Economic Scenario Generator for Insurance and Pension Rational Decision Making Under Uncertainty

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September 1997

Abstract

We develop a stochastic generator for the generation of scenarios of the S & P 500 index, dividend yield, consumer price index, and U.S. Treasury yields. We first create a set of "stylized facts" for these series, and we estimate statistical models for these series. These in-sample statistical models are themselves not suitable for generation of scenarios for decision making, but instead are additional "stylized facts" that assist in model development. The "best" statistical model according to standard statistical model selection criteria can easily lead to a model that is highly unsuitable for generation of scenarios for decision making. We develop a stochastic generator that is suitable for decision making under uncertainty.

Key Words:

Stylized Facts, Double Mean Reverting Process^{TM} [1], ARIMA models, Transfer Functions, Diffusion Models, Insurance, Pension, and Uncertainty.

[1]Double Mean Reverting Process^{TM} and DMRP^{TM}are trademarks of Mark Tenney.

Contents

1 Introduction

The generation of economic scenarios generation is a joint process on the yield curve, and additional economic state variables. Examples of additional economic state variables are inflation, stock price index, the dividend yield on this index, and currency exchange rates. One application of economic scenarios in insurance is the evaluation of equity linked annuities where one needs a joint process on the yield curve and the equity indexs price and dividend returns.The pricing and the hedge strategy depend on this process. Another application would be in proper asset-liability projection of and entire company where inflation is usually required. Also exchange rates are required if the company is international. In pension plans and casualty insurance economic scenarios could be used in the bond and stock asset mix analysis. There has also been increased pressure to conduct various forms of dynamic conditions analysis upon both the life and casualty business. Some form of economic scenario generation is required with this analysis. Also increased sophistication of new asset derivatives such as inflation indexed bonds, places pressure on the industry to develop and use these complex models.

David Wilkie has been instrumental in creating major western economic models for the British actuarial community. See [23, 24, 25, 26, 27]. The process that we propose will fundamentally move away from his general approach. However, one of the goals of our model will be to simulate realistic market behavior while also allowing for the benefit of risk neutral pricing. David Wilkies goals for economic scenarios have always been that simulation of realistic market behavior. Much of his insight in the use of these models will guide us in our overall goals for our models.

In Section 2, we will discuss what we call qualitative and quantitative stylized facts as well as economic phenomenon. Here we will also describe the philosophy of modeling that we will follow. Here we will describe the difference of realistic and risk adjusted scenariosand its importance. In Section 3, we will describe our data sources. In Section 4, we will list our stylized facts, and briefly describe Tenneys DMRP^{TM}model. In Section 5, we will list some observed market phenomenon. In Section 6, we will describe the construction of an risk adjusted U.S. model. In Section 7, we will discuss the concept of Generalized Method of Moments, and the need of re-calibrating our risk neutral model to reflect realistic economic scenarios.In Section 8, we will discuss future research.

2 Our Philosophy of Modeling

Qualitative stylized facts are qualitative non-episode dependent obervations that are true over long time intervals and across economies. We believe that these facts should be contained in the equilibrium model for all industrial economies. Quantitative stylized facts are specific numerical observations of specific market processes. These are less stable than qualitative facts and can change through time. We define market phenomenon as where interesting relationships within the observed data that still need to be explained. Examples of all of these concepts will be discussed in Sections 4 and 5.

There are two approaches to statistical modeling. The first approach attempts to find the true model and determine its parameters by maximum likelihood estimation. This model is selected to pass all in-sample statistical tests no matter how strange the resulting model. The second approach, which is the one that we find an approximate model. Here, the functional form of the model to conform to qualitative stylized facts. We may also constrain the parameters of the model to satisfy the quantitative stylized facts. Finally, we may estimate the model with a criterion function based on multiple measures of in-sample fit. In the above estimations we may create rough quantitative stylized facts from qualitative stylized facts. However, the approximate model may not pass all statistical tests.

We use the term "realistic" when describing economic scenarios as those scenarios that best replicate the above stylized facts and/or the market phenomenon. Since we exist in an risk adverse world, all market prices of assets have risk premia imbedded into their value. Because we are using market data (see Section 3) to determine various stylized facts, our stylized facts will also be imbedded with the market price of risk. We will also use the term "realistic" when describing other scenario generators that include various imbedded risk factors and/or some form of stylized facts.

In a complete market an asset has only one price, because if two different prices existed, then market players would take advantage of the arbitrage opportunity and force the separate prices to the single price. There are two processes by which one finds the market price of a security. The first process is to determine the "hedge portfolio" or defeasing strategy in a realistic setting. The defeasing strategy will replicate the behavior of the underlying security. The prices of the securities that make up the defeasing strategy are then determined by finding their market price. These prices are accumulated and will be the price of the original security, otherwise a arbitrage opportunity is constructed. The other process is to We use the term "risk adjusted" or "risk neutral" when describing the scenarios generation process.

