Portfolio Selection and Repeating Empirical Progressive Optimization (REPO)

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Asset Portfolios

- Designing and future maintenance of an asset portfolio backing a new line of business is critical to proper AL/M for that line of business.

Considerations

- Scenarios
- Asset Universe
- Liability Models
- Profit Measures
- Asset strategies
- Optimization

Scenarios

- Historical Models

Asset Universe

- Types
- Quality
- Defaults
- Models

Liability Models

- An accurate model of the line of business is assumed

Profit Measures

- There must be some type of consideration of trade off between return and risk
  - Utility
  - Global Roe and Lower Second Partial Moments

Asset Strategies

- Initial Investment Strategy
- Renewal Investment Strategy
- Disinvestment Strategy
  - Borrowing
  - Selling off assets

Asset Strategies Continued

- Static
  - Static mix of assets that maximize the profit measures for all scenarios processed
  - Our following example
- Dynamic or Interactive
  - Determination of optimal asset mixes in different economic scenarios

Optimization

- Based on the given profit measure(s), you can obtain an optimal asset strategy by using various search algorithms.

Optimization Problems

- The corporate model software have long runtimes, and hence an experiment is very costly.
- Most complex search algorithms require many different experiments.
- Most simple search algorithms only influence one control variable at a time.
- If the researcher runs out of time, an optimal answer may have not been obtained.

Introducing: REPO

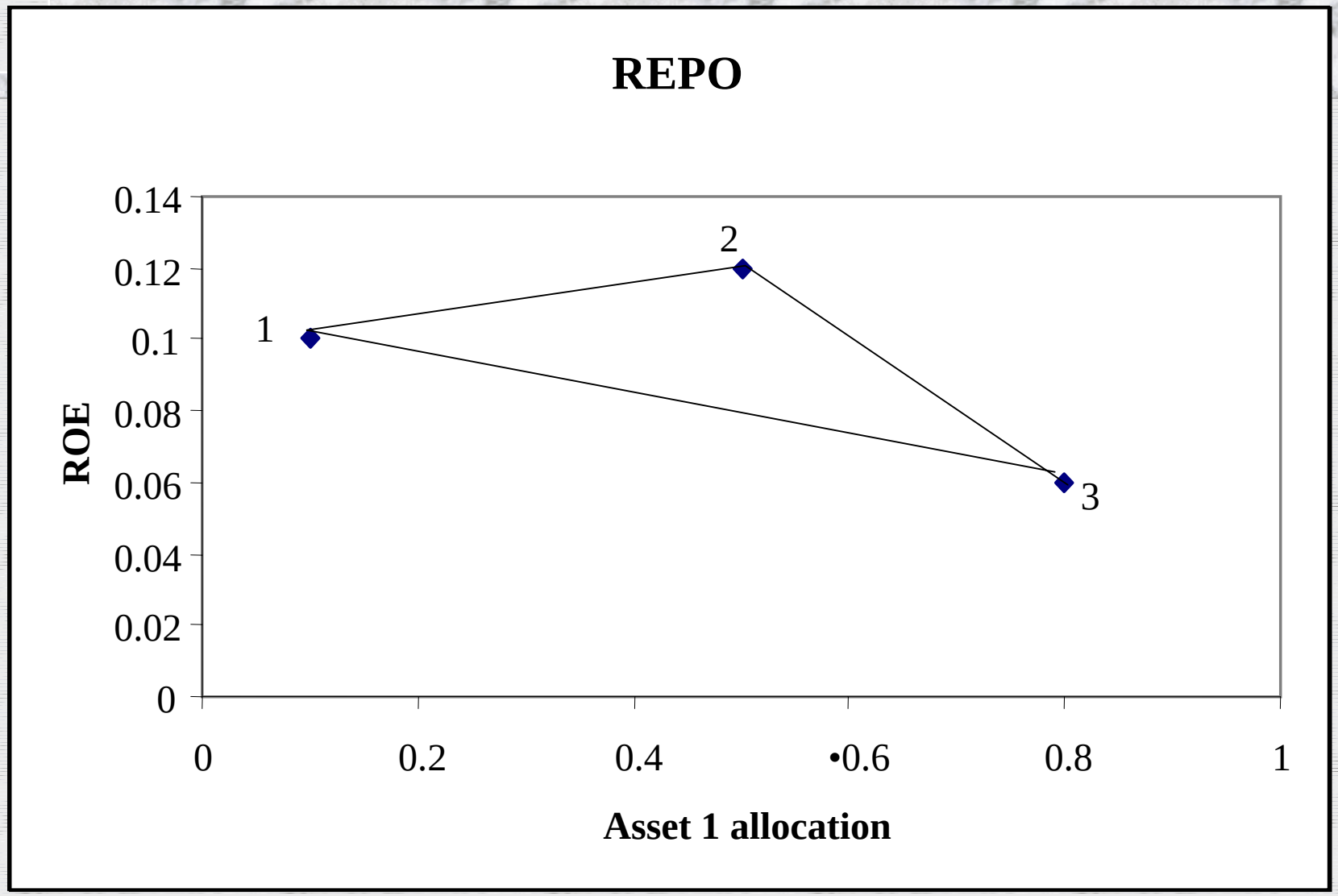
- REPO is short for Repeating Empirical Progressive Optimization
- We will first use a simple two asset portfolio example to demonstrate the algorithm

