consider the following:

Let $\{Y_i\}$ be a random sample of size n. Let $\{X_{in}\}$ be the increasing order statistics of the $\{Y_i\}$. Consider the two following sets of indices of the order statistics:

$A_{12} = \{1, \cdots, [\sqrt{n}]$ and $A_{34} = \{[\sqrt{n}], \cdots, [2\sqrt{n}]\}$, i.e. the first $[\sqrt{n}]$ order statistics and the second set of $[\sqrt{n}]$ with overlap at $[\sqrt{n}]$. Let S_{12} and S_{34} be the slopes of the least square regression lines between $-ln(-ln((n+1-i+0.5)/n))$ and the order statistics $\{X_{in}\}$ for i in A_{12} and A_{34}, respectively. As Castillo et al. [10] shows the ratio S_{12}/S_{34} have large values for distributions with lighter tails than exponential, and small values for heavier than exponential tails and midrange values for those tails with exponential weight. See Craighead [15] for a further discussion and analysis of this ratio in the determination of the tail type.

Other interesting articles on the determination of the three distributions are Wang [102] and Weissman [104].

D Statistical Graphics

D.1 Boxplots

Boxplots are a good exploratory tool, specifically when several are placed side by side for a comparative study. The box in the boxplot shows the limits of the middle half of the data (i.e. from the twenty-fifth percentile to the seventy-fifth percentile). The line inside the box represents the median. The whiskers are drawn to the nearest value not beyond a standard span from the quartiles. The standard span is three halves times the interquartile distance. The interquartile distance is the difference between the third quartile and the first quartile. The extreme points (outliers) are drawn individually. With data sampled from a Normal distribution, approximately 99.3% of the samples fall within the whiskers. Observe that boxplots show the location and spread of data and indicates its skewness, as well. See Hoaglin et al. [36], McGill [55], Tukey [98] and Velleman and Hoaglin [100] for a further discussion of the use of and interpretation of boxplots.

D.2 Funnel of Doubt

The other concept used in tandem with boxplots is the representation of the Funnel of Doubt (denoted FOD). This concept is used by Wilkie [108] to indicate how the dispersion of results increases through time. We use this to display the behavior of the interest rate scenarios over the twenty-year projection used to generate the EVAS values.

E Known and Potential Models

There are several actual or potential methods to maximize information obtained from a corporate model. Briefly these can be summarized as output only, input only and both input and output. The following is a brief survey of these:

1. Output only. At this point we determine and fit the best distribution. The process is to take sample computer output and find the best distribution(s) that fit the data. Recall we do not know the actual underlying distribution, but we are attempting to determine the best one that fits the observed data. The resultant mathematical model is then used to extrapolate the answers to different corporate model problems without having to resort to conducting further scenario testing. These methods are:

   (a) Whole support fit. The sample data is used to approximate the entire distribution trying to cover the entire support. The concept of support of a distribution is the range of values of the random variable being modeled. This is considered to be the domain of the distribution.

      i. Single distribution. One searches for the "best" distribution that fits the data. See Klugman, Panjer, and Willmot [51], De Vylder [19], Hogg and Klugman [38], Panjer and Willmot [66] and Robbins et al. [79]. Our GB2 analysis above is of this form. Also see Carlin and Louis [9] for a further discussion of the potential use of Markov Chain Monte Carlo methods in hierarchical Bayesian model fitting.

      ii. Mixture models. One fits a linear combination of many distributions (commonly these are the same distribution but with differing parameter values). Each coefficients of the linear combination must be positive and these coefficients must sum to one. See Klugman [50], Venables and Ripley [101], Everitt and Hand [25], Titterington et al. [97], and McLachlan and Basford [57] for further discussions of the various algorithms one can use in the modeling of mixtures.

      iii. Semiparametric fitting. A semiparametric model consists of observing a certain key behavior in a specific family of distributions. The semiparametric model makes that behavior central to the algorithm that fits the data. This behavior takes precedence over the fitting of the distribution. The closest actuarial analogy is the Whittaker-Henderson graduation method. See London [52] or Whittaker [107] for a further discussion of this. Currently, semiparametric density fitting appears to be an open area of research.

iv. Nonparametric fitting. In the use of these methods, one does not attempt to determine the best parametric distribution. These methods are designed to use several functional forms to approximate the distribution. Some of these are:

- Kernel. A kernel is a functional form that has properties of specific distributions. In kernel analysis, the statistical tools are used to find the best linear combination of a family of these kernels that best fit the data. Currently, these methods are best used in the graphical presentation of the data. In methods similar to graduation theory, there is an operator to control on how well the data is fit versus the smoothness of the result See Silverman [85], Wegman [103],Venables and Ripley [101], Härdle [31] and Scott [80] for a further discussion of these methods. Note how this method is similar to the use of mixture models.
- Fourier transforms. Fourier transforms are used in a fashion similar to the kernel methods. See Tarter and Lock [92].
- Wavelets. This seems to be one of the late entries into this field. Again there is a kernel like approach, however the kernel itself is designed by the data. See Müller and Vidakovic [62] for a further discussion of their techniques and other references.
- Resampling. Each observed data value is converted to a Dirac δ function where each δ is centered at the specific observation. The empirical distribution is considered to be a "comb" where each tooth of the comb represents a specific Dirac δ function. By resampling from the teeth, the distribution is filled in and simulates the underlying population distribution. See Taylor and Thompson [91] for a further discussion of this method. Also see Efron [22]

for a further discussion of resampling.

(b) Extreme tail fit. The sample data is used to approximate only one of the extreme tails of the distribution.

i. Fitting the distribution. Gnedenko [26] proved that there are only three different types of extreme tails. An extensive reference on this subject is Embrechs et al. [23]. Also, see Gumbel [28] for the traditional description of how to use graphs to fit the extreme tail.

ii. Saddlepoint approximation. If the parametric distribution of the output data is known, the use of saddlepoint approximations can tighten the estimates in the extreme tail. See Jensen [39], and Wood [109] for a further discussion.

iii. Resampling. This is a semiparametric method that assumes certain properties of the exponential extreme distribution, and uses a resampling technique to replicate the tail of the distribution. Zelterman [110], Strawderman and Zelterman [89] and Zelterman and Lindgren [111] and Craighead [14, 15] have examined these resampling techniques on extreme tails. This is also discussed in Appendix B.3. Our prior REV analysis is within this category. Also see Efron [22] for a further discussion of resampling.

2. Input only. The technique to obtain more information is controlled at the construction of the economic scenario files.

(a) Deterministic (Such as the New York 7 scenarios). By the intentional design of specific economic possibilities in the scenarios, one gains insight into how different evolutions of the economic scenarios can influence the corporate model output. Hopefully, this can lead the actuary to the knowledge of what the major risk drivers that impact the business. However one cannot obtain the probabilities of ruin or observe the distribution of results. The results can be very misleading since the use of these scenarios is really more like a sensitivity test.

(b) Monte Carlo (a.k.a. Brute Force. See Sedlak [83]). More information is obtained by processing more scenarios through the corporate model.

(c) Representative scenarios. In using representative scenarios one attempts to reduce the vast universe of scenarios into a restricted set that covers the majority of the possibilities. The probability measure on the restricted set must also be adjusted to reflect condensing the scenarios from the larger universe. Three known methods are:

   i. Cherry picking. Christiansen [11, 12] uses methods that examines a larger set of scenarios and optimally chooses a subset of scenarios that replicate the overall statistics of the larger set of scenarios. Mike Zurcker uses a similar process at Lincoln National.

   ii. Linear Path Space. Ho [34, 35] uses a technique that creates equivalence classes of the various paths of interest rates, and restricts the number of these classes used in the projection process.

   iii. Clustering. Doug George of Avon Consulting starts with a large set of scenarios. He then assigns a distance between any two scenarios in the data set. Next for a specific scenario and a specific cutoff, scenarios that have a distance below the cutoff are considered to be identical to the original scenario. These *identical* scenarios are then eliminated. The benefit of scenario reduction by clustering is very similar to the results of Linear Path Space.

(d) Manipulation of the choice of random numbers. One determines the total number of random numbers that is required to generate one scenario. Say this number is n. The researcher then examines methods to generate random numbers in the n dimensional space from where the random numbers are chosen.[4] One major requirement in the choice of these random numbers is to prevent an improper concentration in any subregion of the n dimensional space. See Knuth [42] or Niederreiter [63] for further discussion.

[4]This n dimensional space is the unit hypercube, except when not generating random numbers using an inverse integral method. The hypercube is the Cartesian product of n separate unit intervals. Most methods of choosing random numbers from a specific distribution is to choose random numbers from some uniform distribution and then use a transformation on these uniform random numbers to create random numbers for the specific distribution. See Knuth [42] for a further discussion of this.

Some of these are:

i. Latin hypercube. A Latin hypercube is the n dimensional space generalization of a Latin square. Latin hypercubes are used in experimental design to make sure that the experiment has no omissions or over emphasis on any specific factor. See Tang [90], Stein [87], Welch, et al. [105] and McKay, Beckman and Conover [56] for further discussions on the use of Latin hypercubes in experimental design and sampling reduction.

ii. Panel methods. If one knows that they are restricted to some mk number of scenarios. One then divides the hypercube into k separate "panels." The union of these panels is the hypercube and the intersections of the panels only occur on the boundaries of the panels. To obtain the mk scenarios one selects m separate random samples from each of the k panels. See Boyle [6] for a further discussion of these methods.

iii. Low discrepancy sequences (LDS). Using deep results from the theory of numbers, random numbers are generated in a deterministic fashion such that a special metric measuring the distance between points already chosen is at a minimum. Because the generated numbers are chosen in a deterministic fashion, they are called Quasi Pseudo random numbers or Quasi Monte Carlo Methods. See Halton [30], Niederreiter [63], Tezuka [93], Lord and Vanderhoof [99], Paskov and Traub [65], Boyle [6], Caflisch and Morokoff [8] and Joy, Boyle and Tan [41] for further insight and references.

iv. Brownian Bridges. This is a method that reduces the dimensional requirements of LDS methods. The order in which the random numbers are selected is critical. By taking a specific order of selecting the random numbers the dimensional requirements are halved. See Caflisch and Morokoff [8] for further details on how they priced a Collateralized Mortgage Obligation using this technique.

v. Latin Supercubes. This is a new area where LDS techniques are combined with other Latin hypercube techniques to address high dimensional scenario requirements. See Owen [64] for further details on this new possibility.

3. Input tied to output. For each of the following, the probabilities of the associated scenarios can be affected just as in the pure input methods.

   (a) Neural networks. A neural network is designed to replicate the output results from the input. A neural network is in the truest sense a 'blackbox' method. Samples of the input and output of the model are processed and the neural network "learns" the best weights to be associated with internal nodes of a network. This process can take many hours of "learning" time. Finally, the model is compared to samples outside of the original learning set of samples to see how well the model replicates the overall process. See Hertz et al. [33], Ripley [77, 78], and White [106] for a further discussion of the use of these types of models.

   (b) One Representative Scenario. Manistre [59, 60] designs one single representative scenario that reflects the aggregate output of a large collection of scenarios. He uses this technique to explain a company's risk position to upper management.

   (c) Numerix's Monte Carlo improvements. The Numerix consulting firm has developed a series of techniques and algorithms that uses input scenarios and existing output and produces more extensive and timely output results. Contact Goldenfeld [27] for a further discussion of their services.

   (d) Stratified or Importance sampling. This is a modified method from Latin Hypercubes. First a preliminary set of scenarios are processed. From the output, one determines the region of the hypercube that requires further sampling. Also see McKay, Beckman and Conover [56] and Welch et al. [105] for other methods modifying the input based on model output.

   (e) LDS stratified sampling. Lord and Vanderhoof [99] determine from a small set of scenarios where the extreme tail is located and obtain the associated restricted region of the random numbers in the hypercube. Using a modified LDS process, they then sample from this restricted region. Using these additional random samples, they construct additional scenarios and process the corporate model on these new scenarios.

   (f) Quantile Regression (QR) and Cumulative Quantile Regression (CQR). Using regression, one approximates the relationship between the input scenarios and the output results. One major difference between these methods and ordinary linear regression is that the quantile regression results are related to specific quantiles instead of the mean. We will examine the potential of QR in our subsequent analysis. See Bassett and Koenker [4], Koenker [43], Koenker and Bassett [44], Koenker and Portnoy [47], Portnoy [67], Portnoy and Koenker [69] for further discussions on QR. See Koenker and D'Orey [45, 46] for computer algorithms to implement QR analysis. Buchinsky [7] also has an excellent overview of the theory and applications of QR. Another potential area of research is CQR. As the name implies, the regression is related to the cumulative quantiles. Where QR models a specific quantile, such as the fifth percentile, CQR models the entire tail up to the specified quantile. Since CQR models the entire tail, this area of research appears to be very promising in the modeling of insolvency. See Rao and Zhao [71] for further references on CQR.

F MLE GB2 Confidence Intervals

Below we will examine the possibility of obtaining a closed formula for the confidence intervals for large sample maximum likelihood estimates. Rice [76] discusses large sample theory for maximum likelihood estimates. Briefly he states:

> The vector of maximum likelihood estimates is asymptotically normally distributed. The mean of the asymptotic distribution is the vector of true parameters, and the elements of the vector of estimates have variances and covariances given by the corresponding elements of the matrix $(1/n)I^{-1}(\theta_0)$, where $I(\theta)$ is a matrix with the ij component

$$E\left[\frac{\partial}{\partial\theta_i}\log f(x|\theta)\frac{\partial}{\partial\theta_j}\log f(X|\theta)\right] = -E\left[\frac{\partial^2}{\partial\theta_i\partial\theta_j}\log f(X|\theta)\right] \tag{61}$$

Using Equation 61, we will see how difficult it will be to obtain closed formulae. Recall that the GB2 distribution has four parameters: $\alpha > 0, \gamma > 0, \lambda > 0$, and $\tau > 0$. Its probability density function is

$$f(x|\alpha,\gamma,\lambda,\tau) = \frac{\lambda^{\tau\alpha}\tau x^{\tau\alpha-1}}{\beta(\alpha,\gamma)(1+(\lambda x)^\tau)^{\alpha+\gamma}} \tag{62}$$

Also $\log f(x|\alpha,\gamma,\lambda,\tau)$ is

$$\tau\alpha\log(\lambda)+\log(\tau)+(\tau\alpha-1)\log(x)-\log(\beta(\alpha,\gamma))-(\alpha+\gamma)\log(1+(\lambda x)^\tau) \tag{63}$$

Recall that the beta function is

$$\beta(\alpha,\gamma) = \frac{\Gamma(\alpha)\Gamma(\gamma)}{\Gamma(\alpha+\gamma)}. \tag{64}$$

The partial derivative of $\log f(x|\alpha,\gamma,\lambda,\tau)$ with respect to α is

$$\begin{aligned}\frac{\partial}{\partial\alpha}\log f(x|\alpha,\gamma,\lambda,\tau) &= -\log((\lambda x)^\tau+1)+\tau\log(x)+\tau\log(\lambda)\\ &\quad -\psi(\alpha)+\psi(\alpha+\gamma)\end{aligned} \tag{65}$$

Here $\psi(x)$ is the digamma function and is defined as

$$\psi(x) = \Gamma'(x)/\Gamma(x) \tag{66}$$

The partial derivative of $\log f(x|\alpha,\gamma,\lambda,\tau)$ with respect to γ is

$$\begin{aligned}\frac{\partial}{\partial\gamma}\log f(x|\alpha,\gamma,\lambda,\tau) &= -\log((\lambda x)^\tau + 1)\\ &\quad - \psi(\gamma) + \psi(\alpha+\gamma)\end{aligned} \tag{67}$$

The partial derivative of $\log f(x|\alpha,\gamma,\lambda,\tau)$ with respect to λ is

$$\frac{\partial}{\partial\lambda}\log f(x|\alpha,\gamma,\lambda,\tau) = \frac{\tau(\alpha - \gamma(\lambda x)^\tau)}{\lambda((\lambda x)^\tau + 1)} \tag{68}$$

The partial derivative of $\log f(x|\alpha,\gamma,\lambda,\tau)$ with respect to τ is

$$\begin{aligned}\frac{\partial}{\partial\tau}\log f(x|\alpha,\gamma,\lambda,\tau) &= ((\lambda x)^\tau(\alpha\log(x) - (\tau(\alpha+\gamma)\log(\lambda x)\\ &\quad - \alpha\tau\log(\lambda) - 1)/\tau) + \alpha\log(x)\\ &\quad + \alpha\log(\lambda) + 1/\tau)/((\lambda x)^\tau + 1)\end{aligned} \tag{69}$$

At this point one is able to use the left hand side of Equation 61. However, to implement the right hand side, we need the following partial derivatives.

The second partial derivative of $\log f(x|\alpha,\gamma,\lambda,\tau)$ with respect to α is

$$\frac{\partial^2}{\partial\alpha^2}\log f(x|\alpha,\gamma,\lambda,\tau) = -\psi'(\alpha) + \psi'(\alpha+\gamma) \tag{70}$$

The second partial derivative of $\log f(x|\alpha,\gamma,\lambda,\tau)$ with respect to γ is

$$\frac{\partial^2}{\partial\gamma^2}\log f(x|\alpha,\gamma,\lambda,\tau) = -\psi'(\gamma) + \psi'(\alpha+\gamma) \tag{71}$$

The second partial derivative of $\log f(x|\alpha,\gamma,\lambda,\tau)$ with respect to λ is

$$\frac{\partial^2}{\partial\lambda^2}\log f(x|\alpha,\gamma,\lambda,\tau) = \frac{\tau(\gamma(\lambda x)^{2\tau} - (\alpha(\tau+1) + \gamma(\tau-1))(\lambda x)^\tau - \alpha)}{\lambda^2((\lambda x)^\tau + 1)^2} \tag{72}$$

The second partial derivative of $\log f(x|\alpha,\gamma,\lambda,\tau)$ with respect to τ is

$$\frac{\partial^2}{\partial\tau^2}\log f(x|\alpha,\gamma,\lambda,\tau) = \frac{-((\lambda x)^{2\tau} + (\lambda x)^{\tau}(\tau^2(\alpha+\gamma)\log(\lambda x)^2 + 2) + 1)}{\tau^2((\lambda x)^{\tau} + 1)^2} \tag{73}$$

The second partial derivative of $\log f(x|\alpha,\gamma,\lambda,\tau)$ with respect to γ and α is

$$\frac{\partial}{\partial\gamma\partial\alpha}\log f(x|\alpha,\gamma,\lambda,\tau) = \psi'(\alpha+\gamma) \tag{74}$$

The second partial derivative of $\log f(x|\alpha,\gamma,\lambda,\tau)$ with respect to α and λ is

$$\frac{\partial}{\partial\alpha\partial\lambda}\log f(x|\alpha,\gamma,\lambda,\tau) = \frac{\tau}{\lambda((\lambda x)^{\tau} + 1)} \tag{75}$$

The second partial derivative of $\log f(x|\alpha,\gamma,\lambda,\tau)$ with respect to α and τ is

$$\frac{\partial}{\partial\alpha\partial\tau}\log f(x|\alpha,\gamma,\lambda,\tau) = \frac{((\lambda x)^{\tau}(\log(x) - \log(\lambda x) + \log(\lambda)) + \log(x) + \log(\lambda))}{(\lambda x)^{\tau} + 1} \tag{76}$$

The second partial derivative of $\log f(x|\alpha,\gamma,\lambda,\tau)$ with respect to γ and λ is

$$\frac{\partial}{\partial\gamma\partial\lambda}\log f(x|\alpha,\gamma,\lambda,\tau) = \frac{-\tau(\lambda x)^{\tau}}{\lambda((\lambda x)^{\tau} + 1)} \tag{77}$$

The second partial derivative of $\log f(x|\alpha,\gamma,\lambda,\tau)$ with respect to γ and τ is

$$\frac{\partial}{\partial\gamma\partial\tau}\log f(x|\alpha,\gamma,\lambda,\tau) = \frac{-(\lambda x)^{\tau}\log(\lambda x)}{(\lambda x)^{\tau} + 1} \tag{78}$$

The second partial derivative of $\log f(x|\alpha,\gamma,\lambda,\tau)$ with respect to λ and τ is

$$\frac{\partial}{\partial\lambda\partial\tau}\log f(x|\alpha,\gamma,\lambda,\tau) = \frac{-(\gamma(\lambda x)^{2\tau} + (\lambda x)^{\tau}(\tau(\alpha+\gamma)\log(\lambda x) - \alpha + \gamma) - \alpha)}{\lambda((\lambda x)^{\tau} + 1)^2} \tag{79}$$

The need to determine the negative expectations of the above second order derivatives in the $I(\theta)$ matrix in Equation 61 is relatively simple for Equations 70 and 71. However, the determination of the remaining expectations for Equation 72 through Equation 79 is extremely complex because of the nonlinear relationships in λx. Because of these complicated formulas, we are unable to obtain closed formulas for the confidence intervals for α, γ, λ, and τ. This implies that various numerical methods will be required to evaluate the expectations and one is only able to approximate the confidence intervals.