Tenney in [15] makes the distinction between realistic and risk neutral pricing. Here he states that when pricing any stream of cash flows, one can use either realistic scenarios or risk adjusted scenarios and obtain the same price. When using realistic scenarios, one must construct the hedge portfolio that replicates the cash flows. When once this hedge portfolio is obtained, one finds the price by amalgamating the market price of each asset in the hedge portfolio.

Now when using risk adjusted scenarios, one can obtain the same market price without the added requirement of determining the hedge portfolio.

3 Data Sources

The original data source for yield curve data is the U.S. Treasury data from 1970 to 1994. See [20, 21, 22]. Mark Tenney's DMRP^{TM}model has the monthly u and θ values that best fit that month end U.S. Treasury yield curve. Here u and θ are the state variables underlying the DMRP^{TM}model. See [17]. Also see Appendix A. Graphs of u and θ are Figures 1 and 2.

The DMRP^{TM}model is calibrated to the 1970 to 1994 period. Because of this calibration, we have restricted all of our data series to correspond to the same period. The source of the month end S & P 500 stock return index is[1]. A graph of the monthly price return on the S & P 500 is Figure 3. See Appendix B.4 for the basic statistics. A graph of the monthly total return (see our defination in Section 6) on the S & P 500 is Figure 4. Review Appendix B.3 for the basic statistics. We also use the quarterly (1970-1987) dividend yield index on the same underlying stock. See [1]. This quarterly index was pro rata allocated to the months in that specific quarter. We also use the monthly dividend yield index from 1987 to 1994. See 5 for a graph of this series. Refer to Appendix B.2 for basic statistics on the series. The monthly U.S. inflation series is the Consumer Price Index–All Urban Consumers. See [22]. See Figure 6 for a graph of this monthly series. See Appendix B.1 for the basic statistics on monthly inflation.