Experiments

- First you must set up the initial experiments (we will start with 3)
  - Experiment 1 (.1,.9) {here .1 is the percentage of asset 1 in the portfolio, .9 etc}
  - Experiment 2 (.5,.5)
  - Experiment 3 (.8,.2)

Conduct the experiments

- Using the different allocations, you will obtain the profit measures that you are interested in maximizing.
- Experiment 1 roe = .10
- Experiment 2 roe = .12
- Experiment 3 roe = .06


REPO
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
ROE
1
2
3
0
0.2
0.4
•0.6
0.8
1
Asset 1 allocation

Sort the experimental results by profit measure

- Experiment 3 (.8, .2) Roe = .06
- Experiment 1 (.1, .9) Roe = .10
- Experiment 2 (.5, .5) Roe = .12

Determination of the worst experiment(s) to throw out

- This is done by finding the maximum difference between the average good experiments and the average bad experiments

Determination of the worst experiment(s), Continued

- Experiment 3 (.8, .2) - .06 .11-.06 = .05
- Experiment 1 (.1, .9) - .10 .12-.08 = .04
- Experiment 2 (.5, .5) - .12
- We will throw out experiment 3

Design of the new experiment

- There are two methods that can be used based on whether you can have negative asset allocations or not.

Negative asset allocations allowed

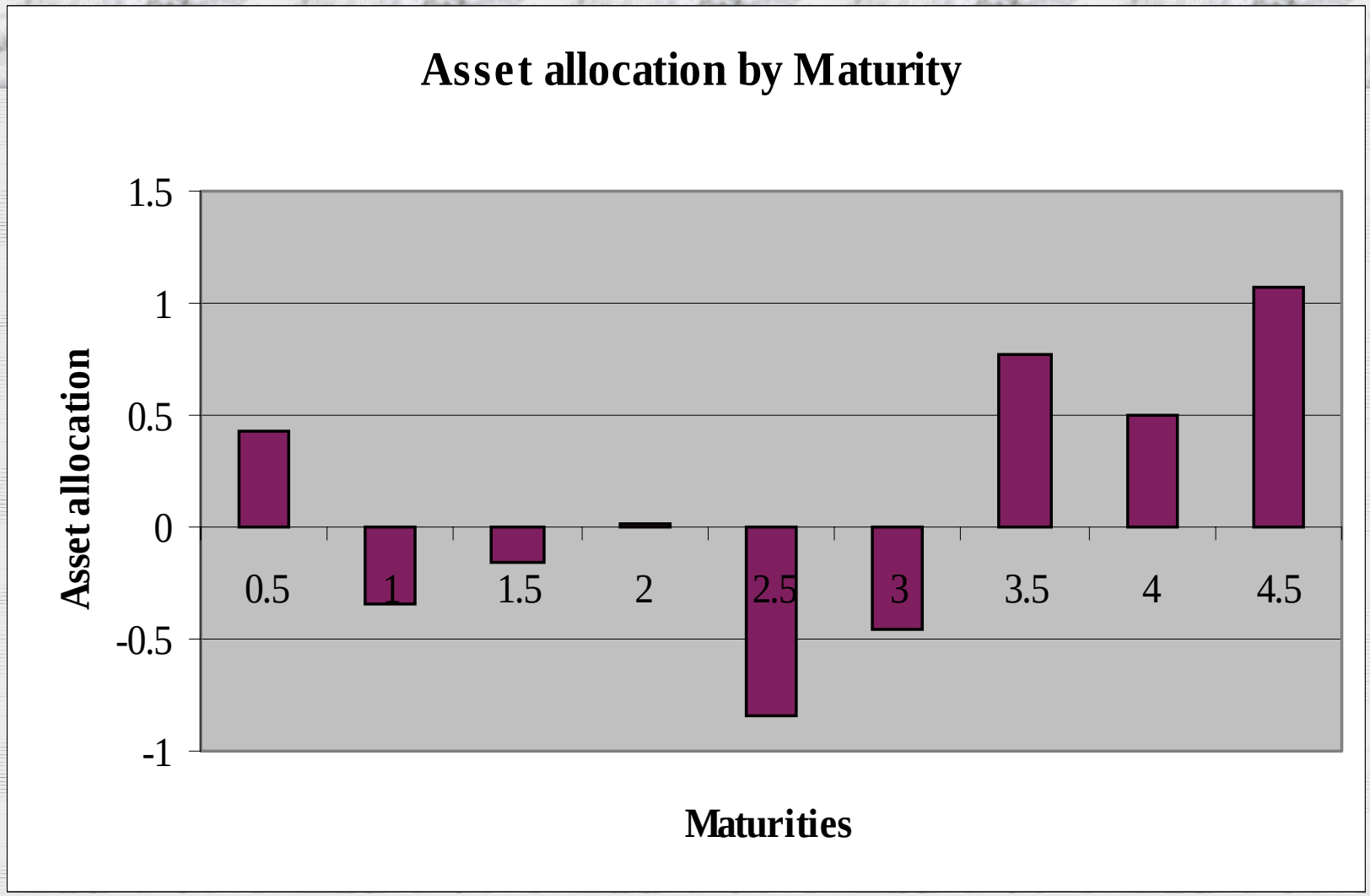
- Take the average of the asset allocations of the good experiments and multiply by two and subtract the bad asset allocation. This will produce new coordinates that may not make sense as an asset allocation. You obtain the new experiment by dividing the new experimental values by the sum of the observed coordinates.

Negative asset allocation allowed

- Experiment 3 (.8, .2) - .06 BAD
- Experiment 1 (.1, .9) - .10
- Experiment 2 (.5, .5) - .12
- Experiment 4 (2*(.1+.5)/2-.8, 2*(.9+.5)/2 -.2) = (-.2,1.2)
- Sum = 1 the new experiment should be (-.2,1.2)

Some Justifications for Negative Asset Allocations

- Maximize economic value
- Negative allocation may be counterbalancing underlying options in the liability
- Derivatives can be used to obtain an equivalency to the negative asset allocation
- If negative asset allocations represent embedded liability options, then an economic price of the options could be calculated


Asset allocation by Maturity
1.5
1
0.5
0
-0.5
-1
Asset allocation
0.5
1
1.5
2
2.5
3
3.5
4
4.5
Maturities

Simplification of asset universe

If you want to eliminate the asset that gives the negative allocation, you may do so, but you must reallocate the other assets so that you have a 100% allocation. This method does reduce the asset universe and does improve model run times.