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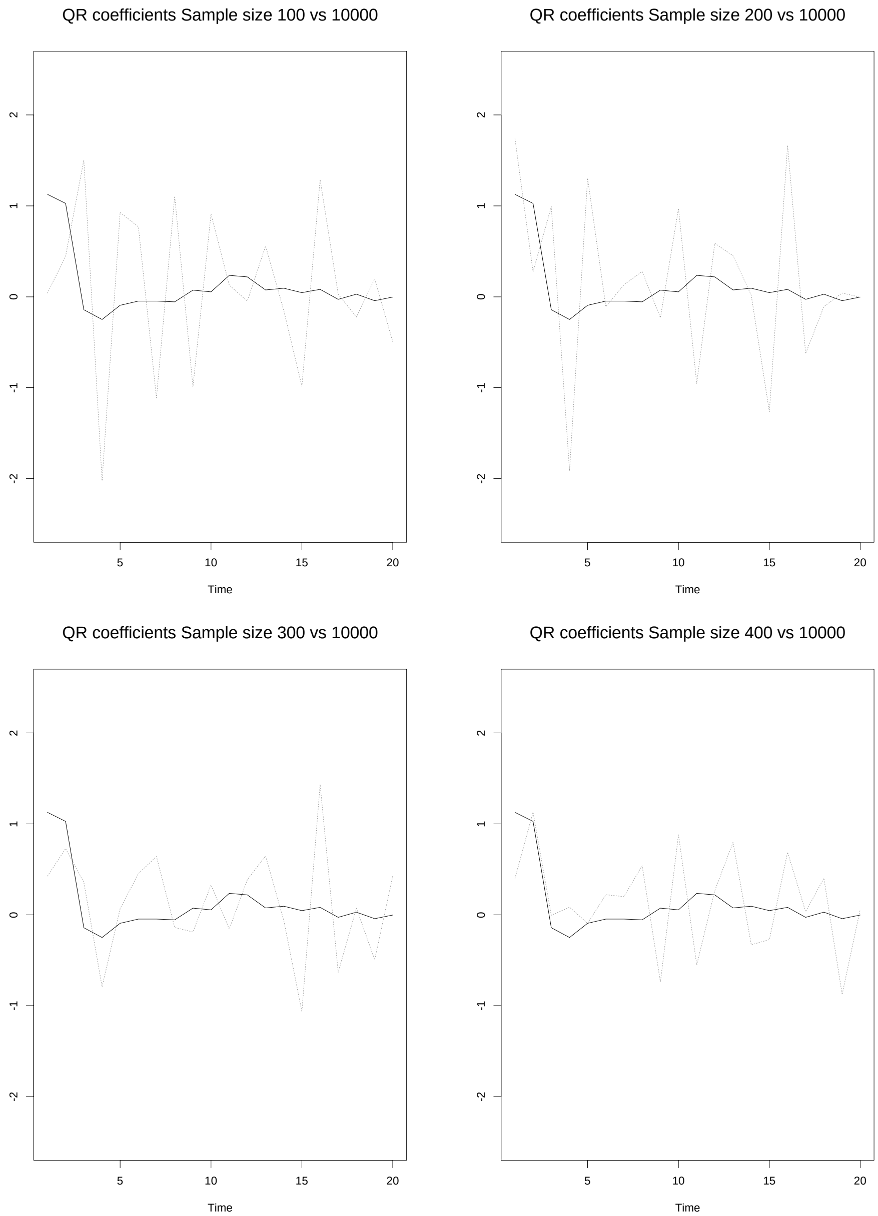

Figure 19: Stability of QR Coefficients vs Sample Size

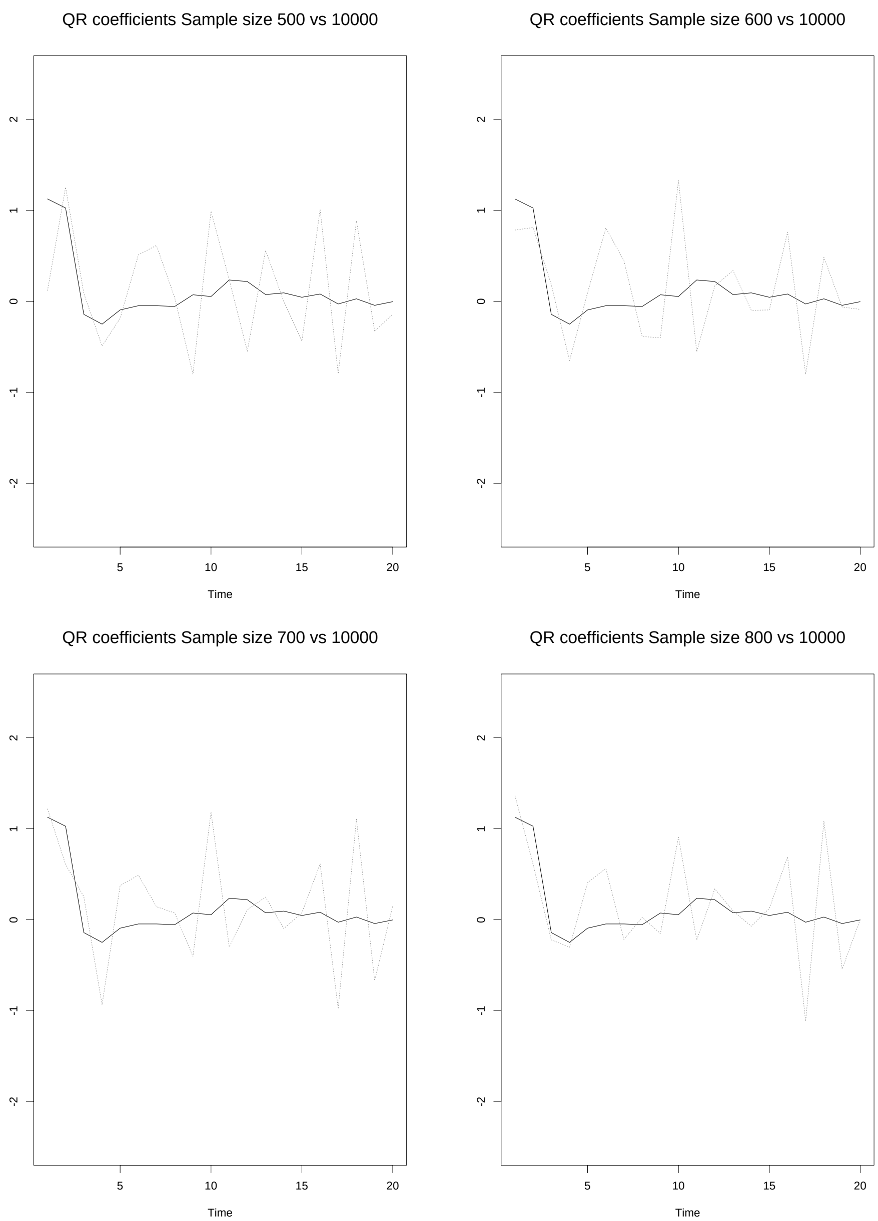

Figure 20: Stability of QR Coefficients vs Sample Size

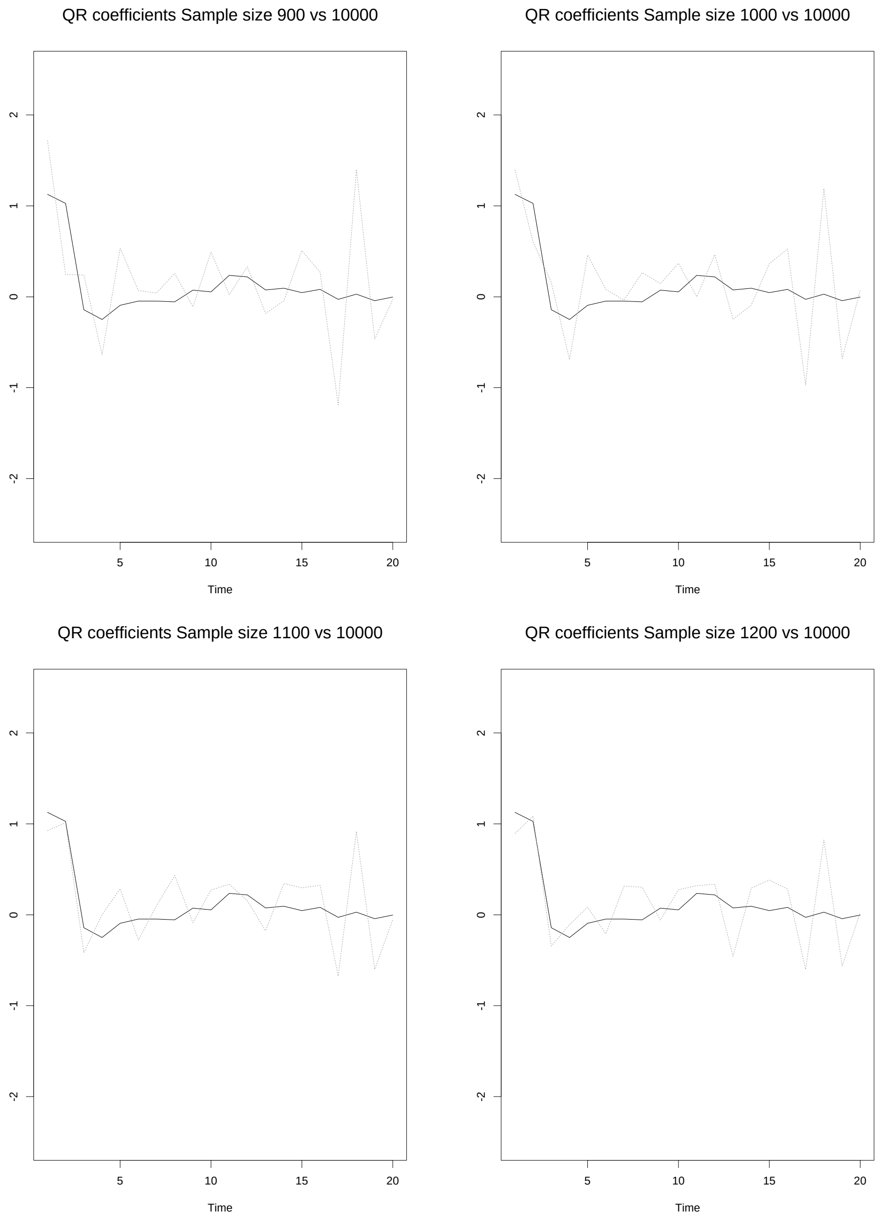

Figure 21: Stability of QR Coefficients vs Sample Size

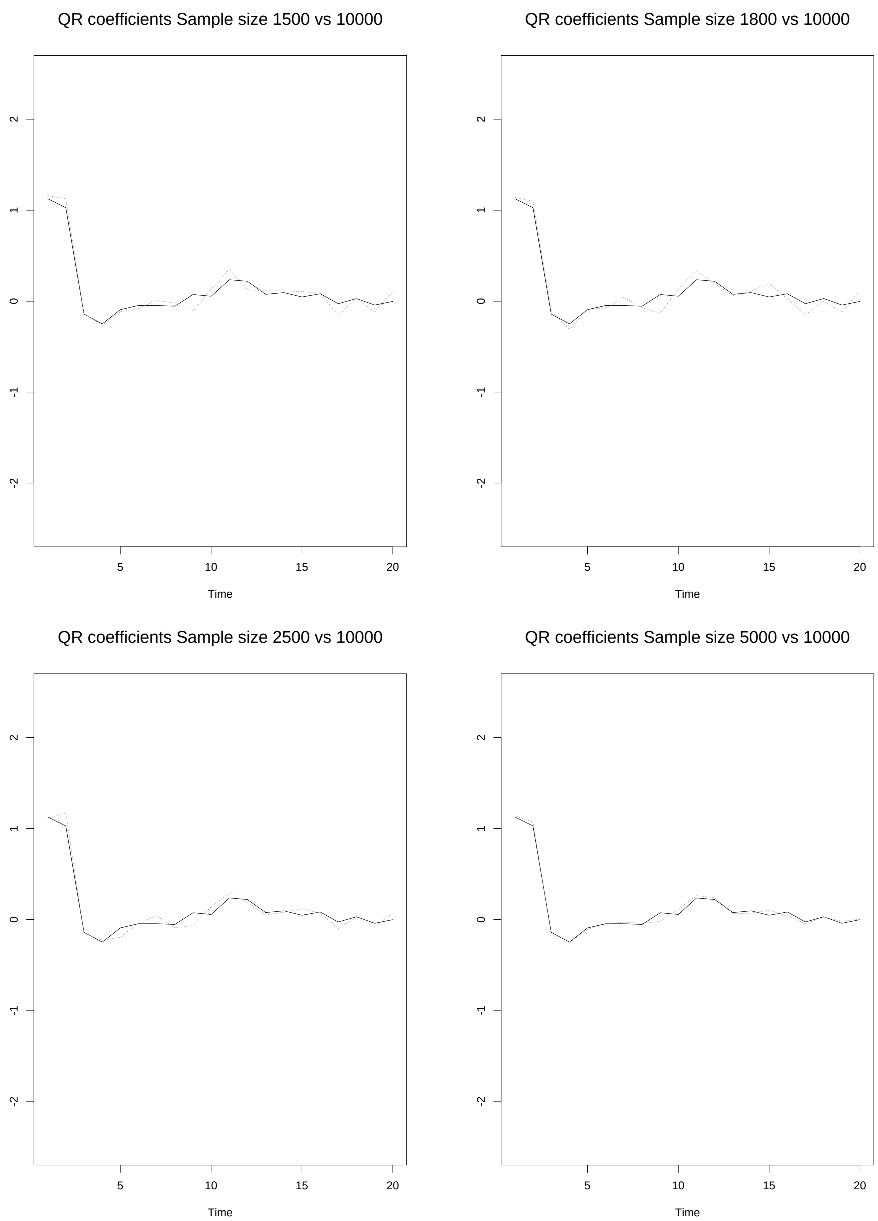

Figure 22: Stability of QR Coefficients vs Sample Size

Epoch	Min	1st Qtr	Median	Mean	3rd Qtr	Max	StDev
0	0.05766	0.05766	0.05766	0.05766	0.05766	0.05766	0
1	0.02757	0.04952	0.0558	0.05761	0.0639	0.131	0.011966
2	0.02404	0.04849	0.056	0.05801	0.06553	0.1437	0.013808
3	0.01987	0.04717	0.05614	0.05835	0.06701	0.1532	0.01573
4	0.01919	0.04669	0.05668	0.05889	0.06833	0.1583	0.017266
5	0.01486	0.04587	0.0564	0.05922	0.06931	0.25	0.018752
6	0.01303	0.04511	0.05648	0.0595	0.07036	0.2448	0.020321
7	0.01333	0.04418	0.05624	0.06002	0.07153	0.2188	0.022162
8	0.01501	0.04353	0.05613	0.06022	0.07218	0.25	0.023283
9	0.0129	0.04329	0.05623	0.06069	0.07322	0.2351	0.024611
10	0.01304	0.04259	0.05647	0.06124	0.07424	0.2409	0.026298
11	0.01091	0.04215	0.05617	0.06173	0.07516	0.2486	0.027419
12	0.01159	0.04197	0.05631	0.06223	0.07614	0.25	0.028773
13	0.008839	0.04122	0.05681	0.0625	0.07659	0.2455	0.029505
14	0.01084	0.04126	0.05673	0.063	0.07702	0.25	0.0311
15	0.008075	0.04077	0.05693	0.06356	0.07826	0.25	0.031994
16	0.009941	0.04047	0.05688	0.06402	0.07967	0.25	0.032936
17	0.01065	0.04029	0.05692	0.06438	0.0804	0.25	0.033719
18	0.009216	0.0395	0.05667	0.06464	0.08102	0.25	0.034989
19	0.008874	0.03918	0.05668	0.06488	0.0823	0.25	0.035375
20	0.006733	0.03919	0.05656	0.0653	0.08274	0.25	0.036229

Table 7: Summary of 1992 Interest Rate file-90 Day Rates

Epoch	Min	1st Qtr	Median	Mean	3rd Qtr	Max	StDev
0	0.04447	0.04447	0.04447	0.04447	0.04447	0.04447	0
1	0.02168	0.03866	0.04394	0.04497	0.05026	0.09802	0.009038
2	0.01752	0.03766	0.04396	0.04533	0.05153	0.1236	0.010825
3	0.01647	0.03683	0.04399	0.04564	0.05246	0.1308	0.012381
4	0.01083	0.03615	0.04417	0.04607	0.05383	0.1288	0.013824
5	0.0125	0.03588	0.04411	0.04656	0.05446	0.155	0.015224
6	0.01216	0.03529	0.04402	0.04682	0.05539	0.157	0.016313
7	0.01143	0.03477	0.04415	0.04724	0.0561	0.1903	0.0177
8	0.01117	0.03428	0.04447	0.04776	0.05731	0.2179	0.019034
9	0.01044	0.0338	0.04436	0.048	0.05791	0.2408	0.020308
10	0.009545	0.03344	0.04446	0.04859	0.05894	0.2136	0.021459
11	0.006834	0.03312	0.04448	0.04884	0.05953	0.2009	0.022358
12	0.00817	0.03254	0.0445	0.04936	0.0608	0.2145	0.023622
13	0.007931	0.03236	0.04424	0.04953	0.06116	0.2401	0.024364
14	0.007837	0.03212	0.04445	0.04982	0.06183	0.221	0.025135
15	0.007195	0.03155	0.04411	0.05013	0.06255	0.25	0.026098
16	0.005955	0.03142	0.04441	0.05059	0.06298	0.2054	0.027301
17	0.006838	0.03099	0.04452	0.05084	0.06334	0.25	0.028024
18	0.006772	0.03064	0.04439	0.05114	0.06421	0.2275	0.028786
19	0.007085	0.03028	0.04417	0.05148	0.06474	0.2363	0.029602
20	0.005651	0.03023	0.04391	0.05163	0.06528	0.25	0.030169

Table 8: Summary of 1993 Interest Rate file-90 Day Rates

Epoch	Min	1st Qtr	Median	Mean	3rd Qtr	Max	StDev
0	0.06315	0.06315	0.06315	0.06315	0.06315	0.06315	0
1	0.02813	0.05357	0.06096	0.06239	0.06983	0.1361	0.012546
2	0.02453	0.05227	0.06103	0.06291	0.07153	0.1716	0.014925
3	0.02286	0.0511	0.06103	0.06336	0.07286	0.1816	0.017191
4	0.01504	0.05016	0.06131	0.06394	0.07469	0.1788	0.019144
5	0.01801	0.0497	0.06117	0.06461	0.07554	0.2152	0.021202
6	0.01688	0.04891	0.06118	0.06495	0.0769	0.2179	0.022607
7	0.01762	0.04842	0.06137	0.06559	0.07776	0.2284	0.024463
8	0.01551	0.04751	0.06169	0.06617	0.0794	0.2246	0.026126
9	0.0145	0.04684	0.06147	0.06644	0.08042	0.2338	0.027565
10	0.01325	0.04635	0.0618	0.06712	0.08169	0.25	0.028849
11	0.01311	0.04595	0.06182	0.06748	0.08273	0.25	0.030015
12	0.01134	0.04512	0.06182	0.06799	0.0843	0.25	0.031151
13	0.0119	0.04489	0.06141	0.06807	0.08469	0.25	0.03193
14	0.01088	0.04464	0.06183	0.06848	0.08585	0.2452	0.032613
15	0.009989	0.04383	0.06128	0.06871	0.08677	0.25	0.033794
16	0.008267	0.04362	0.06173	0.06912	0.08743	0.25	0.034713
17	0.009928	0.04296	0.06169	0.06916	0.08776	0.25	0.035041
18	0.009402	0.04263	0.06137	0.06927	0.08836	0.25	0.035872
19	0.009582	0.04213	0.06146	0.0697	0.08953	0.25	0.036564
20	0.006667	0.04189	0.06084	0.06957	0.08995	0.25	0.036978

Table 9: Summary of 1994 Interest Rate file-90 Day Rates

Epoch	Min	1st Qtr	Median	Mean	3rd Qtr	Max	StDev
0	0.067	0.067	0.067	0.067	0.067	0.067	0
1	0.04139	0.06202	0.06722	0.06758	0.07272	0.1123	0.007928
2	0.03261	0.05996	0.06714	0.06798	0.07505	0.1369	0.011399
3	0.02829	0.0584	0.06721	0.06842	0.07711	0.1413	0.014053
4	0.02367	0.05727	0.06712	0.06893	0.07877	0.166	0.01637
5	0.02279	0.05622	0.06729	0.06942	0.0801	0.1907	0.018564
6	0.01868	0.05505	0.06757	0.06983	0.08132	0.2	0.020556
7	0.01898	0.05418	0.06722	0.0703	0.08293	0.2	0.022552
8	0.01949	0.05321	0.06715	0.07074	0.0841	0.2	0.024311
9	0.01981	0.05251	0.06689	0.07127	0.08544	0.2	0.02612
10	0.01799	0.05197	0.0671	0.07188	0.08662	0.2	0.027832
11	0.01709	0.05158	0.06718	0.07237	0.0875	0.2	0.029319
12	0.01607	0.05079	0.06732	0.07296	0.08882	0.2	0.030872
13	0.01461	0.05009	0.06768	0.07348	0.08962	0.2	0.032139
14	0.01458	0.04976	0.06721	0.07387	0.09026	0.2	0.033479
15	0.01229	0.0496	0.06756	0.07447	0.09139	0.2	0.034694
16	0.01167	0.04882	0.06761	0.0749	0.09256	0.2	0.035816
17	0.01161	0.04849	0.06759	0.07533	0.09411	0.2	0.036886
18	0.0107	0.04782	0.06753	0.07574	0.09521	0.2	0.037881
19	0.01031	0.04736	0.0674	0.07607	0.09622	0.2	0.03879
20	0.01	0.04698	0.06768	0.07636	0.09721	0.2	0.039591

Table 10: Summary of 1992 Interest Rate file-10 Year Rates

Epoch	Min	1st Qtr	Median	Mean	3rd Qtr	Max	StDev
0	0.0583	0.0583	0.0583	0.0583	0.0583	0.0583	0
1	0.0375	0.05373	0.05825	0.05876	0.06331	0.09913	0.007255
2	0.02743	0.05164	0.05827	0.05919	0.06567	0.109	0.010435
3	0.02226	0.05018	0.05815	0.05961	0.06743	0.1258	0.012996
4	0.02066	0.04928	0.05825	0.06013	0.06915	0.1462	0.015146
5	0.02173	0.04849	0.05836	0.06065	0.07037	0.1612	0.017025
6	0.01797	0.04779	0.05846	0.06114	0.07171	0.1622	0.018708
7	0.01616	0.04695	0.05825	0.06168	0.07277	0.2	0.020648
8	0.0163	0.04649	0.05836	0.0622	0.07371	0.2	0.022441
9	0.01296	0.04556	0.05867	0.06268	0.07509	0.2	0.024166
10	0.01372	0.04502	0.05877	0.0633	0.07642	0.2	0.025704
11	0.01049	0.04437	0.05881	0.06385	0.07781	0.2	0.027189
12	0.01154	0.04391	0.05883	0.06438	0.0788	0.2	0.028646
13	0.01221	0.04336	0.0587	0.06462	0.07921	0.2	0.029649
14	0.01089	0.04312	0.05877	0.065	0.07989	0.2	0.030764
15	0.01026	0.04256	0.05841	0.06535	0.08079	0.2	0.03196
16	0.01053	0.04223	0.05822	0.06584	0.08184	0.2	0.033262
17	0.01077	0.04144	0.05852	0.06622	0.0823	0.2	0.034348
18	0.01	0.04103	0.05834	0.06665	0.08334	0.2	0.035288
19	0.01	0.04079	0.05855	0.06704	0.08438	0.2	0.036404
20	0.01	0.04052	0.05852	0.06733	0.08517	0.2	0.037133

Table 11: Summary of 1993 Interest Rate file-10 Year Rates

Epoch	Min	1st Qtr	Median	Mean	3rd Qtr	Max	StDev
0	0.08094	0.08094	0.08094	0.08094	0.08094	0.08094	0
1	0.05206	0.07463	0.08095	0.08163	0.08795	0.1376	0.010052
2	0.03809	0.0717	0.08087	0.0822	0.09119	0.1513	0.014486
3	0.0309	0.06972	0.08071	0.08279	0.09367	0.1747	0.018072
4	0.02868	0.06839	0.08083	0.08341	0.09593	0.2	0.020981
5	0.03101	0.06728	0.08096	0.08411	0.09743	0.2	0.023545
6	0.02495	0.06639	0.08115	0.0848	0.09928	0.2	0.025811
7	0.02244	0.06524	0.08095	0.08559	0.1009	0.2	0.028357
8	0.02263	0.06443	0.08102	0.08618	0.1022	0.2	0.030613
9	0.01799	0.06316	0.08138	0.08672	0.1043	0.2	0.032577
10	0.01904	0.06238	0.08159	0.08745	0.106	0.2	0.034302
11	0.02001	0.06151	0.08166	0.08803	0.1077	0.2	0.035808
12	0.01707	0.06091	0.08178	0.08855	0.1092	0.2	0.037227
13	0.01695	0.06029	0.08151	0.08874	0.1098	0.2	0.038259
14	0.01511	0.05991	0.08173	0.08913	0.1107	0.2	0.039413
15	0.01425	0.0591	0.08111	0.08931	0.1121	0.2	0.040429
16	0.01462	0.05864	0.08102	0.08976	0.1135	0.2	0.04159
17	0.01495	0.0576	0.08128	0.08995	0.1141	0.2	0.042398
18	0.01351	0.05697	0.0809	0.09026	0.1152	0.2	0.043305
19	0.0118	0.05663	0.08135	0.09051	0.1166	0.2	0.04418
20	0.01035	0.05631	0.08111	0.09063	0.1172	0.2	0.044801

Table 12: Summary of 1994 Interest Rate file-10 Year Rates

LOB	Min	1st Qtr	Median	Mean	3rd Qtr	Max	StDev
LOB01	-940593	1025416	1118093	31604426	64465488	91514604	32371854
LOB02	61229648	64999559	66130533	65879205	66853099	68423896	1238074
LOB03	-43825222	6914646	14630399	12446991	20514602	26046948	10187328
LOB04	-301153105	1012849583	1098929231	1062800912	1157362861	1309101803	145860786
LOB06	-216142	227985	282052	269737	327982	407779	77320
LOB07	56737	469354	522133	511765	562659	625805	65256
LOB09	98583	106846	108605	108385	110113	115265	2431
LOB10	8391195	23128408	25767346	24648939	26892036	29113142	3102505
LOB15	10734823	11322301	11447965	11435399	11561062	12195664	176237
LOB16	7831991	8136726	8227832	8271814	8350354	9782921	208740
LOB17	-54318143	20806770	21460222	20238142	21931461	24052036	5158386
LOB19	1682637	1803903	1832805	1829978	1858881	1939934	40262
LOB26	-212968589	179699120	246646466	236090714	305708415	388300895	87786510
LOB27	-699947	289812	419717	395212	540354	743615	195905
LOB28	-237127	298577	399296	387673	487889	648425	130303
LOB29	20093629	38358851	41940267	40495134	43699559	47343807	4304978
LOB31	4640	2747323	2992367	2938960	3188717	3597124	354284
LOB32	8422611	42223010	46872568	46275665	51113718	57585400	6005240
LOB33	6349	121014	127172	126418	133518	179228	12781
LOB34	16877	42160	44862	44296	47124	58465	4380
LOB35	131633	816814	842261	834721	865823	990858	56077
LOB36	601615	2294934	2352425	2321323	2403947	2763659	164004
LOB42	-3088186	2764602	3147876	2931420	3301814	3710221	584258
LOB44	-5167920	4605575	6160664	5943894	7520974	10144204	2024591
LOB45	16619027	29618939	31356240	30551992	32264160	34243364	2511552
LOB46	468725676	579938068	614181432	607898246	640570813	684238955	39438346
LOB47	197637616	297571689	309572574	304483194	317300893	328610628	18058247
LOB48	1358739	2840314	3082531	3019071	3260974	3637965	334594
LOB49	-29216815	152430092	168263721	154911951	176714606	200967704	40408696
LOB50	13524558	17077700	17935354	17765708	18604514	20564868	1143490
LOB51	10128496	11554779	11815531	11784115	12051151	12981062	390580

Table 13: Summary Statistics on 1992 EVAS

LOB	Min	1st Qtr	Median	Mean	3rd Qtr	Max	StDev
LOB01	1582106	3421195	3480885	3427478	3512301	3603407	168897
LOB02	-1792279	603186	1329522	1165531	1741699	3273540	808561
LOB03	-38641594	25139027	34871682	31604426	40777877	45678762	11651304
LOB04	-4624425	1014106220	1075053125	1038924805	1103641621	1141340737	105929075
LOB05	-112123454	68832303	95315932	86362392	110929649	131507083	31528641
LOB06	-288430	268638	338350	316673	387044	438566	89463
LOB07	-355628	-57460	-7248	-14316	33301	304735	72791
LOB09	50014	114668	124596	122868	132984	157237	13965
LOB10	-974522	19135443	23772434	21639293	25472036	27510930	5125355
LOB13	-200999	69775	114857	108542	151425	388301	64564
LOB14	-12107699	5013982	6873805	6025575	7756593	9672965	2588079
LOB15	-17932213	-11086682	-10163053	-10345266	-9402788	-7282213	1311714
LOB16	0	6732434	7209956	7147124	7627788	9204867	672892
LOB17	-19565841	20586859	22267611	21381682	23184956	25377788	3163606
LOB19	857655	1867363	2025699	2002766	2157960	2629199	218513
LOB20	-266250	842575	972637	906035	1030443	1090447	177722
LOB21	403380541	823097366	970123918	904464625	1015991176	1069712417	146637391
LOB22	385159302	798592941	944362856	879331881	989915954	1045522150	145649096
LOB23	16832655	24259381	25380930	25085620	26116063	27658585	1334897
LOB24	358455761	609469042	723508868	670730106	752097364	799535419	107918834
LOB25	371022133	1025101796	1068141620	1052433655	1094531001	1143854011	57085770
LOB26	-270742485	123464605	186767704	166912836	226854431	269328768	75010292
LOB27	-619208	242374	375106	341805	473124	593761	163029
LOB28	-269140	285822	387987	365053	470296	579624	132008
LOB29	-5045398	27935045	36505311	32609735	39332744	41783187	8794651
LOB30	30306948	99871241	117118587	109013277	122930534	130250446	18753296
LOB31	-499199	1199146	1424398	1343973	1565456	1972292	311497
LOB32	10678275	38296019	42662833	41940267	46432745	53564161	5872278
LOB33	-102636	98552	105369	96918	108008	117590	22506
LOB34	-109830	-33772	-11699	-5372	17216	172631	35386
LOB35	-703717	257328	349659	322327	423801	884044	172197
LOB36	-1532783	605699	992743	990858	1393611	3914425	659907
LOB37	-2376615	435739	1246898	1347115	2143509	8457168	1383691
LOB38	-14008	1648394	2547518	2189376	2774969	3179292	737578
LOB39	-44359293	-16336284	-15415797	-15566593	-14605266	-10715974	1499000
LOB42	-5246460	931796	1267633	1069712	1435080	1664102	571744
LOB43	-27743408	998712415	1059659319	1023216840	1087933656	1122177020	106034186
LOB44	-4721814	2483429	3889292	3487168	4790929	6047567	1640200
LOB45	21651859	32923895	34588939	33960620	35594249	37667700	2314476
LOB46	0	661305327	684238955	677955769	700261080	730734532	29836871
LOB47	175897792	295749565	312588504	305614167	322641601	339920363	24263038
LOB48	1264805	2367190	2558199	2509190	2704912	3022527	267395
LOB49	-93619471	71094250	82686728	77314604	90006639	104049560	19645597
LOB50	-83252	2470235	2848797	2797903	3185575	4231726	546354
kOB51	10367257	11620753	11843806	11793540	12013452	12707744	310310

Table 14: Summary Statistics on 1993 EVAS

LOB	Min	1st Qtr	Median	Mean	3rd Qtr	Max	StDev
LOB02	3026611	7904248	10109646	9999691	12176814	18488275	2807380
LOB03	-66036285	10241593	25811328	23571372	41500444	52527435	20776445
LOB04	-637743379	1056831885	1215482332	1129088524	1298734546	1427854019	259178903
LOB05	-186830536	25091903	63334515	53344249	91891595	129402216	49778987
LOB06	-481292	112972	219000	185448	286985	415004	134358
LOB07	-675442	151896	199365	191951	251642	505796	123860
LOB08	-205429	-88530	-73450	-73199	-59627	320442	27438
LOB09	-158839	-91138	-80048	-81838	-70717	-42412	15430
LOB10	-20417213	4514469	13285797	12484691	21570178	27033408	9691645
LOB11	-733247806	71722568	238069918	199271244	371022133	509566385	206425154
LOB12	-732933647	71471241	237975670	199176996	370707974	509566385	206468684
LOB13	-717854	69052	119946	110113	166599	422230	116840
LOB17	-71408409	43165488	53407081	48066373	58119471	67669913	15820396
LOB18	8209	149603	175458	168484	191135	249757	33269
LOB26	-462756649	24130576	110301330	84603099	167541155	269611511	109357715
LOB27	-904779	57051	247149	198454	384845	625177	235721
LOB28	-472496	46150	132952	106437	185574	393956	116730
LOB29	-42254426	6776416	26948585	24259381	43448231	55888939	20809114
LOB33	-80676	153687	157677	155572	161164	179856	12820
LOB34	-56674	8705	36757	37762	63115	166285	38106
LOB35	-247338	415004	475009	482235	546637	917659	108171
LOB36	-851372	1156106	1539066	1596243	1983602	4470487	643638
LOB37	-1761491	1468066	2370332	2517987	3430620	8592257	1518441
LOB38	-3848451	1997739	5023407	4615000	7159690	10417522	2875161
LOB39	-53532745	-3304956	-2414628	-2668469	-1564513	1422827	2435794
LOB40	-347774345	109924339	180515934	159215933	228173900	373849567	97689330
LOB41	-118689384	14209425	32421240	31698673	49919913	109547348	26489215
LOB43	-643084087	1053690292	1213283217	1126261091	1297163750	1428796496	260876265
LOB44	-8959823	980177	3099810	2575792	4693540	6999469	2623132
LOB48	-51270798	919230	6063274	4093496	10049956	74738497	12880889
LOB49	-246520803	27058541	76560621	65345134	115045136	208978766	66347425

Table 15: Summary Statistics on 1994 EVAS

LOB	Min	1st Qtr	Median	Mean	3rd Qtr	Max	StDev
LOB01	0.2	0.3182	0.5951	0.4619	0.5955	0.6041	0.1415
LOB02	0.2	0.2092	0.2133	0.2149	0.2201	0.2421	0.0072
LOB03	0.2	0.2849	0.3753	0.4088	0.4938	1.273	0.1565
LOB04	0.2	0.2464	0.2642	0.2753	0.2905	0.692	0.0446
LOB06	0.2	0.2784	0.3233	0.3354	0.3764	0.812	0.0758
LOB07	0.2	0.2404	0.2662	0.2728	0.3	0.5637	0.0417
LOB09	0.2	0.2179	0.2231	0.2239	0.2293	0.2579	0.0084
LOB10	0.2	0.2305	0.246	0.2613	0.2822	0.4847	0.0426
LOB15	0.2	0.2209	0.2246	0.225	0.2287	0.2479	0.0058
LOB16	0.2	0.2585	0.2635	0.2618	0.2673	0.2797	0.0085
LOB17	0.2	0.2352	0.2431	0.2634	0.254	1.503	0.0858
LOB19	0.2	0.2167	0.222	0.2227	0.228	0.2531	0.0083
LOB26	0.2	0.285	0.3459	0.3568	0.4148	0.8194	0.0905
LOB27	0.2	0.3093	0.3742	0.3874	0.4441	0.9766	0.1054
LOB28	0.2	0.2991	0.3538	0.361	0.4158	0.7463	0.0804
LOB29	0.2	0.2309	0.2456	0.2581	0.2759	0.4303	0.0364
LOB31	0.2	0.2456	0.2674	0.2733	0.2946	0.5995	0.0394
LOB32	0.2	0.245	0.2744	0.2785	0.3067	0.5415	0.0417
LOB33	0.2	0.302	0.3162	0.3179	0.3299	0.5858	0.0285
LOB34	0.2	0.2776	0.2931	0.2969	0.3115	0.4845	0.03
LOB35	0.2	0.2505	0.2599	0.263	0.2702	0.5469	0.0226
LOB36	0.2	0.2521	0.2595	0.264	0.2678	0.5129	0.0237
LOB42	0.2	0.2442	0.2607	0.284	0.302	0.9329	0.063
LOB44	0.2	0.3034	0.3571	0.3655	0.4184	0.8039	0.0798
LOB45	0.2	0.2229	0.2336	0.243	0.254	0.4058	0.0293
LOB46	0.2	0.2255	0.2408	0.2445	0.2609	0.3259	0.0231
LOB47	0.2	0.2139	0.2232	0.2294	0.2378	0.3594	0.022
LOB48	0.2	0.2413	0.261	0.268	0.2877	0.4506	0.0368
LOB49	0.2	0.2482	0.2651	0.2917	0.2966	0.6582	0.0804
LOB50	0.2	0.2382	0.2512	0.2545	0.2678	0.3369	0.0222
LOB51	0.2	0.2287	0.236	0.2369	0.2439	0.2879	0.012

Table 16: Summary Statistics on Adjusted 1992 EVAS

LOB	Min	1st Qtr	Median	Mean	3rd Qtr	Max	StDev
LOB01	0.2	0.2101	0.2137	0.2197	0.2204	0.4244	0.0187
LOB02	0.2	0.3871	0.4375	0.4576	0.5263	0.8191	0.0988
LOB03	0.2	0.2428	0.2947	0.3231	0.3798	0.9384	0.1021
LOB04	0.2	0.2132	0.2232	0.2359	0.2445	0.6016	0.0371
LOB05	0.2	0.2626	0.3101	0.3373	0.3906	0.9411	0.0959
LOB06	0.2	0.247	0.2916	0.3113	0.3551	0.863	0.0816
LOB07	0.2	0.5563	0.6095	0.6188	0.6754	1.067	0.0955
LOB09	0.2	0.2617	0.2831	0.2874	0.3083	0.4728	0.0355
LOB10	0.2	0.2296	0.2543	0.2853	0.3218	0.6142	0.0745
LOB13	0.2	0.444	0.4816	0.4881	0.5281	0.8071	0.0665
LOB14	0.2	0.2793	0.3157	0.3508	0.3927	1.101	0.107
LOB15	0.803	0.8622	0.8834	0.8885	0.9091	1.1	0.0366
LOB16	0.2	0.2685	0.2866	0.2894	0.3074	0.6	0.0292
LOB17	0.2	0.2346	0.249	0.263	0.2755	0.9084	0.0499
LOB19	0.2	0.2717	0.2918	0.2953	0.3159	0.4695	0.0332
LOB20	0.2	0.222	0.2432	0.2676	0.2908	0.6977	0.0652
LOB21	0.2	0.2201	0.2372	0.2618	0.2922	0.4492	0.0548
LOB22	0.2	0.2212	0.2386	0.2635	0.2944	0.4526	0.0557
LOB23	0.2	0.2223	0.2329	0.2372	0.2492	0.3566	0.0193
LOB24	0.2	0.2237	0.2381	0.2644	0.2951	0.4207	0.054
LOB25	0.2	0.2172	0.2265	0.2319	0.2415	0.4702	0.02
LOB26	0.2	0.2631	0.3226	0.3521	0.4166	1.002	0.1114
LOB27	0.2	0.2814	0.3473	0.3699	0.4368	1.017	0.1098
LOB28	0.2	0.2754	0.3321	0.348	0.4027	0.7858	0.0911
LOB29	0.2	0.2236	0.2507	0.2878	0.3326	0.6483	0.0842
LOB30	0.2	0.2225	0.2403	0.2652	0.2933	0.5069	0.0576
LOB31	0.2	0.2825	0.3111	0.3275	0.3568	0.7012	0.0632
LOB32	0.2	0.2533	0.2815	0.2868	0.314	0.5203	0.0438
LOB33	0.2	0.2326	0.2416	0.2703	0.2648	0.9492	0.0766
LOB34	0.2	0.5601	0.6271	0.6124	0.6782	0.8545	0.082
LOB35	0.2	0.4083	0.4419	0.4541	0.4836	0.9183	0.0779
LOB36	0.2	0.4576	0.4986	0.4988	0.5381	0.7566	0.0674
LOB37	0.2	0.4986	0.541	0.5363	0.5794	0.7124	0.0654
LOB38	0.2	0.251	0.2796	0.3246	0.3927	0.6018	0.0928
LOB39	0.7208	0.7647	0.7738	0.7755	0.7842	1.1	0.0169
LOB42	0.2	0.2551	0.2953	0.3429	0.376	1.861	0.1374
LOB43	0.2	0.2122	0.2223	0.2353	0.244	0.6099	0.0378
LOB44	0.2	0.2832	0.3428	0.3693	0.4357	0.9124	0.1085
LOB45	0.2	0.222	0.2329	0.2395	0.2506	0.3701	0.0246
LOB46	0.2	0.2167	0.2253	0.2289	0.2379	0.6	0.0163
LOB47	0.2	0.2205	0.2322	0.2404	0.252	0.3931	0.0285
LOB48	0.2	0.2421	0.2615	0.2679	0.2867	0.4326	0.0354
LOB49	0.2	0.2539	0.2821	0.3027	0.3266	0.9599	0.0755
LOB50	0.2	0.2988	0.3308	0.3356	0.3666	0.6079	0.0516
LOB51	0.2	0.2219	0.2272	0.2287	0.2342	0.2736	0.0098

Table 17: Summary Statistics on adjusted 1993 EVAS

LOB	Min	1st Qtr	Median	Mean	3rd Qtr	Max	StDev
LOB02	0.2	0.3366	0.3813	0.3837	0.429	0.5345	0.0607
LOB03	0.2	0.2838	0.4034	0.4205	0.522	1.103	0.1583
LOB04	0.2	0.2361	0.2595	0.2837	0.3039	0.7787	0.0726
LOB05	0.2	0.316	0.4042	0.4352	0.5224	1.177	0.1539
LOB06	0.2	0.3235	0.389	0.4213	0.4911	1.064	0.1295
LOB07	0.2	0.401	0.4424	0.4483	0.4799	1.134	0.0979
LOB08	0.2	0.6745	0.6917	0.6914	0.7105	0.8565	0.0343
LOB09	0.7335	0.8226	0.8519	0.8576	0.8869	1.1	0.0486
LOB10	0.2	0.2808	0.4034	0.4153	0.5332	0.9021	0.1434
LOB11	0.2	0.3088	0.4131	0.4436	0.5437	1.176	0.1621
LOB12	0.2	0.3089	0.4132	0.4437	0.5439	1.175	0.1621
LOB13	0.2	0.4421	0.4863	0.4957	0.5346	1.28	0.1107
LOB17	0.2	0.2565	0.2845	0.316	0.3448	1.022	0.0935
LOB18	0.2	0.2939	0.319	0.3301	0.3604	0.5869	0.0533
LOB26	0.2	0.3514	0.4364	0.4745	0.5642	1.287	0.1622
LOB27	0.2	0.3537	0.4419	0.4731	0.5635	1.179	0.1508
LOB28	0.2	0.4116	0.465	0.4919	0.5531	1.08	0.1185
LOB29	0.2	0.289	0.4071	0.4264	0.5515	0.9023	0.1489
LOB33	0.2	0.2415	0.2493	0.254	0.2582	0.7794	0.0285
LOB34	0.2	0.4482	0.5116	0.5091	0.5791	0.7364	0.0917
LOB35	0.2	0.3617	0.393	0.3898	0.4192	0.7078	0.0471
LOB36	0.2	0.4225	0.4622	0.4571	0.4965	0.6762	0.0576
LOB37	0.2	0.4403	0.4897	0.4828	0.5317	0.682	0.0707
LOB38	0.2	0.3251	0.4072	0.4228	0.5233	0.7478	0.1104
LOB39	0.2	1.04	1.279	1.35	1.529	15.65	0.6848
LOB40	0.2	0.3559	0.4069	0.4297	0.4824	0.9721	0.1045
LOB41	0.2	0.4177	0.4816	0.4843	0.5481	1.033	0.0967
LOB43	0.2	0.2369	0.2604	0.2847	0.3051	0.78	0.073
LOB44	0.2	0.3317	0.4229	0.4528	0.544	1.112	0.1499
LOB48	0.2	0.5462	0.5676	0.5781	0.5951	0.8743	0.0689
LOB49	0.2	0.3798	0.4535	0.4749	0.5482	1.072	0.127

Table 18: Summary Statistics on adjusted 1994 EVAS

LOB	alpha	gamma	lamda	tau	MLE	100D	p
LOB01	1.44137	12.49156	1.00856	3.11482	-5399.6	0.33907	0
LOB02	73.8615	13.17351	5.65237	9.17463	-35360.5	0.0524	2.80208E-24
LOB03	15.45139	3.06349	6.64678	1.89284	-5891.3	0.04585	1.09023E-18
LOB04	7.42035	0.62143	4.73441	13.78866	-19554.7	0.01856	0.002039412
LOB06	8.81502	1.87909	4.69871	4.14381	-12621.5	0.02438	1.36857E-05
LOB07	7.782	1.81716	4.89826	6.01043	-18126.5	0.03091	1.00267E-08
LOB09	21.50428	6.5683	4.96427	12.00264	-33675.4	0.01769	0.00382365
LOB10	47.79919	0.55325	5.75678	13.85087	-19568.3	0.0865	2.05448E-65
LOB15	75.91405	12.3492	5.20176	11.87497	-37424.1	0.01829	0.002490844
LOB16	8.28344	25.99859	3.50831	13.79604	-33857.1	0.10427	7.28213E-95
LOB17	10.82257	0.99574	5.536	8.04963	-19790.3	0.25296	0
LOB19	38.32135	7.60098	5.21845	11.10518	-33835.3	0.02618	2.22803E-06
LOB26	25.98523	38.92727	1.93306	1.00894	-10368.4	0.02621	2.16373E-06
LOB27	24.49914	9.61497	5.12391	1.46634	-9126.3	0.0275	5.39729E-07
LOB28	13.97195	16.02642	2.61283	1.70935	-11290.9	0.03291	7.83275E-10
LOB29	15.96639	0.8588	5.40922	10.80068	-20768.9	0.08829	3.94705E-68
LOB31	7.22249	1.79636	4.71231	6.82502	-19121.2	0.02253	7.80209E-05
LOB32	13.33398	2.72304	5.26738	4.6439	-17933.5	0.03144	5.2213E-09
LOB33	3.81926	1.95664	3.37201	11.46786	-22172.8	0.07643	3.61934E-51
LOB34	8.93728	2.92488	3.98565	7.42684	-21522.7	0.03662	4.47866E-12
LOB35	11.14562	1.53555	4.53096	13.75634	-25742.6	0.05981	1.70091E-31
LOB36	17.73501	1.39022	4.67154	13.72495	-25879.7	0.13925	7.6719E-169
LOB42	40.07475	0.89656	6.28191	8.00447	-17268.3	0.08429	3.91858E-62
LOB44	8.87780	5.26700	3.44070	2.62490	-11470.0	0.03770	8.55018E-13
LOB45	24.02963	1.7672	5.78115	8.88356	-23125.8	0.06107	8.15307E-33
LOB46	11.89941	3.22448	5.01958	7.04407	-23827.5	0.05119	3.45771E-23
LOB47	45.15769	1.60852	6.14279	10.4657	-25920.0	0.12095	1.7559E-127
LOB48	9.84747	1.77306	4.93378	7.24753	-19795.0	0.03102	8.79858E-09
LOB49	4.43184	0.74625	4.7655	8.33223	-16187.9	0.09611	1.18845E-80
LOB50	16.73973	2.05084	5.08764	9.04962	-24277.4	0.01759	0.004100714
LOB51	8.28443	4.44982	4.46275	12.1209	-30177.3	0.02703	9.01431E-07