Negative asset allocations not allowed

- Take the geometric average of the asset allocations of the good experiments and square and divide by the bad asset allocation. This will produce new coordinates that will not make sense as a asset allocation. You obtain the new experiment by dividing the new experimental values by the sum of the observed coordinates.

Asset universe, Continued

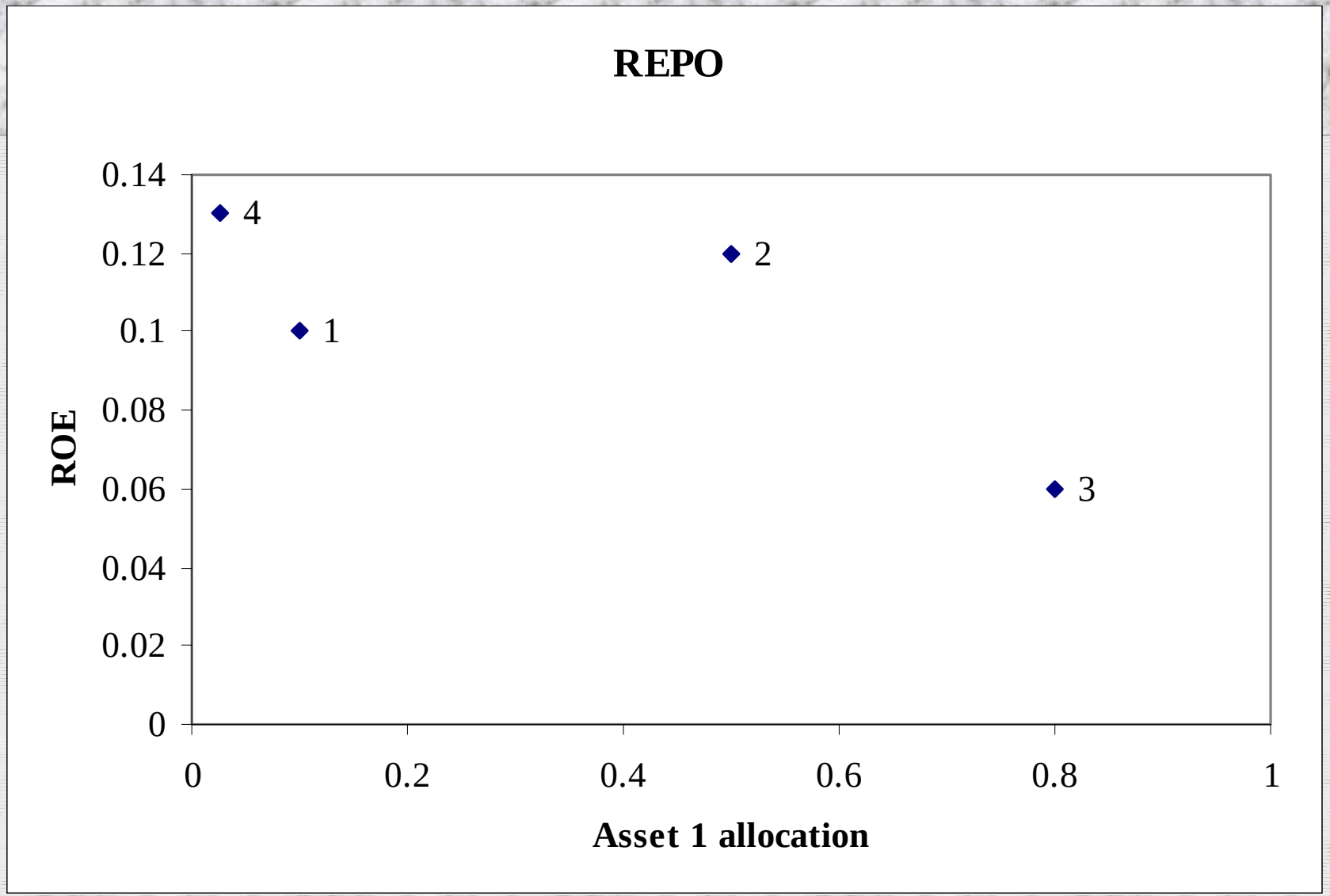
However, if you always want to retain all the assets in the asset universe, but you can’t have negative allocations you should do the following:

Negative asset allocation not allowed

- Experiment 3 (.8, .2) - .06 BAD
- Experiment 1 (.1, .9) - .10
- Experiment 2 (.5, .5) - .12
- Experiment 4
  (((.1*.5)^.5)^2/.8,(.9*.5)^.5)^2/.2) = (.0625,2.25)
- sum = 2.3125 the new experiment should be (0.027027,0.972973)

Conduct the next experiment

- Assume that negative allocations are not allowed
- Experiment 4 (.027027, 0.972973) has an roe of .13


REPO
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
ROE
4
2
1
3
0
0.2
0.4
0.6
0.8
1
Asset 1 allocation

99 Bottles of Beer on the Wall

- Taking the results of experiment 1,2 and 4
- Repeat the process of sorting from lowest to highest and repeat the above steps to design the next experiment.

Repo's Strengths

- 1. Redesigns the next experiment by modifying all variables
- 2. Does not require gradient calculation at each new point which reduces the number of experiments
- 3. If you run out of time, you can stop with the best result that you have obtained so far.

Repo's Strengths, Continued

- 4. Per personal communication by Dr. Hongfei Zhang, REPO is a form of LDS on the output space and not on the input space.
- 5. Experiments are progressive, and designed by the REPO process.
- 6. When recommended experiments begin to repeat former experiments, you stop the process.

Repo's Weaknesses

- 1. Initial experimental design requires a good representation set. This is currently solved by recommended experiments in C. D. Hendrix's paper "Empirical Optimization in Research and Development." Another possibility is to use a LDS process to estimate the starting experiments.

Repo's Weaknesses, Continued

- 2. No guarantee that you will find the global maximum. This is related to the initial experimental design.
- 3. Sometimes multiple experiments are required. Occasionally when most experiments have close Y values, REPO will require a all new experiments except the last. This might increase the required run time to accomplish the next round of experiments.

Repo's Weaknesses, Continued

- 4. For any one experiment REPO requires one Y result value for each experiment. If you have multiple profit measures these must be manipulated until you obtain the single result. One approach to a solution of this is demonstrated below.

Example of actual use of REPO

- Bank Owned Life Insurance
- A life insurance product that is purchased by a bank to fund employee benefits.
- We used a single premium universal life product in our experiments

Boli - Assets

- We used corporate noncallable bonds with a credit rating of ‘A’ with maturities of

  1,3,5,7,10 and 30 years

Results

- Page 1 of the handouts is the analysis of Minimizing PARSTD and maximizing Global ROE.

Results, Continued

- We used the relationship

RS_ROE = (ROE -LBnd)/(UBnd - LBnd)

to adjust the Global Roe to be between 0 and 1.

Results, Continued

- We used the relationship

$$RS\_Parstd=(Parstd- UBnd)/(LBnd - UBnd)$$

to adjust the Partial Lower Moment PARSTD. The reversal of the position of the Upper and Lower bounds, in this formula allows for a minimization problem to be converted to a maximization problem.

Results, Continued

- We placed twice the emphasis on Global roe using (1+RS_ROE)^2 in the following:

  Y= ((1+RS_ROE)^2)*(1+RS_Parstd))^(1/3) - 1
- This rescaled Y will allow it to remain between 0 and 1 and place twice the emphasis on ROE. If this Y is maximized the original ROE is maximized and original Parstd is minimized.

Results, Continued

- On Page 2, you will see the experiments that were designed by REPO
- On Page 3, you will see the experiments resorted by the resultant values of Y.
- On Page 4, you will see the results of Page 1 resorted into increasing importance.

Graph of comparison of spread efficient frontier with the 3/10 year barbell

See Handout

Future Research

- See the results of using the starting yield curve equal to the Tenney implied historical yield curve.
- Using many different starting yield curves, created by starting the valuation 6 months later.
- Obtain a dynamic investment strategy
- Examine the results using NW's utility.