Table 19: Best GB2 parameters on Adj 1992 EVAS

LOB	alpha	gamma	lamda	tau	MLE	100D	p
LOB01	26.94973	1.68495	5.74229	13.67011	-29213.3	0.15393	3.1924E-206
LOB02	216.4666	15.08361	18.86667	1.26775	-9332.8	0.0479	2.34925E-20
LOB03	7.17528	0.3029	5.42252	10.19809	-10716.4	0.04919	1.94274E-21
LOB04	6.07303	1.31566	5.3207	9.09005	-22178.3	0.13923	8.186E-169
LOB05	8.36748	0.45969	5.09014	8.34709	-11063.9	0.05108	4.30321E-23
LOB06	11.28752	0.51339	5.59573	8.08819	-12596.4	0.04696	1.40439E-19
LOB07	0.99126	1.12205	1.59751	11.10605	-9448.4	0.03597	1.14934E-11
LOB09	12.04732	3.27518	4.51534	5.69138	-19612.7	0.02522	5.98319E-06
LOB10	4.95899	0.49243	5.09712	9.67116	-14600.5	0.12855	5.9122E-144
LOB13	0.97351	0.96763	2.07781	13.18082	-13071.1	0.02922	7.70042E-08
LOB14	7.5091	0.53358	4.83852	7.57354	-10914.3	0.02894	1.06371E-07
LOB15	40.78382	5.38624	1.34102	11.88836	-19309.9	0.05614	8.42507E-28
LOB16	7.61826	3.7364	3.89373	6.68055	-21376.1	0.0177	0.003802056
LOB17	22.18458	0.59442	5.41122	13.62514	-20343	0.05267	1.60285E-24
LOB19	8.0357	4.5579	3.82032	5.63175	-20086.8	0.02234	9.23673E-05
LOB20	39.85767	0.50228	6.45236	10.29818	-16820.3	0.12628	6.1487E-139
LOB21	14.3334	0.89361	6.08424	7.73886	-17212.1	0.11099	1.9959E-107
LOB22	47.17718	0.67111	6.86754	8.89522	-17276.9	0.12168	5.0574E-129
LOB23	12.72069	1.48608	5.13997	12.58096	-26139.1	0.05173	1.14048E-23
LOB24	7.5875	0.46841	5.40928	11.72497	-17669.7	0.13573	1.959E-160
LOB25	16.98278	1.31074	5.34239	13.87736	-26568.6	0.07939	3.64543E-55
LOB26	6.63427	0.52543	5.10918	6.73195	-9367.5	0.05882	1.77143E-30
LOB27	14.46447	2.66123	5.85365	2.55726	-9003.2	0.04255	3.75729E-16
LOB28	8.7937	2.24834	4.82004	3.15958	-10579.3	0.03636	6.57184E-12
LOB29	25.66884	0.30147	5.95096	13.42451	-14171.3	0.09981	5.92929E-87
LOB30	9.1505	0.46398	5.44922	12.10359	-17402.3	0.1208	3.5311E-127
LOB31	3.35545	0.64373	3.93619	10.42539	-15071.3	0.02404	1.90798E-05
LOB32	29.81764	3.19695	6.08638	4.34436	-17564.7	0.03505	4.25272E-11
LOB33	9.79839	0.46019	5.10351	13.83016	-18569.3	0.25296	0
LOB34	0.82651	8.95405	1.24354	10.88393	-11433.1	0.0517	1.22066E-23
LOB35	2.26278	1.09634	2.50472	8.67473	-12266.6	0.04928	1.59839E-21
LOB36	1.07492	1.41011	1.94728	11.13618	-12843.2	0.01891	0.001570617
LOB37	1.48364	9.48612	1.4384	8.31608	-13231.9	0.02452	1.20098E-05
LOB38	3.0088	0.30987	4.58073	11.49422	-12119.7	0.10747	9.4522E-101
LOB39	201.3755	18.46031	1.58085	11.90389	-27360.9	0.03351	3.54097E-10
LOB42	43.14292	0.22294	5.75332	13.68624	-10602.8	0.0161	0.011183452
LOB43	3.94827	0.86035	5.19226	11.02503	-22160.1	0.13419	7.9838E-157
LOB44	7.90294	1.69098	4.80778	3.40963	-9173.8	0.04451	1.24232E-17
LOB45	10.45812	0.99723	5.17786	13.37544	-24552.3	0.06326	3.51148E-35
LOB46	11.45297	5.30266	4.81699	8.48471	-27714.2	0.08636	3.28555E-65
LOB47	20.93862	1.06288	5.65213	11.52806	-23530.9	0.06316	4.48356E-35
LOB48	8.82588	1.40836	4.9136	8.25596	-20129.5	0.01543	0.01713598
LOB49	13.75886	0.54945	5.3071	9.48073	-14847.4	0.01999	0.000676577
LOB50	11.66233	7.33216	3.49616	3.22142	-15701.2	0.01695	0.006386356
LOB51	50.14382	4.1308	5.33491	13.07761	-32547	0.04473	8.38247E-18

Table 20: Best GB2 parameters on Adj 1993 EVAS

LOB	alpha	gamma	lamda	tau	MLE	100D	p
LOB02	4.13766	19.87701	1.65736	3.56168	-13814.8	0.03685	3.19193E-12
LOB03	5.56751	6.71915	2.23156	1.58441	-4927.7	0.06299	6.86107E-35
LOB04	51.89633	0.4181	6.09231	11.92538	-16068.4	0.06496	4.44255E-37
LOB05	821.18768	38.20455	1309.41069	0.49025	-5691.5	0.02608	2.4737E-06
LOB06	10.5576	1.61434	4.7562	3.30353	-7667.4	0.0323	1.73411E-09
LOB07	1.496	1.36362	2.32796	6.62749	-9834.8	0.08776	2.52141E-67
LOB08	3.61166	13.6342	1.30293	13.71214	-20163.6	0.03975	3.76585E-14
LOB09	43.4404	3.18235	1.49437	11.12789	-16394.6	0.02323	4.12019E-05
LOB10	5.13567	12.70816	1.33629	1.49365	-5526.4	0.08031	1.88207E-56
LOB11	10.85376	3.24172	4.86746	1.85549	-5055.5	0.05272	1.45277E-24
LOB12	29.80883	4.84135	8.72657	1.47309	-5088.1	0.05999	1.10058E-31
LOB13	1.13903	1.50553	1.94725	7.36362	-8653.3	0.06779	2.43992E-40
LOB17	31.1186	0.29862	5.33705	13.97365	-13743.9	0.03351	3.53003E-10
LOB18	8.76661	3.07049	3.94689	4.55158	-15646.1	0.04389	3.6979E-17
LOB26	20.35063	3.07418	6.0508	2.01996	-5332.1	0.0305	1.66319E-08
LOB27	28.54213	4.26455	6.82651	1.7716	-5712	0.03919	9.06619E-14
LOB28	2.06826	1.10719	2.40527	6.01456	-8009.3	0.03491	5.1575E-11
LOB29	7.13934	13.67231	1.49792	1.31242	-5191.3	0.08699	3.71753E-66
LOB33	17.40804	1.58483	4.8657	13.01151	-26256.6	0.10279	3.37664E-92
LOB34	0.85526	13.51154	1.26952	7.57601	-9851.6	0.03184	3.12495E-09
LOB35	0.89819	1.67521	2.40649	12.99759	-16529.3	0.0309	1.02107E-08
LOB36	0.75886	2.38543	1.94136	12.82263	-14514.1	0.02782	3.77803E-07
LOB37	0.88962	5.58322	1.63185	9.5777	-12525.2	0.01491	0.023404603
LOB38	4.02205	13.85563	1.34945	2.1939	-7989	0.08434	3.26705E-62
LOB39	0.59362	0.63116	0.78134	7.98241	5207.5	0.03477	6.32291E-11
LOB40	12.67944	2.06367	4.13804	3.741	-9500.8	0.02641	1.73973E-06
LOB41	1.73799	5.05531	1.62556	4.86878	-9208.5	0.01755	0.004231699
LOB43	60.54606	0.36195	5.96454	13.63353	-16063.4	0.04664	2.56648E-19
LOB44	11.76203	2.20546	4.95384	2.50166	-5767.8	0.0483	1.08371E-20
LOB48	2.4486	1.60248	1.83778	10.39963	-13359.1	0.10157	4.90507E-90
LOB49	59.55362	5.69641	8.70963	1.75752	-7135.1	0.02333	3.72749E-05

Table 21: Best GB2 parameters on Adj 1994 EVAS

LOB	year	alpha	gamma	lamda	tau	MLE	100D	p
LOB19	92	82.36452	9.22298	5.72172	9.28260	-33833.45	0.01120	0.16399
LOB19	93	8.81239	5.44246	3.78724	4.91251	-20078.07	0.01263	0.08228
LOB49	93	5.42760	0.45206	4.78350	11.06494	-14834.85	0.01100	0.17849
LOB50	93	5.05949	4.62918	3.08076	4.37046	-15689.27	0.00967	0.30828
LOB50	93	5.42212	5.67511	2.99303	4.08075	-15682.26	0.01262	0.08266
LOB50	93	4.07241	4.55630	2.94526	4.74766	-15677.25	0.01293	0.07058
LOB50	93	78.76245	9.26235	7.54152	2.38088	-15680.60	0.01321	0.06084
LOB37	94	1.05083	8.40161	1.52962	8.17449	-12516.59	0.01074	0.19863
LOB41	94	3.51584	39.99310	0.87336	2.91379	-9206.80	0.00754	0.62021
LOB41	94	2.07597	4.20112	1.75381	4.64240	-9204.01	0.01127	0.15791
LOB41	94	2.50577	4.49438	1.78534	4.14946	-9192.92	0.01240	0.09241

Table 22: GB2 LOBs that pass KS-test on Adj 1994 EVAS

LOB	CS Ratio	D for Median	p for Median	D for Mean	p for Mean
LOB01	1.007912548	3.910000026	0	3.939999938	0
LOB02	0.971023115	0.879999974	1.42027E-61	0.900000015	2.1943E-64
LOB03	1.531627356	0.319999838	1.64159E-08	0.340000272	1.48914E-09
LOB04	0.928172331	0.929999781	1.01457E-68	0.950000024	1.08956E-71
LOB06	1.497773303	0.519999981	8.90298E-22	0.519999981	8.90298E-22
LOB07	0.809950426	1.070000008	7.88334E-91	1.080000028	1.58128E-92
LOB09	1.087034313	0.580000067	5.47009E-27	0.569999993	4.42661E-26
LOB10	1.772995018	0.689999983	5.0924E-38	0.700000003	4.06761E-39
LOB15	1.101137067	1.209999993	4.9186E-116	1.230000034	6.8938E-120
LOB16	1.128583131	1.080000031	1.58128E-92	1.100000072	5.70468E-96
LOB17	0.710545707	1.309999943	6.2125E-136	1.340000081	3.2788E-142
LOB19	2.113922622	0.54000003	1.88592E-23	0.570000017	4.42659E-26
LOB26	1.176923504	0.510000014	5.79226E-21	0.540000224	1.88585E-23
LOB27	1.51111676	0.620000029	8.86729E-31	0.669999981	7.15659E-36
LOB28	0.991775592	0.439999723	1.03248E-15	0.450000286	2.0468E-16
LOB29	1.816189887	1.390000004	5.4617E-153	1.350000072	2.4639E-144
LOB31	1.018334638	0.629999888	9.13634E-32	0.629999983	9.13614E-32
LOB32	1.146157069	0.859999919	7.94836E-59	0.859999919	7.94836E-59
LOB33	0.728368787	0.480000091	1.28243E-18	0.450000167	2.04684E-16
LOB34	0.881715892	1.5	4.3162E-178	1.5	4.3162E-178
LOB35	0.744713695	0.509999776	5.79252E-21	0.570000011	4.42659E-26
LOB36	0.685981619	0.320000196	1.64152E-08	0.300000167	1.56453E-07
LOB42	0.649873903	0.510000017	5.79226E-21	0.489999977	2.19838E-19
LOB44	1.499336326	0.550000095	2.59911E-24	0.600000083	7.48987E-29
LOB45	1.689321235	0.620000023	8.8673E-31	0.64000001	9.07684E-33
LOB46	1.141731485	0.839999998	3.84589E-56	0.790000045	1.0482E-49
LOB47	1.605194417	0.810000026	3.11633E-52	0.869999999	3.42147E-60
LOB48	1.513835294	0.63000001	9.13608E-32	0.65000003	8.6959E-34
LOB49	0.203416715	8.809999824	0	8.820000291	0
LOB50	1.784910797	0.760000038	4.92276E-46	0.739999902	1.15107E-43
LOB51	1.41904429	0.799999973	5.82034E-51	0.83999998	3.84591E-56

Table 23: KS analysis of REV on Adj 1992 EVAS

LOB	CS Ratio	D for Median	p for Median	D for Mean	p for Mean
LOB01	0.802113144	0.789999974	1.04822E-49	0.819999886	1.60905E-53
LOB02	1.88506058	0.489999723	2.19848E-19	0.510000157	5.79211E-21
LOB03	0.807219944	0.649999976	8.69601E-34	0.669999975	7.1566E-36
LOB04	0.51125598	0.619999987	8.86738E-31	0.630000007	9.13609E-32
LOB05	0.451144075	0.310000062	5.16081E-08	0.329999781	5.03513E-09
LOB06	0.472499887	0.290000272	4.57362E-07	0.290000272	4.57362E-07
LOB07	1.284344847	1.650000066	2.1147E-215	1.650000066	2.1147E-215
LOB09	0.656163617	0.280000055	1.28931E-06	0.260000014	9.18661E-06
LOB10	1.095524636	0.900000143	2.1942E-64	0.889999974	5.68502E-63
LOB13	1.231165884	1.309999973	6.2124E-136	1.339999938	3.279E-142
LOB14	0.631179753	0.560000017	3.45419E-25	0.57	4.4266E-26
LOB15	0.84774617	0.630000004	9.13609E-32	0.589999998	6.51835E-28
LOB16	1.148529119	1.17	1.6184E-108	1.159999986	1.1192E-106
LOB17	0.677430072	1.579999948	1.5091E-197	1.620000124	1.1789E-207
LOB19	1.325531149	0.509999979	5.7923E-21	0.489999938	2.1984E-19
LOB20	0.597605937	0.540000014	1.88593E-23	0.590000004	6.51834E-28
LOB21	1.161695175	1.080000043	1.58128E-92	1.060000002	3.78979E-89
LOB22	1.166587308	1.160000002	1.1192E-106	1.119999921	1.7796E-99
LOB23	0.634619298	0.57	4.4266E-26	0.529999998	1.31955E-22
LOB24	1.444485719	1.170000136	1.6183E-108	1.150000095	7.4633E-105
LOB25	0.836982772	0.970000029	1.01178E-74	1.000000015	2.17969E-79
LOB26	0.630193569	0.320000015	1.64156E-08	0.339999998	1.48919E-09
LOB27	0.687820293	0.330000067	5.03496E-09	0.339999938	1.4892E-09
LOB28	0.668648524	0.329999998	5.035E-09	0.340000018	1.48918E-09
LOB29	0.677958362	0.429999781	5.02174E-15	0.450000119	2.04686E-16
LOB30	1.097989956	1.099999988	5.70487E-96	1.080000043	1.58128E-92
LOB31	1.332894336	1.920000017	1.6337E-291	1.930000037	1.4897E-294
LOB32	1.215710286	1.270000041	8.7537E-128	1.289999932	7.931E-132
LOB33	1.803960304	1.569999957	4.635E-195	1.600000018	1.4344E-202
LOB34	0.942170639	1.659999937	5.1477E-218	1.679999977	2.7346E-223
LOB35	1.64167026	1.909999996	1.7276E-288	1.920000017	1.6337E-291
LOB36	1.5350164	2.080000043	0	2.080000043	0
LOB37	2.410955712	1.369999945	1.2476E-148	1.389999986	5.4617E-153
LOB38	0.636600317	1.030000103	3.38498E-84	0.979999852	2.91971E-76
LOB39	0.232388519	0.890000063	5.68486E-63	0.949999982	1.08957E-71
LOB42	0.611800197	0.9	2.19431E-64	0.920000041	2.93124E-67
LOB43	0.500657479	0.669999939	7.15666E-36	0.7	4.06761E-39
LOB44	0.664934571	0.490000004	2.19837E-19	0.510000007	5.79227E-21
LOB45	0.792993077	1.350000054	2.4639E-144	1.379999965	8.4062E-151
LOB46	1.048350918	0.29000001	4.57375E-07	0.309999752	5.16099E-08
LOB47	2.058418387	0.900000036	2.19428E-64	0.930000022	1.01448E-68
LOB48	1.255210108	0.619999969	8.86741E-31	0.65000003	8.6959E-34
LOB49	0.69360143	0.589999878	6.51852E-28	0.609999907	8.29933E-30
LOB50	0.687783612	0.290000105	4.5737E-07	0.299999976	1.56457E-07
LOB51	1.110729604	1.230000067	6.8937E-120	1.239999938	7.7285E-122

Table 24: KS analysis of REV on Adj 1993 EVAS

LOB	CS Ratio	D for Median	p for Median	D for Mean	p for Mean
LOB02	1.513598387	1.390000057	5.4615E-153	1.330000067	4.2074E-140
LOB03	1.053670129	0.619999996	8.86736E-31	0.579999998	5.47016E-27
LOB04	0.933317453	0.329999828	5.0351E-09	0.360000134	1.16806E-10
LOB05	1.05228659	0.349999809	4.24735E-10	0.349999833	4.24734E-10
LOB06	0.652175458	0.439999932	1.03245E-15	0.420000041	2.35515E-14
LOB07	1.696654662	2.529999912	0	2.529999912	0
LOB08	1.024680511	0.960000098	3.38106E-73	0.970000064	1.01176E-74
LOB09	1.019842278	0.609999973	8.29921E-30	0.569999987	4.42661E-26
LOB10	0.724838836	1.010000008	5.63958E-81	0.939999999	3.38569E-70
LOB11	1.038915189	0.35999999	1.16808E-10	0.379999733	7.92213E-12
LOB12	1.040184686	0.35999999	1.16808E-10	0.379999733	7.92213E-12
LOB13	1.653921216	2.419999987	0	2.419999987	0
LOB17	0.755184063	0.750000001	7.66565E-45	0.700000002	4.06761E-39
LOB18	0.946338707	1.090000093	3.05851E-94	1.06000005	3.78972E-89
LOB26	0.799831107	0.490000129	2.19832E-19	0.45999999	3.91325E-17
LOB27	1.179192086	0.410000157	1.06513E-13	0.410000157	1.06513E-13
LOB28	0.973120705	0.450000042	2.04689E-16	0.490000248	2.19828E-19
LOB29	0.8718261	0.950000001	1.08957E-71	0.919999996	2.93128E-67
LOB33	0.986219699	2.960000098	0	2.999999988	0
LOB34	1.116765073	1.010000026	5.63954E-81	0.970000148	1.01173E-74
LOB35	1.331296498	2.099999928	0	2.099999928	0
LOB36	1.291259489	1.319999993	5.2064E-138	1.330000013	4.2075E-140
LOB37	1.443024532	1.940000057	1.3099E-297	1.949999928	1.1108E-300
LOB38	0.809900078	0.439999998	1.03244E-15	0.439999998	1.03244E-15
LOB39	0.135295598	3.91999979	0	4.030000257	0
LOB40	0.449148913	0.489999986	2.19838E-19	0.460000014	3.91323E-17
LOB41	0.653880646	1.449999958	1.9147E-166	1.490000039	9.9101E-176
LOB43	0.92029423	0.360000134	1.16806E-10	0.389999747	1.95357E-12
LOB44	0.927429517	0.520000011	8.90293E-22	0.470000058	7.21406E-18
LOB48	2.546210406	0.879999852	1.42032E-61	0.899999785	2.19446E-64
LOB49	0.565595845	0.579999999	5.47016E-27	0.519999996	8.90296E-22

Table 25: KS analysis of REV on Adj 1994 EVAS

LOB	95%	Lower CI	Upper CI	GB2	Rev	Lower CI	Upper CI	Resid	QR
LOB01	0.59735	0.59721	0.59748	0.70733	0.59789	0.59762	0.59822	-3.8e-007	0.59845
LOB02	0.22833	0.22777	0.2288	0.22666	0.22828	0.22719	0.22956	-3.02e-006	0.22182
LOB03	0.70737	0.69143	0.72361	0.70995	0.71268	0.67428	0.75758	-4.82e-006	0.93056
LOB04	0.36017	0.35518	0.36487	0.3502	0.36849	0.35388	0.38474	-3.22e-006	0.34271
LOB06	0.4771	0.471	0.48584	0.48168	0.482	0.46605	0.50093	2.23e-006	0.49367
LOB07	0.34687	0.34371	0.3491	0.35591	0.3483	0.34302	0.3544	-2.22e-006	0.3179
LOB09	0.23925	0.23855	0.23965	0.23851	0.2388	0.23762	0.24016	-2.56e-006	0.24241
LOB10	0.34777	0.34528	0.35076	0.34458	0.34967	0.34371	0.35808	-1.4e-007	0.2625
LOB15	0.23504	0.23475	0.23541	0.23496	0.23517	0.23442	0.23612	-2.35e-006	0.23469
LOB16	0.2716	0.27138	0.27183	0.27412	0.27126	0.27079	0.27175	8.9e-007	0.26121
LOB17	0.38125	0.36622	0.39912	0.35173	0.42314	0.38212	0.47315	1.08e-006	0.22506
LOB19	0.23763	0.23705	0.23808	0.23728	0.23737	0.23645	0.23854	4.44e-006	0.23284
LOB26	0.52097	0.51456	0.52678	0.52021	0.52236	0.50651	0.54123	5.9e-007	0.49351
LOB27	0.58315	0.57222	0.59217	0.59111	0.58986	0.56893	0.61312	-2.7e-006	0.42704
LOB28	0.50019	0.49507	0.50429	0.50302	0.50279	0.49258	0.51533	1.06e-006	0.34164
LOB29	0.33588	0.33302	0.33872	0.33126	0.33185	0.32621	0.33796	1e-008	0.36271
LOB31	0.34266	0.33866	0.34663	0.3439	0.34613	0.33833	0.35487	2e-007	0.34695
LOB32	0.34971	0.34747	0.35289	0.36333	0.35367	0.34747	0.35972	-8.3e-007	0.36478
LOB33	0.3679	0.36406	0.37152	0.36865	0.36573	0.35867	0.37485	0.02299155	0.34951
LOB34	0.35062	0.34831	0.35449	0.35122	0.34881	0.3425	0.35682	0.02355019	0.29424
LOB35	0.30125	0.29751	0.30487	0.29738	0.30142	0.29528	0.31245	4.17e-006	0.2473
LOB36	0.31139	0.30625	0.31574	0.30427	0.31072	0.30387	0.31911	-3.21e-006	0.23375
LOB42	0.41349	0.40558	0.42241	0.38442	0.41142	0.39203	0.43253	2.06e-006	0.53396
LOB44	0.50344	0.49781	0.51039	0.51275	0.50836	0.49632	0.52443	1.4e-007	0.49434
LOB45	0.30691	0.30406	0.31014	0.28748	0.30647	0.30007	0.31349	-9e-008	0.27803
LOB46	0.28678	0.28553	0.28803	0.28792	0.28531	0.28318	0.28811	7e-007	0.27669
LOB47	0.27666	0.27319	0.27955	0.27196	0.27799	0.27136	0.28623	-4.1e-006	0.28478
LOB48	0.34005	0.33746	0.34317	0.33432	0.34001	0.33361	0.34696	2.95e-006	0.24807
LOB49	0.49614	0.46257	0.53949	0.40987	0.59484	0.59469	0.59554	-1.96e-006	0.23294
LOB50	0.29721	0.29556	0.29916	0.29964	0.29528	0.29201	0.3001	-2.19e-006	0.29597
LOB51	0.2588	0.258	0.25944	0.25824	0.25908	0.257	0.26146	-3.99e-006	0.21954

Table 26: 95% Table on 1992 Adj EVAS

LOB	95%	Lower CI	Upper CI	GB2	Rev	Lower CI	Upper CI	Resid	QR
LOB01	0.25721	0.25466	0.26067	0.24636	0.26106	0.25396	0.26715	2.02e-006	0.27334
LOB02	0.63948	0.63232	0.64476	0.6394	0.63606	0.62534	0.64921	8.1e-007	0.5417
LOB03	0.52403	0.51288	0.53405	0.60825	0.52853	0.50761	0.54853	2.04e-006	0.60657
LOB04	0.30968	0.30412	0.31487	0.28925	0.31387	0.30137	0.32837	-3.77e-006	0.3257
LOB05	0.52424	0.51597	0.53209	0.56871	0.52672	0.50821	0.54641	-1.89e-006	0.47611
LOB06	0.46908	0.46116	0.47495	0.50925	0.4706	0.45269	0.4877	-6e-007	0.41352
LOB07	0.79154	0.7824	0.79854	0.79034	0.79616	0.78388	0.80995	-3.02e-006	0.62572
LOB09	0.35257	0.34993	0.35514	0.34838	0.35204	0.34666	0.35715	-3.4e-007	0.29292
LOB10	0.44432	0.43954	0.44924	0.44305	0.43918	0.42744	0.45245	-3.94e-006	0.44001
LOB13	0.60852	0.60306	0.61322	0.60529	0.61046	0.59917	0.62332	-5.7e-007	0.55855
LOB14	0.57113	0.56051	0.58194	0.58042	0.57497	0.5503	0.59524	-1.9e-006	0.67449
LOB15	0.95649	0.95369	0.95996	0.95483	0.95612	0.94988	0.96455	-2.26e-006	0.8989
LOB16	0.34157	0.33951	0.3436	0.3425	0.34023	0.33446	0.34549	3.9e-006	0.28842
LOB17	0.34425	0.33753	0.35261	0.34026	0.362	0.34309	0.38285	-5.07e-006	0.38684
LOB19	0.35522	0.35298	0.35844	0.35115	0.35425	0.34847	0.36116	-5.09e-006	0.28664
LOB20	0.40472	0.39894	0.41144	0.40461	0.40534	0.3905	0.42592	-3.31e-006	0.43541
LOB21	0.37802	0.37537	0.38023	0.35864	0.374	0.36767	0.3811	-2.2e-007	0.38961
LOB22	0.38173	0.37876	0.38403	0.27421	0.37718	0.37042	0.38448	-7.2e-007	0.37481
LOB23	0.27343	0.27203	0.27476	0.38644	0.27336	0.27019	0.27755	2.87e-006	0.26828
LOB24	0.37839	0.37626	0.3807	0.37689	0.37434	0.36938	0.38057	4.76e-006	0.28001
LOB25	0.2723	0.27041	0.27465	0.26738	0.27233	0.26855	0.27814	-2.98e-006	0.28843
LOB26	0.56898	0.56145	0.57779	0.622	0.57161	0.55089	0.59727	-2.52e-006	0.56958
LOB27	0.57506	0.56625	0.58343	0.58257	0.57829	0.56033	0.59933	-2.2e-006	0.53103
LOB28	0.5212	0.51433	0.5283	0.53513	0.52043	0.50304	0.53808	7.7e-007	0.4341
LOB29	0.46441	0.45892	0.46923	0.46033	0.46316	0.45262	0.47718	4e-007	0.50557
LOB30	0.38958	0.38477	0.39326	0.3829	0.38398	0.37405	0.39349	2.49e-006	0.38337
LOB31	0.45762	0.44981	0.4638	0.45046	0.46268	0.45228	0.47391	3.25e-006	0.43415
LOB32	0.36659	0.36341	0.37052	0.36764	0.36972	0.36182	0.38105	-5.7e-007	0.34662
LOB33	0.44352	0.43109	0.45699	0.37637	0.45272	0.4291	0.4769	5.63e-006	0.23748
LOB34	0.71632	0.71484	0.71751	0.72986	0.71732	0.71332	0.72068	-1.48e-006	0.64755
LOB35	0.61205	0.60278	0.61968	0.59632	0.61695	0.60313	0.63726	4.5e-007	0.32597
LOB36	0.61582	0.61081	0.62158	0.61816	0.61738	0.60842	0.62537	-3.73e-006	0.61643
LOB37	0.63879	0.63609	0.64113	0.63819	0.63934	0.63295	0.64757	4.76e-006	0.44413
LOB38	0.50512	0.50326	0.50697	0.56879	0.50243	0.4981	0.50863	1.66e-006	0.44321
LOB39	0.8029	0.80191	0.8041	0.80254	0.80455	0.80127	0.80861	-2.93e-006	0.80453
LOB42	0.6332	0.61617	0.65128	0.62906	0.65364	0.6216	0.69858	-2.78e-006	0.59684
LOB43	0.3102	0.30448	0.31558	0.29982	0.31482	0.3013	0.32732	-2.15e-006	0.28155
LOB44	0.57809	0.57195	0.58534	0.58488	0.58085	0.56325	0.60272	1.53e-006	0.31614
LOB45	0.28899	0.28679	0.29178	0.28757	0.29085	0.28657	0.29627	-7e-008	0.28981
LOB46	0.25966	0.25836	0.26103	0.25516	0.25965	0.25757	0.2624	1.75e-006	0.26944
LOB47	0.30118	0.29704	0.30495	0.2927	0.30067	0.29347	0.30995	-2.61e-006	0.31007
LOB48	0.33597	0.33213	0.3401	0.33441	0.3366	0.33085	0.34365	4.89e-006	0.27863
LOB49	0.44834	0.43877	0.45826	0.45078	0.45792	0.44076	0.47527	8.9e-007	0.21171
LOB50	0.42837	0.42452	0.43177	0.42857	0.42848	0.42202	0.43694	2.83e-006	0.37289
LOB51	0.24706	0.24633	0.2479	0.24618	0.2474	0.24616	0.249	2.2e-007	0.23033

Table 27: 95% Table on 1993 Adj EVAS

LOB	95%	Lower CI	Upper CI	GB2	Rev	Lower CI	Upper CI	Resid	QR
LOB02	0.48737	0.48492	0.48973	0.4824	0.48187	0.47507	0.48911	7.1e-007	0.44577
LOB03	0.71492	0.70439	0.72587	0.72711	0.71167	0.68315	0.73793	-1.07e-006	0.73908
LOB04	0.4367	0.42752	0.44479	0.42683	0.44056	0.41775	0.46489	-3.1e-007	0.24223
LOB05	0.73574	0.72157	0.74583	0.72526	0.73552	0.7038	0.76381	-1.44e-006	0.42347
LOB06	0.67935	0.66872	0.69045	0.68924	0.6766	0.65325	0.70131	6.2e-007	0.40059
LOB07	0.63145	0.62123	0.64637	0.62273	0.65631	0.6358	0.68186	1.71e-006	0.45011
LOB08	0.73989	0.73812	0.74165	0.74149	0.74291	0.73834	0.74808	-3.78e-006	0.72499
LOB09	0.948	0.94404	0.95186	0.94812	0.94649	0.93647	0.95455	-2.91e-006	0.792
LOB10	0.65339	0.64656	0.65975	0.69552	0.65374	0.63118	0.68301	-2.36e-006	0.55806
LOB11	0.7538	0.74208	0.76565	0.78923	0.75145	0.7254	0.78361	-3.62e-006	0.55303
LOB12	0.75387	0.74224	0.76572	0.76655	0.75103	0.72919	0.78301	6.98e-006	0.82856
LOB13	0.69995	0.68964	0.71802	0.67042	0.7309	0.70499	0.75923	-5.5e-006	0.58634
LOB17	0.50482	0.4946	0.51784	0.50375	0.51005	0.47976	0.54725	-6.5e-007	0.42166
LOB18	0.43539	0.43255	0.43952	0.42851	0.42934	0.42069	0.43912	4.9e-007	0.4572
LOB26	0.79452	0.78246	0.80698	0.80467	0.79331	0.76001	0.82958	-1.34e-006	0.5789
LOB27	0.76114	0.74909	0.77298	0.7792	0.76069	0.73415	0.79843	2.32e-006	0.40443
LOB28	0.72816	0.71691	0.73636	0.72692	0.72424	0.69401	0.75529	-3.2e-007	0.40963
LOB29	0.67503	0.6682	0.68042	0.70744	0.6739	0.65936	0.69118	2.2e-006	0.64667
LOB33	0.28522	0.28083	0.29123	0.28959	0.30696	0.29381	0.32122	-7.5e-007	0.23268
LOB34	0.64796	0.646	0.65075	0.64626	0.64486	0.63885	0.65135	5.51e-006	0.57048
LOB35	0.45991	0.45676	0.46321	0.4667	0.46809	0.46131	0.47614	-5.13e-006	0.30736
LOB36	0.54297	0.53997	0.54547	0.54373	0.54485	0.5391	0.55156	-4.73e-006	0.41661
LOB37	0.58839	0.58504	0.59053	0.58607	0.59041	0.58494	0.59673	-3.01e-006	0.45305
LOB38	0.59892	0.59563	0.60247	0.6155	0.59869	0.59153	0.60755	7e-007	0.54294
LOB39	1.93778	1.90274	1.97413	2.14258	2.5438	2.07406	3.13532	-0.02734862	0.88813
LOB40	0.63664	0.62802	0.64418	0.6209	0.63645	0.61978	0.65566	-3.2e-006	0.70399
LOB41	0.64705	0.64163	0.65236	0.64163	0.65503	0.64113	0.67372	-1e-006	0.65201
LOB43	0.43791	0.42903	0.44659	0.42547	0.44197	0.41983	0.4643	-1.58e-006	0.36809
LOB44	0.73821	0.72834	0.74988	0.75952	0.73758	0.71044	0.76578	4.6e-006	0.57959
LOB48	0.72929	0.71963	0.73871	0.69397	0.7189	0.69899	0.74596	-3.53e-006	0.76427
LOB49	0.71316	0.70495	0.72304	0.71894	0.71506	0.68886	0.74666	1.42e-006	0.74917

Table 28: 95% Table on 1994 Adj EVAS

LOB	99%	Lower CI	Upper CI	GB2	Rev	Lower CI	Upper CI	Resid	QR
LOB01	0.59900	0.59881	0.59924	0.81763	0.59979	0.59909	0.60070	-2.2e-006	0.59975
LOB02	0.23434	0.23358	0.23511	0.23267	0.23541	0.23230	0.23981	3.6e-007	0.21923
LOB03	0.91494	0.88761	0.95050	0.99073	0.94391	0.83301	1.08432	-8.14e-006	0.869
LOB04	0.43822	0.42603	0.45570	0.42269	0.45708	0.41658	0.50662	3.68e-006	0.34817
LOB06	0.56927	0.55455	0.58903	0.60284	0.59437	0.55009	0.65112	-3.99e-006	0.59662
LOB07	0.38598	0.37988	0.39164	0.41734	0.38653	0.37209	0.40462	-4.15e-006	0.38713
LOB09	0.24531	0.24456	0.24642	0.24613	0.24678	0.24335	0.25029	-2.82e-006	0.2311
LOB10	0.38497	0.37919	0.38856	0.42518	0.39111	0.37135	0.41984	-1.78e-006	0.34044
LOB15	0.23993	0.23942	0.24047	0.23994	0.23989	0.23764	0.24250	3.1e-007	0.23314
LOB16	0.27460	0.27428	0.27497	0.27909	0.27429	0.27315	0.27594	2.72e-006	0.26657
LOB17	0.68761	0.62313	0.73386	0.43114	0.71198	0.59716	0.87940	-1.1e-007	0.25726
LOB19	0.24331	0.24269	0.24393	0.24462	0.24437	0.24148	0.24789	-1.69e-006	0.23385
LOB26	0.60551	0.59151	0.62355	0.61584	0.62943	0.58292	0.68828	-5.6e-007	0.60511
LOB27	0.70684	0.68776	0.73115	0.72665	0.73173	0.67850	0.81177	-2.2e-007	0.4137
LOB28	0.56623	0.55477	0.57902	0.58418	0.58054	0.5510	0.61855	-1.36e-006	0.42255
LOB29	0.35901	0.35674	0.36168	0.39454	0.37105	0.35373	0.39457	3.62e-006	0.33525
LOB31	0.39482	0.38752	0.40274	0.39625	0.40430	0.38128	0.43327	3.94e-006	0.39443
LOB32	0.38773	0.38115	0.39588	0.42155	0.40136	0.38403	0.42210	1.56e-006	0.36942
LOB33	0.41406	0.40875	0.42301	0.39918	0.41270	0.39350	0.43886	0.02334047	0.39072
LOB34	0.39680	0.38940	0.40304	0.38388	0.39206	0.37564	0.41788	0.04653712	0.2911
LOB35	0.35037	0.34316	0.36269	0.32219	0.34366	0.32408	0.37133	-1.58e-006	0.2424
LOB36	0.36177	0.35465	0.36874	0.33212	0.36558	0.34185	0.39734	-1.45e-006	0.235
LOB42	0.52359	0.51082	0.54514	0.48199	0.52710	0.47702	0.59236	-4.55e-006	0.30066
LOB44	0.58160	0.57177	0.59440	0.60011	0.59579	0.56173	0.65090	-4.41e-006	0.52612
LOB45	0.33991	0.33519	0.34656	0.32075	0.35031	0.33254	0.37379	-1.63e-006	0.34544
LOB46	0.30032	0.29831	0.30233	0.31403	0.30357	0.29648	0.31342	2.62e-006	0.29414
LOB47	0.30628	0.30237	0.30899	0.30077	0.31948	0.30041	0.34509	1.52e-006	0.30211
LOB48	0.38289	0.37632	0.39048	0.38245	0.38890	0.36909	0.41089	-2.87e-006	0.23405
LOB49	0.59457	0.59437	0.59478	0.53159	0.59594	0.59543	0.59779	2.75e-006	0.59339
LOB50	0.31563	0.31404	0.31731	0.32952	0.31976	0.30942	0.33614	4.72e-006	0.25783
LOB51	0.27060	0.26915	0.27160	0.26915	0.27202	0.26592	0.27965	-5.26e-006	0.23728

Table 29: 99% Table on 1992 Adj EVAS

LOB	99%	Lower CI	Upper CI	GB2	Rev	Lower CI	Upper CI	Resid	QR
LOB01	0.30025	0.29353	0.30686	0.26531	0.30485	0.28549	0.32783	-1.89e-006	0.25289
LOB02	0.70605	0.70041	0.71293	0.75802	0.72091	0.68133	0.76751	2.6e-006	0.55357
LOB03	0.64533	0.63161	0.66626	1.02414	0.66299	0.60948	0.73458	-5.6e-007	0.65732
LOB04	0.38608	0.37534	0.39919	0.33254	0.39863	0.36106	0.44327	-2.92e-006	0.20531
LOB05	0.63931	0.61753	0.65633	0.86516	0.65250	0.59746	0.71831	3.84e-006	0.56138
LOB06	0.57459	0.56157	0.59103	0.75049	0.58950	0.53528	0.65370	3.07e-006	0.49692
LOB07	0.87237	0.86304	0.88755	0.90379	0.89143	0.85353	0.93827	2.74e-006	0.7529
LOB09	0.38971	0.38410	0.39412	0.38742	0.39203	0.37765	0.40720	4.14e-006	0.32607
LOB10	0.49885	0.49140	0.50478	0.62127	0.51161	0.47911	0.55301	1.12e-006	0.48326
LOB13	0.66295	0.65764	0.67483	0.68869	0.67900	0.64891	0.71723	5.36e-006	0.54961
LOB14	0.73018	0.70584	0.75038	0.86461	0.73849	0.67811	0.82086	5.34e-006	0.86793
LOB15	0.99500	0.98923	0.99975	0.98908	1.00478	0.98462	1.03119	-3.71e-006	0.88118
LOB16	0.36925	0.36444	0.37278	0.37256	0.37563	0.36253	0.39533	-4e-008	0.28841
LOB17	0.47980	0.45767	0.50114	0.41516	0.47153	0.42275	0.55068	-7.7e-007	0.20823
LOB19	0.38462	0.38100	0.39001	0.38343	0.39103	0.37557	0.41156	-4.02e-006	0.27483
LOB20	0.50148	0.48521	0.51777	0.55236	0.50245	0.45447	0.57299	1.15e-006	0.55046
LOB21	0.40559	0.40240	0.40855	0.45355	0.41737	0.39929	0.43813	1.45e-006	0.39051
LOB22	0.40929	0.40627	0.41220	0.30007	0.42204	0.40560	0.44638	-1.18e-006	0.35462
LOB23	0.29263	0.29006	0.29536	0.51806	0.29647	0.28651	0.31250	-5.4e-007	0.2697
LOB24	0.39909	0.39717	0.40155	0.49383	0.40863	0.39082	0.43063	-8.3e-007	0.33279
LOB25	0.29669	0.29435	0.29948	0.29296	0.29732	0.28627	0.31359	-3.88e-006	0.30549
LOB26	0.68954	0.67539	0.71197	0.98066	0.71154	0.65174	0.78813	4.8e-006	0.55296
LOB27	0.70048	0.67912	0.72109	0.76638	0.71300	0.67007	0.77067	-3.3e-006	0.49016
LOB28	0.60630	0.59642	0.62356	0.69038	0.62033	0.58137	0.67491	7.2e-007	0.46538
LOB29	0.52369	0.51901	0.52983	0.68514	0.53877	0.50819	0.57904	-9.5e-007	0.44246
LOB30	0.42916	0.42560	0.43560	0.51001	0.44195	0.41679	0.47172	-3.56e-006	0.39031
LOB31	0.53325	0.52370	0.54615	0.57286	0.54585	0.51508	0.58326	3.5e-007	0.54438
LOB32	0.41199	0.40535	0.41944	0.42263	0.41700	0.39626	0.45749	4.88e-006	0.42156
LOB33	0.59336	0.57321	0.61569	0.48469	0.62493	0.55283	0.72369	3.14e-006	0.24585
LOB34	0.73218	0.72959	0.73482	0.76782	0.73925	0.73060	0.74932	5.7e-006	0.70739
LOB35	0.70738	0.69698	0.72359	0.70934	0.72510	0.68098	0.77975	-1.34e-006	0.33064
LOB36	0.67001	0.66382	0.67656	0.69014	0.67656	0.65432	0.70666	-4.99e-006	0.28772
LOB37	0.67193	0.66836	0.67467	0.67551	0.68284	0.66430	0.71141	-1.89e-006	0.49438
LOB38	0.52723	0.52480	0.53140	0.89371	0.53505	0.52114	0.55605	-3.03e-006	0.42148
LOB39	0.82093	0.81803	0.82435	0.81563	0.82349	0.81406	0.83769	-0.00047965	0.8186
LOB42	0.86288	0.83904	0.90133	1.06603	0.93826	0.84336	1.06118	-7.34e-006	0.36147
LOB43	0.38821	0.37667	0.40128	0.35580	0.39975	0.36313	0.44672	1.28e-006	0.20995
LOB44	0.68416	0.66926	0.69886	0.78804	0.69952	0.65321	0.77889	-4.84e-006	0.58385
LOB45	0.32244	0.31724	0.32633	0.32499	0.32331	0.30930	0.34210	-2.42e-006	0.33072
LOB46	0.27575	0.27387	0.27758	0.26884	0.27731	0.26967	0.28637	3.33e-006	0.27794
LOB47	0.33890	0.33590	0.34232	0.33451	0.34552	0.32263	0.37873	1.28e-006	0.34768
LOB48	0.37779	0.37340	0.38144	0.38629	0.38075	0.36181	0.40083	1.58e-006	0.31667
LOB49	0.58892	0.57437	0.60812	0.61411	0.61092	0.56096	0.67447	-3.5e-006	0.21828
LOB50	0.47581	0.47110	0.48327	0.48069	0.48471	0.46299	0.51136	-1.33e-006	0.38639
LOB51	0.25698	0.25605	0.25805	0.25579	0.25690	0.25298	0.26182	1.41e-006	0.23668

Table 30: 99% Table on 1993 Adj EVAS

LOB	99%	Lower CI	Upper CI	GB2	Rev	Lower CI	Upper CI	Resid	QR
LOB02	0.51016	0.50784	0.51212	0.52692	0.51653	0.50031	0.53740	-2.9e-006	0.44374
LOB03	0.84941	0.83398	0.87589	0.94578	0.89195	0.81853	0.97698	-5.39e-006	0.57885
LOB04	0.56475	0.54394	0.58083	0.58946	0.58483	0.52452	0.66287	-2.55e-006	0.45717
LOB05	0.90451	0.88083	0.92750	0.93695	0.93925	0.84645	1.03753	-2.44e-006	0.62226
LOB06	0.82097	0.80415	0.84361	0.94869	0.85329	0.78295	0.93231	-3.78e-006	0.7389
LOB07	0.78949	0.76429	0.80817	0.75198	0.81047	0.73977	0.89301	-3.211e-005	0.43941
LOB08	0.76650	0.76216	0.77093	0.76080	0.77163	0.75910	0.78846	5e-008	0.74282
LOB09	0.99024	0.98460	0.99504	1.00114	1.00435	0.98115	1.04076	-4.37e-006	0.82332
LOB10	0.73703	0.72692	0.75347	0.86892	0.77153	0.71417	0.86849	3.97e-006	0.67981
LOB11	0.91151	0.88988	0.92787	1.09678	0.94109	0.87570	1.04436	8.08e-006	0.69626
LOB12	0.91153	0.88986	0.92784	1.04225	0.94260	0.86813	1.03716	3.2e-007	0.254
LOB13	0.87901	0.84846	0.90187	0.78504	0.90868	0.83351	0.99913	3.364e-005	0.53113
LOB17	0.68856	0.65854	0.71206	0.74084	0.71621	0.63488	0.82155	9.6e-007	0.72771
LOB18	0.47796	0.47179	0.48284	0.49303	0.49036	0.46579	0.52096	-4.61e-006	0.4531
LOB26	0.97005	0.94724	0.99723	1.09720	1.01480	0.92673	1.13058	1.54e-006	0.64426
LOB27	0.91328	0.89645	0.93441	1.02948	0.94781	0.87439	1.05886	-2.94e-006	0.82621
LOB28	0.85712	0.84062	0.87846	0.93176	0.89328	0.81818	0.99356	1.31e-006	0.376
LOB29	0.75934	0.74595	0.76932	0.89209	0.78766	0.74133	0.84705	-1.03e-006	0.68419
LOB33	0.38778	0.37602	0.40883	0.31441	0.38428	0.35196	0.42615	1.25e-006	0.26951
LOB34	0.67306	0.67109	0.67579	0.69166	0.68195	0.66707	0.70151	-4.57e-006	0.2407
LOB35	0.51165	0.50412	0.51843	0.50728	0.51489	0.49510	0.53667	-1.51e-006	0.23766
LOB36	0.57798	0.57417	0.58458	0.58220	0.58514	0.56745	0.60538	9.1e-007	0.37932
LOB37	0.62494	0.61998	0.62910	0.62425	0.62731	0.61111	0.64575	-3.86e-006	0.44716
LOB38	0.64071	0.63575	0.64696	0.71951	0.64821	0.62643	0.67335	3.93e-006	0.57944
LOB39	3.70901	3.32466	4.46378	2.95314	4.25940	3.03450	6.36360	-0.0027755	0.88698
LOB40	0.74635	0.73107	0.76687	0.78093	0.77294	0.72742	0.85213	-2.44e-006	0.83632
LOB41	0.72102	0.70840	0.73017	0.71890	0.74464	0.70478	0.79888	-9.3e-007	0.70598
LOB43	0.56825	0.54889	0.58447	0.58955	0.58759	0.52552	0.66494	1.1e-006	0.62876
LOB44	0.88302	0.86506	0.90344	1.05154	0.91757	0.84592	1.01365	2.92e-006	0.74796
LOB48	0.79863	0.79355	0.80228	0.77003	0.82964	0.77752	0.91147	1.95e-006	0.84293
LOB49	0.84679	0.82527	0.86086	0.90324	0.87301	0.80563	0.98615	-7.21e-006	0.72344

Table 31: 99% Table on 1994 Adj EVAS

LOB	NINETY DAY	TEN YEAR	DELTA 90 DAY	DELTA 10 YEAR	SPREAD	DELTA SPREAD
LOB01	.01251	.01247	.01251	.01247	.00726	.00726
LOB02	.83245	.82733	.83245	.82733	.29556	.29556
LOB03	.32267	.32838	.32267	.32838	.17919	.17919
LOB04	.59097	.60253	.59097	.60253	.33826	.33826
LOB06	.64190	.64631	.64190	.64631	.28792	.28792
LOB07	.21482	.20391	.21482	.20391	.10996	.10996
LOB09	.62727	.76875	.62727	.76875	.17279	.17279
LOB10	.45293	.45402	.45293	.45402	.20796	.20796
LOB15	.58557	.73496	.58557	.73496	.21050	.21050
LOB16	.43794	.46148	.43794	.46148	.09621	.09621
LOB17	.46246	.45962	.46246	.45962	.33090	.33090
LOB19	.70766	.86017	.70766	.86017	.23877	.23877
LOB26	.74755	.75808	.74755	.75808	.29170	.29170
LOB27	.77706	.78735	.77706	.78735	.29676	.29676
LOB28	.75077	.77706	.75077	.77706	.26153	.26153
LOB29	.36853	.35932	.36853	.35932	.19530	.19530
LOB31	.24765	.23934	.24765	.23934	.11200	.11200
LOB32	.24911	.24271	.24911	.24271	.15119	.15119
LOB34	.21342	.23823	.21342	.23823	.08916	.08916
LOB33	.48260	.52621	.48260	.52621	.30081	.30081
LOB35	.30715	.34479	.30715	.34479	.16921	.16921
LOB36	.38915	.41286	.38915	.41286	.25943	.25943
LOB42	.47314	.47980	.47314	.47980	.20213	.20213
LOB44	.71438	.72907	.71438	.72907	.27177	.27177
LOB45	.39601	.38700	.39601	.38700	.24418	.24418
LOB46	.69590	.64017	.69590	.64017	.23398	.23398
LOB47	.28412	.29046	.28412	.29046	.16657	.16657
LOB48	.82074	.79132	.82074	.79132	.32780	.32780
LOB49	.13775	.13723	.13775	.13723	.07030	.07030
LOB50	.77365	.82673	.77365	.82673	.19686	.19686
LOB51	.82644	.85637	.82644	.85637	.17814	.17814

Table 32: R^1 Coefficients for 1992 Quantile Regressions

LOB	NINETY DAY	TEN YEAR	DELTA 90 DAY	DELTA 10 YEAR	SPREAD	DELTA SPREAD
LOB01	.59589	.59297	.59589	.59297	.50852	.50852
LOB02	.81871	.82214	.81871	.82214	.53342	.53342
LOB03	.32250	.32957	.32250	.32957	.24914	.24914
LOB04	.67556	.68186	.67556	.68186	.56540	.56540
LOB05	.67142	.67597	.67142	.67597	.49407	.49407
LOB06	.73988	.74856	.73988	.74856	.52753	.52753
LOB07	.24437	.25396	.24437	.25396	.22894	.22894
LOB09	.86356	.84215	.86356	.84215	.49597	.49597
LOB10	.46010	.47089	.46010	.47089	.35684	.35684
LOB13	.15328	.16065	.15328	.16065	.14992	.14992
LOB14	.56097	.56319	.56097	.56319	.46131	.46131
LOB15	.93592	.77845	.93592	.77845	.43689	.43689
LOB16	.84740	.77943	.84740	.77943	.28057	.28057
LOB17	.68661	.68594	.68661	.68594	.56705	.56705
LOB19	.92130	.81818	.92130	.81818	.38605	.38605
LOB20	.33529	.33812	.33529	.33812	.28246	.28246
LOB21	.51733	.54833	.51733	.54833	.36904	.36904
LOB24	.50813	.54686	.50813	.54686	.34699	.34699
LOB22	.51395	.54518	.51395	.54518	.36672	.36672
LOB23	.82247	.71809	.82247	.71809	.42789	.42789
LOB25	.25711	.25358	.25711	.25358	.20474	.20474
LOB26	.66123	.66474	.66123	.66474	.48610	.48610
LOB27	.71420	.72390	.71420	.72390	.48831	.48831
LOB28	.78755	.81679	.78755	.81679	.53475	.53475
LOB29	.52253	.54745	.52253	.54745	.36682	.36682
LOB30	.52485	.55379	.52485	.55379	.37119	.37119
LOB31	.16559	.18317	.16559	.18317	.15874	.15874
LOB32	.04760	.05206	.04759	.05206	.02249	.02249
LOB33	.30084	.31612	.30084	.31612	.25762	.25762
LOB34	.62484	.75562	.62484	.75562	.38510	.38510
LOB35	.50909	.52989	.50909	.52989	.42587	.42587
LOB36	.64887	.69203	.64887	.69203	.51906	.51906
LOB37	.73883	.80600	.73883	.80600	.56003	.56003
LOB38	.48988	.52341	.48988	.52341	.32166	.32166
LOB39	.16090	.46486	.16090	.46486	.18662	.18662
LOB42	.51064	.53274	.51064	.53274	.34300	.34300
LOB43	.67451	.67975	.67451	.67975	.56544	.56544
LOB44	.63552	.64269	.63552	.64269	.46481	.46481
LOB45	.14951	.15890	.14951	.15890	.11973	.11973
LOB46	.47620	.44976	.47620	.44976	.32136	.32136
LOB47	.11164	.12156	.11164	.12156	.09354	.09354
LOB48	.88937	.89213	.88937	.89213	.64314	.64314
LOB49	.73110	.72459	.73110	.72459	.58966	.58966
LOB50	.79046	.72587	.79046	.72587	.45278	.45278
LOB51	.84376	.90258	.84376	.90258	.46310	.46310

Table 33: R^1 Coefficients for 1993 Quantile Regressions

LOB	NINETY DAY	TEN YEAR	DELTA 90 DAY	DELTA 10 YEAR	SPREAD	DELTA SPREAD
LOB02	.76772	.87522	.76772	.87522	.57219	.57219
LOB03	.69569	.69229	.69569	.69229	.41328	.41328
LOB04	.67019	.66309	.67019	.66309	.49433	.49433
LOB05	.76094	.78289	.76094	.78289	.46732	.46732
LOB06	.74754	.76448	.74754	.76448	.49816	.49816
LOB07	.20060	.22761	.20060	.22761	.18152	.18152
LOB08	.05804	.07413	.05804	.07413	.06804	.06804
LOB09	.95271	.75837	.95271	.75837	.36736	.36736
LOB10	.61598	.65613	.61598	.65613	.27610	.27610
LOB11	.75215	.77450	.75215	.77450	.45789	.45789
LOB12	.75225	.77469	.75225	.77469	.45777	.45777
LOB13	.14930	.17649	.14930	.17649	.13876	.13876
LOB17	.69208	.68615	.69208	.68615	.49195	.49195
LOB18	.79635	.80626	.79635	.80626	.54734	.54734
LOB26	.76228	.78001	.76228	.78001	.49732	.49732
LOB27	.72798	.74049	.72798	.74049	.45925	.45925
LOB28	.79643	.84203	.79643	.84203	.49559	.49559
LOB29	.59921	.65083	.59921	.65083	.27670	.27670
LOB33	.18843	.20653	.18843	.20653	.16605	.16605
LOB34	.57343	.70514	.57343	.70514	.31939	.31939
LOB35	.65222	.71878	.65222	.71878	.49095	.49095
LOB36	.73778	.86191	.73778	.86191	.52060	.52060
LOB37	.74799	.87636	.74799	.87636	.53354	.53354
LOB38	.48187	.50195	.48187	.50195	.27847	.27847
LOB39	.44608	.42940	.44608	.42939	.29596	.29596
LOB40	.60114	.57952	.60114	.57952	.42203	.42203
LOB41	.05815	.09303	.05815	.09303	.03774	.03774
LOB43	.67082	.66342	.67082	.66342	.49458	.49458
LOB44	.73114	.74128	.73114	.74128	.46861	.46861
LOB48	.68536	.68277	.68536	.68277	.49528	.49528
LOB49	.67669	.63633	.67669	.63633	.42149	.42149

Table 34: R^1 Coefficients for 1994 Quantile Regressions

LOB	Ninety Day $\chi^2(\nu=1)$	Ten Year $\chi^2(\nu=1)$	Spread $\chi^2(\nu=1)$
LOB01	6.5756E+07	6.5759E+07	6.4532E+07
LOB02	9.1853E+07	1.2302E+08	5339900
LOB03	183410	229310	56451
LOB04	436640	771060	222280
LOB06	576930	724220	169050
LOB07	397490	301560	242520
LOB09	1.6627E+07	5.3798E+07	2904000
LOB10	949870	976300	211590
LOB15	3.1813E+07	1.0226E+08	9448700
LOB16	3.1844E+07	3.5785E+07	1.8145E+07
LOB17	167280	142830	111750
LOB19	3.5451E+07	1.6565E+08	4174700
LOB26	1683200	1535500	196820
LOB27	1269500	1167000	130010
LOB28	1257600	1893200	181570
LOB29	729270	595390	332450
LOB31	325970	354760	260820
LOB32	356100	465790	334160
LOB34	477170	462880	308090
LOB33	1461900	2374700	1190200
LOB35	1272600	1235000	971720
LOB36	818670	765930	550860
LOB42	194450	208160	123730
LOB44	1312100	1416900	193160
LOB45	1453600	618640	390370
LOB46	1.3792E+07	4631100	805410
LOB47	1796800	1527700	575440
LOB48	4948500	3605900	317520
LOB49	149870	161300	72619
LOB50	5929500	1.5962E+07	1012600
LOB51	3.4835E+07	5.126E+07	1792000

Table 35: Wald Estimators for 1992 Quantile Regressions

LOB	Ninety Day $\chi^2(\nu = 1)$	Ten Year $\chi^2(\nu = 1)$	Spread $\chi^2(\nu = 1)$
LOB01	1746800	1813300	1481100
LOB02	3054000	2965900	438830
LOB03	235050	282200	138270
LOB04	808100	885550	591310
LOB05	1441000	1326300	222650
LOB06	1601800	1628500	217540
LOB07	267930	374240	243290
LOB09	9261800	9258600	710320
LOB10	484870	553280	178210
LOB13	242740	382910	331730
LOB14	833050	453190	166860
LOB15	6.3792E+08	3.242E+07	6686000
LOB16	1.7289E+07	7158700	608570
LOB17	776810	501890	462010
LOB19	3.1153E+07	8819700	551330
LOB20	496540	435810	211610
LOB21	465190	873730	240580
LOB24	406520	953520	305930
LOB22	460460	903540	231060
LOB23	1.4212E+07	7187100	1487800
LOB25	2665600	1568100	1205200
LOB26	657910	1296000	206370
LOB27	1081400	1695400	162910
LOB28	1799900	5214600	205140
LOB29	488630	848830	184590
LOB30	583720	1193300	219330
LOB31	125540	110810	134730
LOB32	419730	306320	283330
LOB33	59327	56480	43643
LOB34	4881900	1.8825E+07	2815600
LOB35	310210	333580	317810
LOB36	1298500	1424300	842120
LOB37	5332700	6850300	2107100
LOB38	340260	489470	163900
LOB39	1.1851E+07	4.2242E+07	1.0577E+07
LOB42	132560	99678	49811
LOB43	967920	863430	506070
LOB44	797570	1208000	176460
LOB45	589460	542430	496190
LOB46	6067700	3849700	1597700
LOB47	816900	477100	698540
LOB48	1.0668E+07	1.4065E+07	1297100
LOB49	620510	552790	259580
LOB50	3319600	2308000	564390
LOB51	5.3236E+07	1.6045E+08	6117100

Table 36: Wald Estimators for 1993 Quantile Regressions

LOB	Ninety Day $\chi^2(\nu=1)$	Ten Year $\chi^2(\nu=1)$	Spread $\chi^2(\nu=1)$
LOB02	3647400	3.211E+07	1081500
LOB03	1059400	690780	107480
LOB04	303980	270440	144170
LOB05	406730	1037100	123270
LOB06	564020	809200	188380
LOB07	170000	118660	127790
LOB08	2641600	2475300	2312500
LOB09	6.205E+08	1.7366E+07	2267700
LOB10	889120	1688700	155500
LOB11	756790	982960	152230
LOB12	744940	994880	153690
LOB13	144310	130520	129210
LOB17	288960	249570	90735
LOB18	5886300	4948700	665980
LOB26	525990	656170	205370
LOB27	483670	404090	157080
LOB28	886390	1724400	241080
LOB29	617210	1544900	130000
LOB33	1215100	1958300	890900
LOB34	2145900	6035900	954490
LOB35	3226400	5991000	1012200
LOB36	3356400	2.7452E+07	1560500
LOB37	2908800	2.2715E+07	1689800
LOB38	555370	597210	286330
LOB39	291110	221740	90870
LOB40	365970	318970	236390
LOB41	161680	219860	182960
LOB43	327150	270330	150600
LOB44	861420	531040	139110
LOB48	2855800	2434300	1854600
LOB49	594120	324380	155210

Table 37: Wald Estimators for 1994 Quantile Regressions

LOB	Lowest(%)	2nd Lowest(%)	3rd Lowest(%)	Highest(%)	2nd Highest(%)	3rd Highest(%)
LOB01						
LOB02				9(10.2)	10(8.8)	8(7.8)
LOB03	-2(14.2)	-1(11.7)	-3(9.1)	7(8.9)	14(7.6)	8(7.2)
LOB04				1(16.9)	2(15.6)	11(10.0)
LOB06	-16(2.4)			7(16.1)	5(13.9)	6(12.8)
LOB07	-6(6.9)			1(31.9)	2(17.9)	3(9.3)
LOB09	-1(21.8)	-2(14.4)	-11(5.6)	17(9.3)	18(6.6)	16(6.3)
LOB10				4(14.7)	5(12.8)	2(12.8)
LOB15	-1(8.0)	-11(6.8)	-10(6.7)	3(16.8)	17(6.7)	18(6.6)
LOB16	-10(8.9)	-11(8.4)	-8(7.2)	3(16.3)	2(6.8)	4(5.2)
LOB17				14(21.7)	10(12.5)	11(10.4)
LOB19	-1(13.9)	-11(6.4)	-10(5.5)	3(14.7)	17(9.2)	19(6.9)
LOB26	-16(1.7)			7(11.3)	2(10.4)	6(9.8)
LOB27				4(17.4)	3(15.1)	5(11.9)
LOB28	-16(1.3)			3(13.2)	4(12.7)	1(11.1)
LOB29	-3(8.0)			1(19.9)	7(12.4)	2(7.0)
LOB31	-12(5.7)			1(23.6)	3(11.4)	2(11.3)
LOB32				1(21.4)	2(12.4)	7(10.6)
LOB33	-2(11.4)	-3(11.2)	-6(10.3)	18(2.2)	14(1.8)	
LOB34	-8(11.9)	-10(10.0)	-9(9.5)	1(12.9)	3(7.4)	2(6.3)
LOB35	-2(19.5)	-1(12.8)	-4(9.8)	16(4.2)	18(3.7)	14(2.7)
LOB36	-2(12.4)	-4(12.0)	-6(11.3)			
LOB42	-5(11.6)	-4(11.4)	-6(7.5)	12(10.6)	13(9.3)	11(8.8)
LOB44				2(13.8)	3(12.8)	7(9.6)
LOB45	-19(2.9)			7(17.2)	5(15.9)	14(10.0)
LOB46				1(12.0)	4(9.1)	4(9.1)
LOB47	-2(11.2)	-5(5.9)	-4(5.7)	10(11.1)	14(8.5)	8(7.9)
LOB48				1(6.9)	11(6.7)	15(6.6)
LOB49				10(13.8)	6(9.7)	
LOB50	-3(9.9)	-1(9.0)	-2(8.8)	19(11.2)	18(9.0)	17(8.1)
LOB51	-1(18.3)	-2(13.2)	-4(9.7)	17(6.8)	18(6.3)	16(6.2)

Table 38: Ninety-day rate sensitivity for Adjusted 1992 EVAS

LOB	Lowest(%)	2nd Lowest(%)	3rd Lowest(%)	Highest(%)	2nd Highest(%)	3rd Highest(%)
LOB01				6(14.5)	5(12.0)	10(11.6)
LOB02				9(9.1)	3(7.2)	8(7.0)
LOB03	-2(18.1)	-1(13.2)		10(12.8)	6(9.3)	8(8.7)
LOB04	-2(7.7)	-1(4.8)		12(11.6)	10(11.6)	9(10.8)
LOB05				6(14.6)	5(11.5)	4(10.5)
LOB06				6(16.0)	5(13.8)	7(11.4)
LOB07	-6(12.7)	-4(11.5)	-7(10.1)	1(17.2)		
LOB09				3(15.2)	4(12.7)	2(10.8)
LOB10				3(13.9)	7(13.8)	5(11.4)
LOB13	-6(11.7)	-7(11.0)	-4(9.2)	1(16.8)	2(10.4)	
LOB14	-19(2.0)			6(15.4)	8(12.1)	7(10.3)
LOB15				1(18.7)	2(14.7)	3(10.0)
LOB16	-14(1.2)	-15(1.2)	-16(1.1)	1(18.7)	2(18.0)	3(17.6)
LOB17				6(15.2)	12(8.7)	9(8.4)
LOB19				1(17.7)	3(16.8)	2(16.5)
LOB20				4(14.0)	3(13.4)	10(9.7)
LOB21	-2(9.7)	-1(8.9)		6(15.4)	5(12.7)	4(12.6)
LOB22	-2(9.9)	-1(9.2)		6(15.4)	5(12.6)	4(12.4)
LOB23				1(20.5)	3(16.4)	2(14.6)
LOB24	-2(10.8)	-1(10.3)		6(15.8)	5(11.5)	4(11.3)
LOB25	-2(7.9)			6(15.3)	10(10.2)	8(9.6)
LOB26				6(17.4)	5(13.1)	7(12.1)
LOB27				3(14.7)	5(12.4)	4(11.4)
LOB28				3(11.7)	2(11.0)	6(8.8)
LOB29	-2(12.0)			6(15.6)	5(14.4)	4(12.7)
LOB30	-1(9.0)	-2(8.8)		5(15.1)	4(14.7)	6(13.1)
LOB31	-4(15.9)	-5(11.4)	-6(10.6)	1(14.6)	10(5.8)	
LOB32	-3(16.8)	-4(16.5)	-2(11.4)	10(9.3)	8(9.0)	6(8.3)
LOB33	-3(17.3)	-2(16.2)	-4(13.5)			
LOB34	-9(19.1)	-10(14.2)	-8(10.6)	15(2.5)		
LOB35	-3(15.9)	-2(15.2)	-4(9.5)			
LOB36	-5(15.2)	-4(14.0)	-6(12.5)			
LOB37	-7(15.3)	-6(14.8)	-8(13.0)			
LOB38	-2(8.2)			6(13.4)	9(12.0)	7(11.8)
LOB39	-7(6.8)	-11(5.6)	-8(4.8)	1(23.5)	2(20.3)	3(10.4)
LOB42	-3(13.0)	-2(10.9)	-4(10.9)	11(8.1)	10(7.1)	9(7.1)
LOB43	-2(7.6)	-1(4.5)		10(11.8)	12(11.2)	6(10.7)
LOB44				1(12.7)	7(12.6)	6(12.1)
LOB45	-2(18.5)	-3(8.8)		6(11.2)	8(9.3)	10(8.4)
LOB46				1(13.8)	7(10.6)	6(10.3)
LOB47	-2(15.6)			9(10.8)	12(9.3)	10(9.0)
LOB48				3(6.9)	5(6.4)	14(6.0)
LOB49				12(14.1)	10(11.6)	9(10.0)
LOB50	-12(2.5)	-13(1.4)		1(14.3)	6(11.0)	7(9.6)
LOB51	-1(11.6)	-2(10.0)	-4(8.9)	17(8.3)	18(7.7)	16(6.3)

Table 39: Ninety-day rate sensitivity for Adjusted 1993 EVAS

LOB	Lowest(%)	2nd Lowest(%)	3rd Lowest(%)	Highest(%)	2nd Highest(%)	3rd Highest(%)
LOB02				8(10.4)	7(10.4)	6(8.9)
LOB03				5(12.1)	1(11.6)	3(11.6)
LOB04				3(10.8)	4(10.7)	5(9.9)
LOB05				3(13.1)	2(12.4)	5(11.9)
LOB06				5(14.4)	6(12.7)	3(11.8)
LOB07	-2(21.5)	-3(19.6)	-1(12.1)	8(3.7)		
LOB08	-5(11.8)	-6(9.2)	-7(7.3)	1(19.4)	3(11.4)	2(8.1)
LOB09				1(20.6)	2(16.2)	3(11.4)
LOB10				1(19.2)	2(17.7)	3(16.4)
LOB11				3(13.2)	4(13.1)	5(12.7)
LOB12				3(13.3)	4(13.1)	5(12.7)
LOB13	-2(20.8)	-3(19.7)	-1(9.8)	8(4.7)		
LOB17				6(13.0)	5(10.7)	4(10.3)
LOB18				3(20.3)	2(11.3)	1(9.0)
LOB26				5(13.9)	4(12.7)	6(12.1)
LOB27				1(13.9)	2(11.4)	6(11.3)
LOB28				2(12.3)	5(12.2)	4(11.0)
LOB29				2(18.3)	3(17.4)	1(16.5)
LOB33	-4(14.9)	-3(14.4)	-6(11.9)			
LOB34	-8(23.7)	-7(13.6)	-9(13.2)	15(4.6)	13(4.3)	12(3.6)
LOB35	-2(11.3)	-5(11.2)	-4(10.8)			
LOB36	-4(19.0)	-3(14.6)	-5(14.5)			
LOB37	-6(18.7)	-5(16.7)	-7(12.9)	1(2.1)		
LOB38	-1(5.7)	-16(2.0)		6(12.1)	8(10.9)	9(10.9)
LOB39				16(12.2)	12(11.5)	5(9.8)
LOB40				2(13.5)	1(12.8)	12(8.9)
LOB41	-6(10.5)	-7(9.4)		2(16.8)	1(14.0)	3(13.3)
LOB43				3(11.0)	4(10.4)	5(9.9)
LOB44				6(12.8)	5(12.2)	1(11.4)
LOB48	-1(7.5)	-4(7.2)	-3(4.7)	19(13.0)	17(8.9)	18(6.6)
LOB49				1(12.4)	3(8.4)	12(8.0)

Table 40: Ninety-day rate sensitivity for Adjusted 1994 EVAS

LOB	Lowest(%)	2nd Lowest(%)	3rd Lowest(%)	Highest(%)	2nd Highest(%)	3rd Highest(%)
LOB01						
LOB02	-3(4.9)	-4(4.5)		9(13.0)	1(12.0)	10(7.3)
LOB03	-1(13.2)	-2(12.0)	-3(10.2)	12(7.6)	10(7.3)	7(7.2)
LOB04	-3(5.0)	-19(4.1)		1(20.3)	2(17.0)	12(7.6)
LOB06	-1(8.7)			4(14.3)	7(12.6)	6(11.4)
LOB07	-6(12.0)			1(23.4)	2(21.3)	
LOB09	-1(29.1)	-2(17.0)	-11(5.1)	3(10.4)	17(5.4)	6(4.5)
LOB10	-1(7.3)	-10(5.5)		2(14.6)	4(12.8)	3(9.3)
LOB15	-1(24.5)	-11(7.4)	-10(4.3)	3(24.9)	6(6.2)	17(6.2)
LOB16	-11(7.2)	-10(6.9)	-12(6.2)	3(29.6)	17(5.9)	2(5.1)
LOB17						
LOB19	-1(29.9)	-11(7.2)	-2(3.3)	3(21.4)	6(6.2)	18(5.8)
LOB26	-19(2.2)			2(13.4)	1(12.6)	7(10.6)
LOB27	-1(6.3)	-19(3.5)		3(19.9)	4(18.0)	5(7.2)
LOB28	-16(2.3)			1(19.6)	4(14.8)	3(13.9)
LOB29	-3(12.3)			1(15.7)	2(12.9)	9(8.3)
LOB31	-12(9.0)	-19(6.9)		1(18.8)	2(14.7)	4(7.4)
LOB32				2(19.5)	1(11.7)	
LOB33	-3(13.5)	-1(11.7)	-2(11.4)	18(5.3)		
LOB34	-10(14.6)	-8(11.4)	-19(5.5)	4(10.8)	1(9.6)	17(4.7)
LOB35	-1(19.4)	-2(17.9)	-3(9.5)	18(7.6)	14(3.6)	
LOB36	-4(13.4)	-2(12.6)	-19(7.2)	18(5.1)		
LOB42	-6(13.1)	-4(9.5)	-5(8.4)	12(11.1)	13(5.5)	
LOB44				2(18.7)	3(15.2)	1(12.8)
LOB45				7(14.4)	6(11.5)	
LOB46				1(14.5)	4(12.0)	5(11.0)
LOB47	-3(12.7)	-1(10.5)	-2(10.2)	15(9.8)	10(8.2)	9(7.5)
LOB48				9(9.2)	18(8.1)	17(8.0)
LOB49	-3(9.8)			18(15.9)	11(11.8)	4(11.4)
LOB50	-3(12.3)	-1(12.1)	-2(11.2)	19(9.0)	17(6.4)	18(5.9)
LOB51	-1(21.2)	-2(17.1)	-4(9.5)	17(6.5)	18(5.8)	16(5.3)

Table 41: Ten-year rate sensitivity for Adjusted 1992 EVAS

LOB	Lowest(%)	2nd Lowest(%)	3rd Lowest(%)	Highest(%)	2nd Highest(%)	3rd Highest(%)
LOB01	-16(7.3)	-2(5.8)		7(17.9)	1(11.5)	17(5.5)
LOB02				8(11.0)	9(10.9)	3(8.7)
LOB03	-1(24.3)	-2(14.4)		9(12.2)	7(11.5)	12(6.4)
LOB04	-1(8.7)	-4(4.8)		7(14.0)	10(13.5)	12(10.8)
LOB05				7(17.2)	5(11.7)	4(11.1)
LOB06	-1(12.3)			5(13.9)	7(13.8)	4(11.9)
LOB07	-6(12.8)	-5(11.3)		2(19.1)		
LOB09	-1(22.3)	-11(4.2)		3(13.1)	2(11.3)	5(7.2)
LOB10	-1(10.4)			3(18.6)	7(12.2)	9(10.4)
LOB13	-6(13.6)			2(22.6)		
LOB14				7(15.5)	6(13.3)	9(13.1)
LOB15				1(18.8)	2(16.9)	4(10.4)
LOB16	-15(2.2)	-17(1.7)		3(20.0)	2(19.0)	1(17.7)
LOB17				9(17.6)	15(8.3)	6(7.7)
LOB19				3(19.6)	2(18.6)	1(14.8)
LOB20	-1(16.3)	-18(4.7)		3(11.8)	7(8.7)	9(8.0)
LOB21	-1(15.8)	-2(15.4)		5(14.2)	6(10.2)	4(9.4)
LOB22	-1(16.3)	-2(15.3)		5(14.3)	6(10.3)	4(9.5)
LOB23				3(23.9)	1(14.2)	2(10.4)
LOB24	-2(17.5)	-1(14.0)		5(14.2)	6(10.0)	4(9.9)
LOB25				7(18.3)	9(13.0)	
LOB26				7(16.9)	5(15.4)	6(12.9)
LOB27	-1(13.3)			3(15.2)	2(13.3)	7(11.5)
LOB28				3(15.0)	2(13.4)	1(11.1)
LOB29	-2(28.1)			5(12.3)	4(12.0)	6(11.6)
LOB30	-2(17.3)	-1(14.6)		3(14.4)	4(14.3)	5(12.4)
LOB31	-4(20.5)	-6(10.2)	-11(7.5)	2(11.6)		
LOB32	-4(15.1)	-3(11.9)	-11(5.6)	7(8.8)		
LOB33	-2(16.1)	-1(10.8)	-6(10.2)			
LOB34	-9(28.0)	-10(19.8)	-4(7.6)	1(4.8)	12(3.3)	16(3.0)
LOB35	-2(18.7)	-6(11.7)	-8(11.5)			
LOB36	-5(21.2)	-6(17.6)	-4(15.5)			
LOB37	-6(20.2)	-7(17.2)	-8(11.7)			
LOB38	-2(14.1)			9(15.1)	6(14.1)	8(10.0)
LOB39	-6(5.8)	-5(4.0)		1(48.2)	2(12.7)	19(6.0)
LOB42	-3(17.9)	-5(10.5)	-2(10.2)	11(10.1)	9(10.0)	12(7.0)
LOB43	-1(8.1)	-4(4.7)		7(15.1)	10(13.8)	12(11.5)
LOB44	-3(8.5)			1(15.0)	7(13.7)	2(11.7)
LOB45	-2(18.4)	-1(10.0)		7(14.3)	9(9.5)	
LOB46				7(16.1)	1(13.4)	5(12.5)
LOB47				9(15.2)		
LOB48				3(9.6)	16(8.7)	9(8.3)
LOB49				12(14.4)	9(13.0)	10(8.2)
LOB50	-2(6.7)	-12(5.3)	-13(3.1)	4(14.3)	5(12.5)	8(9.6)
LOB51	-1(15.6)	-2(12.8)	-4(9.5)	17(7.2)	18(7.1)	3(6.2)

Table 42: Ten-year rate sensitivity for Adjusted 1993 EVAS

LOB	Lowest(%)	2nd Lowest(%)	3rd Lowest(%)	Highest(%)	2nd Highest(%)	3rd Highest(%)
LOB02				7(12.2)	8(10.5)	6(10.1)
LOB03				1(14.1)	5(11.9)	2(11.5)
LOB04				3(16.7)	9(8.6)	7(7.8)
LOB05				3(14.4)	2(14.0)	5(11.2)
LOB06	-1(9.5)			5(14.4)	3(11.1)	6(9.5)
LOB07	-2(17.5)	-1(16.9)	-3(11.2)	7(9.9)		
LOB08	-5(26.1)			3(22.8)	19(9.8)	
LOB09				1(22.3)	2(18.1)	4(10.0)
LOB10	-14(1.6)			1(27.3)	2(17.9)	3(15.2)
LOB11				3(14.7)	5(12.6)	4(9.9)
LOB12				3(14.8)	5(12.6)	4(9.9)
LOB13	-1(17.2)	-2(15.5)	-3(10.8)	7(10.9)		
LOB17				7(17.9)	4(10.7)	3(10.1)
LOB18	-4(3.8)			3(27.3)	1(15.6)	7(7.9)
LOB26				5(14.8)	3(14.5)	6(11.2)
LOB27				2(16.3)	1(13.0)	6(11.7)
LOB28				2(17.5)	5(11.0)	1(10.8)
LOB29				1(21.5)	2(21.3)	3(17.1)
LOB33	-4(12.8)	-6(10.9)	-5(9.0)	14(5.9)	19(4.4)	
LOB34	-8(33.7)	-9(14.5)	-6(6.7)	1(8.3)	15(6.5)	12(5.0)
LOB35	-5(18.7)	-2(15.2)	-8(12.7)			
LOB36	-4(25.5)	-5(18.8)	-3(12.7)	1(4.1)	2(3.8)	12(.9)
LOB37	-6(21.2)	-5(17.3)	-8(12.0)	1(9.5)	11(2.1)	
LOB38	-1(14.9)			2(11.3)	8(10.1)	6(8.8)
LOB39				5(12.3)	6(9.8)	
LOB40	-6(8.1)	-8(4.7)		3(15.6)	12(8.4)	19(8.1)
LOB41	-4(14.8)	-6(12.6)	-8(6.0)	3(19.5)	2(14.7)	9(6.9)
LOB43				3(16.6)	9(9.2)	5(8.5)
LOB44				1(16.6)	5(14.3)	6(13.6)
LOB48	-3(9.0)	-4(8.9)	-1(7.8)	19(15.0)	17(11.4)	13(5.4)
LOB49				4(8.7)	16(8.3)	7(6.7)

Table 43: Ten-year rate sensitivity for Adjusted 1994 EVAS

LOB	Delta 90-Day Highest(%)	2nd Highest(%)	3rd Highest(%)	Delta 10-Year Highest(%)	2ndHighest(%)	3rd Highest(%)
LOB01	5(11.4)	6(10.8)	4(10.5)	6(11.5)	4(11.1)	5(11.0)
LOB02	1(9.3)	2(8.7)	4(8.4)	1(9.4)	5(8.5)	6(8.2)
LOB03	5(13.2)	6(12.4)	4(12.0)	4(13.2)	5(12.3)	6(11.5)
LOB04	1(14.6)	2(12.0)	3(9.6)	1(17.4)	2(11.7)	5(8.8)
LOB06	2(15.0)	1(14.4)	3(14.4)	4(15.2)	2(14.7)	3(14.4)
LOB07	1(29.4)	2(16.9)	3(9.8)	1(32.6)	2(18.8)	7(7.9)
LOB09	14(7.8)	13(7.6)	15(7.3)	3(8.3)	-1(8.3)	14(7.4)
LOB10	1(16.0)	2(15.1)	3(12.9)	2(16.5)	1(14.7)	3(13.0)
LOB15	2(9.7)	3(8.8)	14(8.0)	3(12.9)	2(12.6)	14(7.9)
LOB16	-7(11.7)	-6(11.3)	-5(10.8)	-7(11.9)	-6(10.8)	-5(10.6)
LOB17	1(8.7)	6(8.6)	5(8.6)	1(8.9)	4(8.5)	7(8.1)
LOB19	2(8.5)	3(8.4)	14(6.4)	3(11.3)	2(10.4)	14(6.6)
LOB26	1(15.9)	2(14.5)	3(12.7)	1(17.1)	2(14.7)	3(12.2)
LOB27	1(16.5)	2(16.3)	3(15.0)	2(17.1)	3(16.5)	1(15.8)
LOB28	1(17.4)	2(15.4)	3(13.4)	1(20.0)	2(15.8)	3(13.5)
LOB29	1(13.3)	4(10.2)	2(10.0)	1(13.9)	2(10.4)	4(10.2)
LOB31	1(31.4)	2(19.8)	3(14.2)	1(31.0)	2(19.9)	3(11.2)
LOB32	1(18.2)	2(13.5)	3(10.8)	1(17.7)	2(14.2)	6(9.8)
LOB33	-1(18.2)	-2(16.7)	-3(14.3)	-1(20.1)	-2(17.1)	-3(14.3)
LOB34	-7(14.0)	-8(13.1)	-6(12.9)	-8(14.6)	-7(12.3)	-6(10.5)
LOB35	-1(22.7)	-2(18.8)	-3(12.8)	-1(26.0)	-2(18.4)	-3(11.4)
LOB36	-1(18.3)	-2(17.3)	-3(14.6)	-1(20.4)	-2(17.9)	-3(14.5)
LOB42	8(12.4)	9(12.3)	7(11.4)	8(13.3)	9(12.9)	7(12.4)
LOB44	1(17.3)	2(15.6)	3(13.1)	1(19.1)	2(16.5)	3(12.6)
LOB45	1(14.1)	2(14.0)	3(13.4)	4(14.1)	3(13.0)	2(12.4)
LOB46	1(15.2)	2(13.3)	3(12.0)	1(15.3)	2(13.1)	3(11.9)
LOB47	7(10.8)	6(10.7)	8(9.7)	6(11.2)	4(10.8)	5(10.3)
LOB48	1(9.7)	2(9.0)	3(8.4)	1(9.2)	2(9.1)	3(8.4)
LOB49	3(10.5)	1(10.5)	2(10.4)	4(11.5)	1(10.6)	2(10.3)
LOB50	5(6.9)	6(6.8)	7(6.6)	5(7.9)	6(7.6)	7(7.0)
LOB51	12(7.8)	11(7.7)	10(7.6)	12(7.6)	13(7.3)	14(7.3)

Table 44: Delta rate sensitivities for 1992 Adjusted EVAS

LOB	Delta 90-Day Highest(%)	2nd Highest(%)	3rd Highest(%)	Delta 10-Year Highest(%)	2ndHighest(%)	3rd Highest(%)
LOB01	1(13.6)	4(13.2)	3(12.9)	1(13.9)	4(12.7)	3(12.5)
LOB02	1(10.9)	2(10.2)	3(9.5)	1(11.0)	2(10.3)	3(9.6)
LOB03	5(13.0)	4(12.5)	6(12.3)	4(13.7)	5(13.3)	3(12.8)
LOB04	5(10.8)	4(10.7)	3(10.3)	5(11.5)	4(10.6)	6(10.5)
LOB05	1(15.4)	2(14.6)	3(14.0)	1(15.5)	2(14.9)	3(14.3)
LOB06	3(14.3)	2(14.3)	1(13.9)	3(15.0)	2(14.9)	4(14.0)
LOB07	-3(15.4)	-4(14.5)	-2(14.0)	-3(17.5)	-4(15.7)	-5(14.2)
LOB09	1(13.9)	2(13.9)	3(12.4)	2(15.8)	3(13.2)	1(10.7)
LOB10	1(14.6)	2(14.3)	3(14.2)	3(15.7)	2(15.6)	1(13.7)
LOB13	-3(16.4)	-4(16.0)	-2(13.1)	-4(18.1)	-3(17.9)	-5(16.3)
LOB14	4(12.2)	1(12.1)	3(12.0)	4(12.7)	3(12.2)	5(12.1)
LOB15	1(17.2)	2(14.0)	3(11.4)	1(17.1)	2(13.9)	3(11.0)
LOB16	1(28.0)	2(22.0)	3(16.2)	1(28.8)	2(22.9)	3(16.5)
LOB17	4(11.2)	3(11.1)	1(11.0)	4(11.2)	3(11.1)	1(10.5)
LOB19	1(22.5)	2(18.5)	3(14.8)	1(23.2)	2(19.7)	3(15.3)
LOB20	3(14.6)	2(13.8)	1(12.7)	3(16.1)	2(15.4)	4(11.2)
LOB21	3(15.1)	4(13.7)	2(13.3)	3(17.5)	4(15.2)	2(13.3)
LOB22	3(15.2)	4(13.7)	2(13.3)	3(17.5)	4(15.2)	2(13.3)
LOB23	1(19.5)	2(15.3)	3(12.3)	1(18.7)	2(15.8)	3(13.7)
LOB24	3(14.9)	4(13.6)	2(12.8)	3(17.3)	4(15.5)	5(12.8)
LOB25	1(11.8)	3(11.7)	4(11.7)	3(12.1)	5(12.1)	6(11.3)
LOB26	1(14.5)	2(13.4)	3(13.2)	1(14.6)	4(13.2)	3(13.2)
LOB27	1(16.2)	2(16.2)	3(14.6)	2(17.9)	1(15.0)	3(15.0)
LOB28	1(16.0)	2(14.6)	3(12.8)	1(17.0)	2(15.0)	3(12.5)
LOB29	3(15.9)	4(14.4)	2(13.6)	3(19.6)	4(16.6)	5(13.1)
LOB30	3(16.9)	2(15.0)	4(14.4)	3(20.9)	4(16.2)	2(15.2)
LOB31	-3(16.5)	-4(15.8)	-2(15.2)	-4(21.9)	-3(19.4)	-2(11.8)
LOB32	6(14.3)	5(13.6)	7(10.1)	7(13.8)	5(13.6)	6(12.3)
LOB33	-1(23.9)	-2(20.8)	-3(16.5)	-1(26.3)	-2(21.6)	-3(14.6)
LOB34	-2(13.3)	-1(13.2)	-3(12.6)	-2(12.9)	-3(12.2)	-1(12.0)
LOB35	-1(16.2)	-2(15.6)	-3(13.2)	-1(17.3)	-2(17.0)	-3(12.9)
LOB36	-1(14.3)	-2(14.1)	-3(13.1)	-1(14.1)	-2(14.1)	-3(13.5)
LOB37	-1(12.5)	-2(12.4)	-3(11.9)	-2(12.9)	-1(12.5)	-3(12.1)
LOB38	3(12.0)	4(11.8)	5(11.4)	4(12.8)	3(12.7)	5(12.2)
LOB39	1(19.3)	2(9.2)	14(7.6)	1(30.0)	-4(7.5)	-5(7.1)
LOB42	8(12.1)	7(11.5)	6(11.1)	9(12.1)	7(11.9)	8(11.8)
LOB43	5(10.7)	4(10.7)	3(10.4)	5(11.3)	4(10.5)	6(10.5)
LOB44	1(15.4)	2(13.4)	3(12.3)	1(16.8)	2(13.5)	4(12.7)
LOB45	5(16.3)	6(15.4)	4(14.7)	6(16.3)	7(14.9)	5(14.2)
LOB46	1(16.3)	2(14.0)	3(12.7)	1(15.2)	3(13.6)	2(12.6)
LOB47	7(13.0)	8(12.5)	6(11.4)	7(15.0)	6(12.6)	9(12.2)
LOB48	1(9.8)	2(9.4)	3(9.0)	1(9.9)	2(9.4)	3(9.1)
LOB49	1(9.2)	2(9.0)	3(9.0)	2(8.9)	4(8.8)	1(8.7)
LOB50	1(15.3)	2(12.9)	3(12.1)	4(13.3)	3(13.1)	1(13.0)
LOB51	10(8.1)	11(8.1)	9(7.8)	10(8.0)	9(8.0)	12(7.7)

Table 45: Delta rate sensitivities for 1993 Adjusted EVAS

LOB	Highest(%)	Delta 90-Day 2nd Highest(%)	3rd Highest(%)	Highest(%)	Delta 10-Year 2ndHighest(%)	3rd Highest(%)
LOB02	1(10.6)	2(10.3)	3(9.9)	1(11.4)	2(10.3)	3(9.9)
LOB03	1(17.6)	2(15.5)	3(13.7)	1(18.3)	2(15.5)	3(13.3)
LOB04	1(14.3)	2(13.7)	3(13.0)	2(14.0)	3(13.8)	1(13.3)
LOB05	1(18.4)	2(16.4)	3(14.1)	1(18.6)	2(16.8)	3(14.0)
LOB06	2(14.8)	1(14.6)	3(14.2)	2(15.3)	3(14.6)	1(13.5)
LOB07	-1(18.6)	-2(15.6)	-3(10.1)	-1(22.0)	-2(15.4)	-3(8.5)
LOB08	-5(10.7)	-4(10.4)	1(7.6)	-4(16.8)	-5(15.0)	15(7.9)
LOB09	1(19.3)	2(15.3)	3(12.2)	1(19.4)	2(15.0)	3(11.5)
LOB10	1(25.8)	2(20.7)	3(16.0)	1(28.5)	2(20.3)	3(14.9)
LOB11	1(17.9)	2(16.3)	3(14.4)	1(17.9)	2(16.2)	3(14.5)
LOB12	1(17.9)	2(16.3)	3(14.4)	1(17.8)	2(16.3)	3(14.5)
LOB13	-1(18.4)	-2(15.7)	-3(9.8)	-1(22.7)	-2(15.0)	-3(8.1)
LOB17	1(15.0)	2(14.7)	3(13.8)	2(15.1)	3(14.5)	1(14.4)
LOB18	1(14.2)	2(12.9)	3(11.3)	1(15.7)	2(12.9)	3(11.8)
LOB26	1(15.4)	2(14.8)	3(13.9)	1(15.2)	2(14.9)	3(14.5)
LOB27	1(17.1)	2(14.7)	3(12.8)	1(17.7)	2(15.2)	3(12.0)
LOB28	1(17.2)	2(15.7)	3(13.5)	1(18.3)	2(16.3)	3(13.0)
LOB29	1(25.6)	2(21.2)	3(16.4)	1(28.0)	2(21.6)	3(15.3)
LOB33	-1(20.4)	-2(18.7)	-3(16.5)	-1(20.8)	-2(17.6)	-3(15.3)
LOB34	-3(12.3)	-4(12.0)	-2(12.0)	-2(11.3)	-3(11.1)	-6(10.9)
LOB35	-1(16.4)	-2(14.6)	-3(12.7)	-1(17.0)	-2(14.8)	-3(12.1)
LOB36	-1(15.0)	-2(14.9)	-3(14.2)	-3(15.8)	-2(15.1)	-1(14.4)
LOB37	-2(13.2)	-1(12.9)	-3(12.8)	-2(13.6)	-3(13.5)	-4(12.7)
LOB38	2(13.6)	3(12.7)	1(12.7)	2(14.7)	3(12.5)	4(11.8)
LOB39	1(10.0)	2(9.7)	3(9.6)	1(9.5)	2(9.5)	5(9.5)
LOB40	1(11.3)	2(9.8)	3(8.2)	1(10.7)	2(10.2)	3(8.8)
LOB41	1(18.5)	-4(14.6)	-5(12.6)	-4(21.6)	1(15.1)	9(9.2)
LOB43	1(14.3)	2(13.6)	3(13.0)	2(14.0)	3(13.7)	1(13.2)
LOB44	1(15.8)	2(14.0)	3(13.2)	1(16.6)	2(13.8)	3(13.3)
LOB48	10(7.0)	11(6.8)	9(6.7)	10(7.1)	13(6.9)	11(6.9)
LOB49	1(11.4)	2(10.0)	3(9.3)	2(10.5)	1(10.5)	3(9.6)

Table 46: Delta rate sensitivities for 1994 Adjusted EVAS

LOB	Spread Highest(%)	Spread 2nd Highest(%)	Spread 3rd Highest(%)	Delta Spread Highest(%)	Delta Spread 2ndHighest(%)	Delta Spread 3rd Highest(%)
LOB01	19(8.8)	13(8.6)	16(8.1)	2(7.5)	3(7.5)	1(7.5)
LOB02	19(9.9)	17(8.4)	15(7.4)	4(7.3)	3(7.3)	2(7.3)
LOB03	15(10.3)	12(10.2)	19(9.2)	8(7.2)	6(7.2)	5(7.2)
LOB04	15(8.6)	17(6.8)	9(6.6)	2(8.2)	3(8.2)	1(8.0)
LOB06	9(8.2)	10(8.1)	8(7.9)	2(8.5)	3(8.4)	1(8.4)
LOB07	-1(12.4)	5(8.7)	15(6.7)	2(9.6)	3(9.4)	4(8.8)
LOB09	19(8.3)	17(7.5)	1(7.1)	4(7.3)	1(7.3)	5(7.2)
LOB10	5(7.2)	8(7.1)	9(7.1)	2(8.7)	3(8.6)	1(8.5)
LOB15	5(9.4)	4(8.3)	7(7.8)	4(8.8)	2(8.7)	1(8.7)
LOB16	4(16.4)	5(12.1)	6(10.3)	-9(10.2)	-8(10.2)	-10(9.4)
LOB17	15(11.9)	19(9.0)	12(7.9)	1(7.6)	2(7.6)	3(7.5)
LOB19	5(8.2)	19(7.2)	17(6.8)	4(8.2)	3(8.0)	2(8.0)
LOB26	9(7.7)	13(7.6)	11(7.0)	2(8.5)	1(8.4)	3(8.4)
LOB27	9(7.7)	8(7.6)	13(6.9)	2(8.7)	1(8.7)	3(8.7)
LOB28	11(7.2)	15(7.2)	9(7.0)	2(8.7)	1(8.6)	3(8.5)
LOB29	15(8.0)	19(7.6)	-1(7.4)	3(7.9)	2(7.8)	4(7.7)
LOB31	-1(10.6)	12(9.3)	5(7.4)	2(10.8)	3(10.3)	4(9.9)
LOB32	-1(7.7)	11(7.7)	9(7.3)	2(8.8)	3(8.7)	4(8.6)
LOB33	-7(9.3)	-5(8.6)	-3(8.0)	-1(11.3)	-2(11.1)	-3(10.6)
LOB34	-10(15.2)	-9(12.0)	-11(11.9)	-1(11.5)	-2(10.3)	-3(10.1)
LOB35	-3(10.9)	-7(10.5)	-5(9.7)	-2(12.2)	-1(11.9)	-3(11.7)
LOB36	-7(9.8)	-5(9.4)	-6(7.8)	-1(11.0)	-2(10.8)	-3(10.3)
LOB42	19(10.4)	18(10.0)	15(9.8)	9(6.9)	10(6.9)	8(6.8)
LOB44	11(7.2)	13(7.1)	9(7.1)	2(8.6)	1(8.5)	3(8.5)
LOB45	9(8.3)	13(7.9)	-1(7.0)	5(7.8)	4(7.7)	6(7.6)
LOB46	17(7.6)	15(7.5)	19(7.3)	4(7.7)	5(7.7)	3(7.6)
LOB47	-1(10.5)	15(10.0)	19(8.9)	5(7.2)	6(7.1)	4(7.1)
LOB48	19(8.9)	17(8.5)	16(7.7)	3(7.4)	2(7.4)	4(7.4)
LOB49	12(9.8)	15(8.6)	2(7.8)	2(8.5)	1(8.5)	4(8.0)
LOB50	17(8.3)	19(8.2)	18(7.5)	5(7.1)	6(7.0)	7(6.9)
LOB51	19(12.7)	18(9.7)	17(9.4)	1(6.4)	8(6.3)	9(6.3)

Table 47: Spread Sensitivity for 1992 Adjusted EVAS

LOB	Spread Highest(%)	Spread 2nd Highest(%)	Spread 3rd Highest(%)	Delta Spread Highest(%)	Delta Spread 2ndHighest(%)	Delta Spread 3rd Highest(%)
LOB01	13(10.1)	11(9.8)	9(9.7)	1(9.8)	2(9.8)	3(9.4)
LOB02	17(7.0)	11(6.8)	14(6.7)	1(8.5)	2(8.3)	3(8.2)
LOB03	11(10.8)	9(9.9)	13(8.5)	4(8.4)	5(8.2)	7(8.1)
LOB04	11(9.6)	13(9.0)	9(8.5)	1(8.5)	2(8.5)	4(8.4)
LOB05	8(9.3)	7(8.6)	9(7.4)	1(11.0)	2(10.6)	3(10.1)
LOB06	8(9.4)	7(9.0)	9(7.9)	1(10.6)	2(10.3)	3(9.9)
LOB07	-7(10.8)	-6(10.5)	-5(10.3)	-1(11.9)	-2(11.3)	-3(10.6)
LOB09	8(8.7)	11(7.2)	7(7.0)	3(8.7)	2(8.6)	4(8.6)
LOB10	8(10.3)	9(9.5)	7(8.5)	1(11.1)	2(10.9)	3(10.3)
LOB13	-7(11.5)	-6(10.8)	-8(10.5)	-1(12.4)	-2(11.2)	-3(10.4)
LOB14	7(9.6)	8(9.6)	11(9.1)	1(10.0)	2(9.9)	3(9.6)
LOB15	8(7.5)	9(7.4)	17(7.0)	3(8.7)	2(8.6)	4(8.4)
LOB16	5(11.0)	8(11.0)	6(10.7)	2(13.4)	3(13.2)	1(12.8)
LOB17	8(9.5)	11(9.4)	9(8.8)	1(9.4)	2(9.3)	3(9.2)
LOB19	8(10.3)	7(9.1)	9(8.4)	2(10.8)	3(10.7)	1(10.5)
LOB20	7(10.2)	13(8.5)	8(7.8)	2(9.6)	1(9.5)	3(9.2)
LOB21	7(9.9)	8(9.6)	9(9.3)	1(10.8)	2(10.6)	3(10.3)
LOB22	7(9.9)	8(9.8)	9(9.2)	1(10.8)	2(10.6)	3(10.3)
LOB23	8(9.7)	6(7.9)	9(7.5)	3(9.9)	2(9.7)	4(9.5)
LOB24	7(10.7)	8(9.6)	6(9.2)	1(10.8)	2(10.7)	3(10.3)
LOB25	11(10.4)	8(8.6)	7(8.4)	5(8.6)	6(8.5)	3(8.5)
LOB26	8(8.7)	7(8.4)	9(8.0)	1(10.7)	2(10.4)	3(9.8)
LOB27	8(8.7)	7(8.6)	5(7.2)	1(11.3)	2(10.9)	3(10.4)
LOB28	8(8.1)	7(7.7)	9(7.5)	1(10.8)	2(10.4)	3(9.9)
LOB29	8(11.0)	7(10.6)	9(9.5)	1(10.7)	2(10.6)	3(10.1)
LOB30	7(10.4)	8(9.6)	9(8.9)	1(11.5)	2(11.2)	3(10.7)
LOB31	-6(13.5)	-4(12.2)	-5(11.5)	-1(12.8)	-2(12.2)	-3(12.0)
LOB32	-1(13.2)	-5(10.2)	-6(10.1)	7(12.7)	8(11.8)	9(10.8)
LOB33	-5(12.2)	-6(11.1)	-4(10.4)	-1(14.7)	-2(13.6)	-3(12.3)
LOB34	-9(16.1)	-10(13.6)	-8(10.8)	-1(13.2)	-2(13.0)	-3(12.5)
LOB35	-7(9.7)	-6(8.6)	-4(8.5)	-1(12.0)	-2(11.6)	-3(10.8)
LOB36	-5(10.0)	-4(9.2)	-7(8.7)	-1(11.8)	-2(11.6)	-3(11.1)
LOB37	-7(11.5)	-6(10.4)	-8(10.1)	-1(11.0)	-2(11.0)	-3(10.7)
LOB38	8(10.7)	7(9.3)	9(9.2)	1(9.7)	2(9.7)	3(9.6)
LOB39	2(19.7)	1(16.0)	3(15.9)	1(19.4)	2(15.3)	3(10.4)
LOB42	-3(10.3)	-4(9.2)	-5(7.7)	8(8.6)	7(8.5)	9(8.3)
LOB43	11(9.5)	13(8.9)	9(8.3)	1(8.6)	2(8.5)	4(8.4)
LOB44	8(8.7)	9(7.9)	7(7.9)	1(10.7)	2(10.3)	3(9.8)
LOB45	-1(12.3)	13(9.5)	11(9.3)	7(9.6)	6(9.5)	5(8.8)
LOB46	9(9.2)	8(9.0)	17(8.2)	3(8.8)	4(8.7)	2(8.6)
LOB47	13(10.7)	-1(10.3)	-3(10.0)	7(11.0)	8(9.9)	6(9.8)
LOB48	17(7.2)	19(6.7)	13(6.5)	1(8.4)	2(8.3)	3(8.1)
LOB49	11(8.9)	13(7.9)	17(7.4)	2(8.0)	1(8.0)	3(7.9)
LOB50	-1(9.4)	9(8.1)	8(8.1)	4(9.1)	3(8.9)	5(8.9)
LOB51	17(8.9)	19(8.9)	18(8.7)	9(6.9)	10(6.9)	11(6.8)

Table 48: Spread Sensitivity for 1993 Adjusted EVAS

LOB	Highest(%)	Spread 2nd Highest(%)	3rd Highest(%)	Highest(%)	Delta Spread 2ndHighest(%)	3rd Highest(%)
LOB02	13(6.3)	11(6.3)	17(6.2)	1(9.1)	2(8.8)	3(8.5)
LOB03	8(8.5)	7(8.4)	6(7.2)	1(11.4)	2(11.0)	3(10.3)
LOB04	7(10.9)	11(8.5)	8(8.3)	1(10.0)	2(10.0)	3(9.7)
LOB05	7(8.0)	8(7.9)	5(7.4)	1(11.7)	2(11.2)	3(10.5)
LOB06	7(9.4)	8(8.4)	9(7.8)	1(10.7)	2(10.4)	3(10.0)
LOB07	-3(13.9)	-5(12.6)	-6(11.3)	-1(12.3)	-2(11.4)	-3(10.7)
LOB08	-5(15.4)	-1(12.1)	-4(9.9)	-1(12.3)	-2(9.3)	-3(8.7)
LOB09	8(8.2)	11(7.0)	9(6.9)	3(9.0)	2(9.0)	4(8.7)
LOB10	3(8.9)	8(8.7)	6(8.6)	1(12.4)	2(12.2)	3(11.2)
LOB11	8(8.6)	7(8.4)	9(7.4)	1(11.5)	2(11.1)	3(10.4)
LOB12	8(8.6)	7(8.4)	6(7.4)	1(11.5)	2(11.2)	3(10.4)
LOB13	-3(13.4)	-5(11.4)	-4(11.0)	-1(12.7)	-2(11.4)	-3(10.4)
LOB17	7(10.1)	9(8.3)	8(7.9)	1(10.6)	2(10.5)	3(10.1)
LOB18	9(6.7)	11(6.5)	4(6.4)	1(9.5)	2(9.2)	3(8.8)
LOB26	7(8.6)	8(8.1)	9(7.5)	1(10.9)	2(10.6)	3(10.1)
LOB27	8(7.9)	7(7.5)	6(7.4)	1(11.1)	2(10.8)	3(10.2)
LOB28	7(7.9)	8(7.7)	9(7.4)	1(11.2)	2(10.7)	3(10.2)
LOB29	6(8.4)	5(8.4)	7(8.4)	1(12.6)	2(12.0)	3(11.1)
LOB33	-4(11.4)	-5(11.1)	-6(10.0)	-1(13.9)	-2(13.4)	-3(12.4)
LOB34	-8(18.8)	-9(16.4)	-7(14.8)	-3(13.2)	-2(13.2)	-1(13.2)
LOB35	-6(10.1)	-4(8.9)	-3(8.9)	-1(12.1)	-2(12.0)	-3(11.4)
LOB36	-4(11.7)	-5(9.5)	-6(8.7)	-1(12.2)	-2(11.9)	-3(11.3)
LOB37	-6(12.0)	-7(11.3)	-5(9.8)	-1(11.4)	-2(11.3)	-3(11.0)
LOB38	8(9.5)	7(8.1)	10(7.3)	1(10.5)	2(10.1)	3(9.9)
LOB39	13(8.8)	9(8.7)	17(7.6)	2(7.9)	1(7.9)	3(7.8)
LOB40	13(9.3)	17(8.9)	11(8.2)	2(7.5)	3(7.4)	4(7.3)
LOB41	-5(17.6)	-6(14.6)	-7(10.7)	10(8.7)	11(8.4)	12(7.8)
LOB43	7(10.8)	11(8.4)	13(8.2)	2(10.0)	1(10.0)	3(9.6)
LOB44	8(8.2)	7(8.1)	9(7.6)	1(10.8)	2(10.6)	3(10.0)
LOB48	17(10.8)	16(9.7)	19(9.0)	7(6.7)	8(6.6)	6(6.6)
LOB49	17(7.9)	11(7.6)	13(7.5)	2(8.2)	3(8.1)	1(8.0)

Table 49: Spread Sensitivity for 1994 Adjusted EVAS

LOB	χ Size	ϵ 0.001 Size	ϵ 0.002 Size	ϵ 0.003 Size	ϵ 0.004 Size	ϵ 0.005 Size	ϵ 0.006 Size	ϵ 0.007 Size	Smallest Size
LOB01	8000	6000	6000	1000	1000	1000	1000	1000	1000
LOB02	6000	1000	1000	1000	1000	1000	1000	1000	1000
LOB03	4000	6000	6000	6000	6000	6000	6000	6000	4000
LOB04	8000	7000	7000	7000	7000	7000	7000	7000	7000
LOB06	6000	8000	8000	8000	8000	8000	8000	8000	6000
LOB07	4000	10000	10000	10000	10000	10000	10000	10000	4000
LOB09	4000	1500	1500	1500	1500	1500	1500	1500	1500
LOB10	8000	10000	10000	10000	10000	10000	10000	10000	8000
LOB15	4000	4000	4000	2000	2000	2000	2000	2000	2000
LOB16	7000	1500	1500	1500	1500	1500	1500	1500	1500
LOB17	10000	10000	10000	10000	10000	10000	10000	10000	10000
LOB19	7000	9000	7000	7000	7000	7000	4000	4000	4000
LOB26	7000	2000	2000	2000	2000	2000	2000	2000	2000
LOB27	5000	10000	10000	10000	10000	10000	10000	10000	5000
LOB28	7000	2000	2000	2000	2000	2000	2000	2000	2000
LOB29	2500	8000	8000	8000	8000	8000	8000	8000	2500
LOB31	5000	5000	5000	5000	5000	5000	5000	5000	5000
LOB32	4000	10000	10000	10000	10000	10000	10000	10000	4000
LOB33	4000	7000	7000	7000	7000	7000	7000	7000	4000
LOB34	10000	9000	9000	9000	9000	9000	9000	9000	9000
LOB35	4000	10000	10000	10000	10000	10000	10000	10000	4000
LOB36	4000	4000	4000	4000	4000	4000	4000	4000	4000
LOB42	7000	10000	10000	10000	10000	10000	10000	10000	7000
LOB44	5000	4000	4000	4000	4000	4000	4000	4000	4000
LOB45	4000	10000	10000	10000	10000	10000	10000	10000	4000
LOB46	4000	4000	4000	4000	4000	4000	4000	4000	4000
LOB47	3000	10000	10000	10000	10000	10000	10000	10000	3000
LOB48	8000	1000	1000	1000	1000	1000	1000	1000	1000
LOB49	10000	10000	10000	10000	10000	10000	10000	10000	10000
LOB50	6000	7000	7000	7000	7000	7000	5000	5000	5000
LOB51	6000	5000	5000	5000	5000	4000	4000	4000	4000

Table 50: Sub-sample Size for QR 95% 1992 Adj EVAS

LOB	χ Size	ϵ 0.001 Size	ϵ 0.002 Size	ϵ 0.003 Size	ϵ 0.004 Size	ϵ 0.005 Size	ϵ 0.006 Size	ϵ 0.007 Size	Smallest Size
LOB01	8000	7000	7000	7000	7000	7000	5000	5000	5000
LOB02	6000	1000	1000	1000	1000	1000	1000	1000	1000
LOB03	5000	8000	8000	8000	8000	8000	8000	8000	5000
LOB04	7000	8000	8000	8000	8000	8000	8000	8000	7000
LOB05	5000	10000	10000	10000	10000	10000	10000	10000	5000
LOB06	7000	10000	10000	10000	10000	10000	10000	10000	7000
LOB07	6000	10000	10000	10000	10000	10000	10000	10000	6000
LOB09	6000	8000	5000	5000	5000	4000	4000	4000	4000
LOB10	5000	10000	10000	10000	10000	10000	10000	10000	5000
LOB13	6000	10000	10000	10000	10000	10000	10000	10000	6000
LOB14	7000	10000	10000	10000	10000	10000	10000	10000	7000
LOB15	8000	1000	1000	1000	1000	1000	1000	1000	1000
LOB16	7000	7000	7000	7000	7000	7000	5000	5000	5000
LOB17	9000	10000	10000	10000	10000	10000	10000	10000	9000
LOB19	5000	4000	4000	4000	4000	4000	4000	4000	4000
LOB20	5000	9000	9000	9000	9000	9000	9000	9000	5000
LOB21	4000	6000	6000	6000	6000	6000	6000	6000	4000
LOB22	4000	6000	6000	6000	6000	6000	6000	6000	4000
LOB23	1000	1500	1500	1500	1500	1500	1500	1500	1000
LOB24	10000	10000	10000	10000	10000	10000	10000	10000	10000
LOB25	2500	10000	10000	10000	10000	10000	10000	10000	2500
LOB26	5000	10000	10000	10000	10000	10000	10000	10000	5000
LOB27	5000	10000	10000	10000	10000	10000	10000	10000	5000
LOB28	7000	5000	5000	5000	5000	5000	5000	5000	5000
LOB29	4000	10000	10000	10000	10000	10000	10000	10000	4000
LOB30	6000	10000	10000	10000	10000	10000	10000	10000	6000
LOB31	5000	9000	9000	9000	9000	9000	9000	9000	5000
LOB32	4000	10000	10000	10000	10000	10000	10000	10000	4000
LOB33	10000	10000	10000	10000	10000	10000	10000	10000	10000
LOB34	1000	10000	10000	10000	10000	10000	10000	10000	1000
LOB35	4000	10000	10000	10000	10000	10000	10000	10000	4000
LOB36	3000	3000	3000	3000	3000	3000	3000	3000	3000
LOB37	6000	8000	8000	8000	8000	8000	8000	8000	6000
LOB38	5000	8000	8000	8000	8000	8000	8000	8000	5000
LOB39	6000	9000	9000	9000	9000	9000	9000	9000	6000
LOB42	6000	6000	6000	6000	6000	6000	6000	6000	6000
LOB43	7000	8000	8000	8000	8000	8000	8000	8000	7000
LOB44	5000	7000	7000	7000	7000	7000	7000	7000	5000
LOB45	4000	8000	8000	8000	8000	8000	8000	8000	4000
LOB46	3000	10000	10000	9000	9000	9000	9000	7000	3000
LOB47	4000	7000	7000	7000	7000	7000	7000	7000	4000
LOB48	6000	1000	1000	1000	1000	1000	1000	1000	1000
LOB49	9000	10000	10000	10000	10000	10000	10000	10000	9000
LOB50	7000	2500	2500	2500	2500	2500	2500	2500	2500
LOB51	4000	1500	1500	1500	1500	1500	1500	1000	1000

Table 51: Sub-sample Size for QR 95% 1993 Adj EVAS

LOB	χ Size	ϵ 0.001 Size	ϵ 0.002 Size	ϵ 0.003 Size	ϵ 0.004 Size	ϵ 0.005 Size	ϵ 0.006 Size	ϵ 0.007 Size	Smallest Size
LOB02	5000	1000	1000	1000	1000	1000	1000	1000	1000
LOB03	4000	5000	5000	5000	5000	5000	5000	5000	4000
LOB04	6000	10000	10000	10000	10000	10000	10000	10000	6000
LOB05	5000	7000	7000	7000	7000	7000	7000	7000	5000
LOB06	6000	10000	10000	10000	10000	10000	10000	10000	6000
LOB07	5000	9000	9000	9000	9000	9000	9000	9000	5000
LOB08	5000	5000	5000	5000	5000	5000	5000	5000	5000
LOB09	6000	1000	1000	1000	1000	1000	1000	1000	1000
LOB10	4000	10000	9000	9000	9000	9000	9000	9000	4000
LOB11	1000	10000	10000	10000	10000	10000	10000	10000	1000
LOB12	5000	10000	10000	10000	10000	10000	10000	10000	5000
LOB13	8000	8000	8000	8000	8000	8000	8000	8000	8000
LOB17	7000	10000	10000	10000	10000	10000	10000	10000	7000
LOB18	5000	2000	2000	2000	2000	2000	2000	2000	2000
LOB26	7000	5000	5000	5000	5000	5000	5000	5000	5000
LOB27	6000	7000	7000	7000	7000	7000	7000	7000	6000
LOB28	7000	8000	8000	8000	8000	8000	8000	8000	7000
LOB29	4000	8000	8000	8000	8000	8000	8000	8000	4000
LOB33	4000	10000	10000	10000	10000	10000	10000	10000	4000
LOB34	8000	10000	10000	10000	10000	10000	10000	10000	8000
LOB35	3000	10000	10000	10000	8000	8000	8000	8000	3000
LOB36	3000	6000	6000	6000	6000	6000	6000	6000	3000
LOB37	5000	9000	9000	9000	9000	9000	9000	9000	5000
LOB38	5000	5000	5000	5000	5000	5000	5000	5000	5000
LOB39	10000	10000	10000	10000	10000	10000	10000	10000	10000
LOB40	7000	10000	10000	10000	10000	10000	10000	10000	7000
LOB41	6000	10000	10000	10000	10000	10000	10000	10000	6000
LOB43	7000	10000	10000	10000	10000	10000	10000	10000	7000
LOB44	5000	7000	7000	7000	7000	7000	7000	7000	5000
LOB48	7000	7000	7000	7000	7000	7000	7000	7000	7000
LOB49	8000	8000	8000	8000	8000	8000	8000	8000	8000

Table 52: Sub-sample Size for QR 95% 1994 Adj EVAS

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