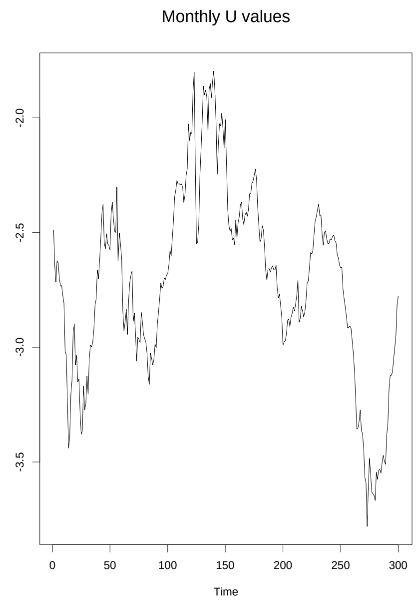

Figure 1: Monthly u values

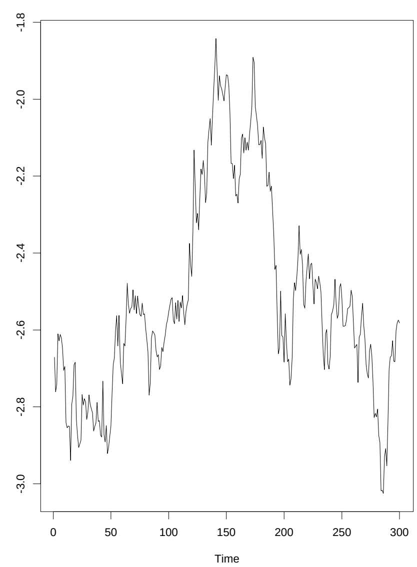

Figure 2: Monthly θ values

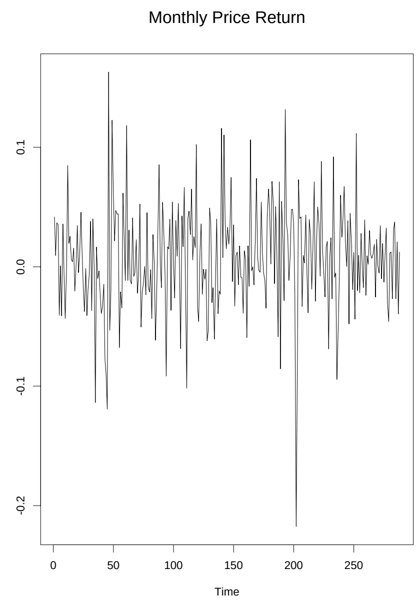

Figure 3: Monthly Price Return

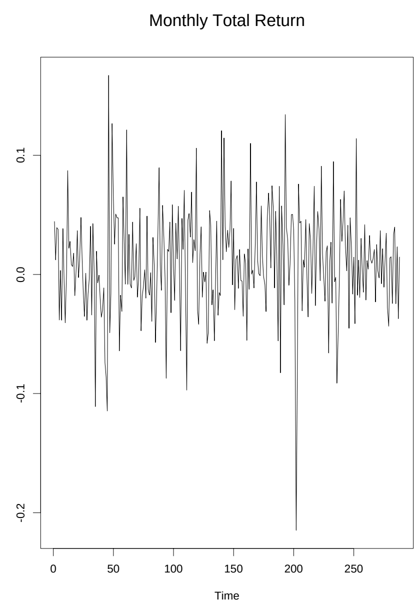

Figure 4: Monthly Total Return

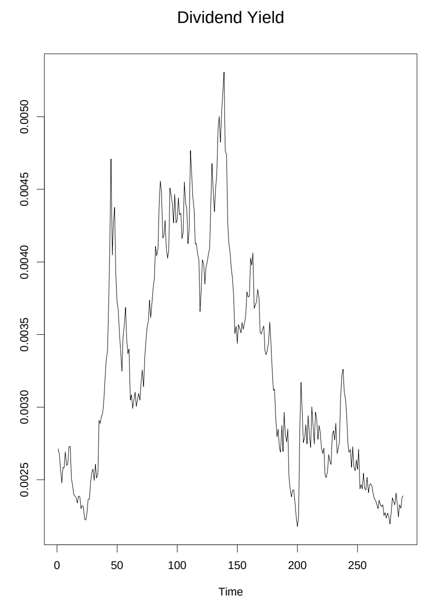

Figure 5: Monthly Dividend Yield

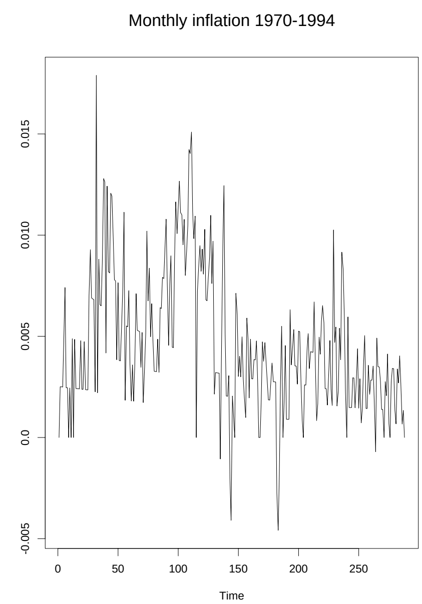

Figure 6: Monthly Inflation

4 Stylized Facts

In [2, 3, 4, 7, 8, 9, 10, 18] there has been various stylized facts described and discussed. Our list of qualitative stylized facts for economic scenarios are:

1. Interest rates do not go to zero or infinity but stay within a reasonable range.
2. Interest rates can spend up to several years within a narrow band or trading range before breaking out to a higher or lower range.
3. Short- and long-term interest rates are correlated but not perfectly.
4. The volatility of long-term rates is less than that of short-term rate.
5. Volatility is higher for higher levels of rates.
6. Interest rates neither increase or decrease rapidly with significant frequency.
7. Higher absolute inflation rate levels are often associated with higher absolute inflation rate volatility. See [27].
8. The rate of inflation has trended upward from the bottom of the depression to the early 1980s and then significantly lower up to 1996. See [27].
9. The stock price return is weakly correlated with interest rates.[2]
10. The stock index tends to trend up.
11. Stock prices can cluster in trading ranges or narrow bands (sometimes for extended periods)

Our list of quantitative stylized facts are:

1. Yield curves can have a variety of shapes. They are normally positively sloped, but can have 'humps'.
2. Significant inversions are infrequent (less than 13%) and of relatively limited durations (less than 27 months).

[2]This similar to Vishwanath Tirupatturs results at Lincoln National.

3. The mean, variance, and geometric mean of the study period should be replicated for each data series.

Other quantitative stylized facts will be discussed in Section 5.

5 Market Phenomenon

In [2, 3, 4, 7, 8, 9] there has been various phenonomon described and discussed. A summary of those results are below. (Note: Spreads referred below are spreads between different maturities on the yield curve.)

Generally, as rates rise:

1. Spreads narrow[3]
2. The yield curve flattens
3. S & P 500 total returns decline
4. S & P 500 dividend yields increase
5. Inflation increases

Generally, as rates fall:

1. Spreads widen
2. The yield curve steepens
3. S & P 500 returns increase
4. S & P 500 dividend yields decrease
5. Inflation decreases

[3]This was first observed by Chris Harper at Nationwide Life Insurance Company.

Series	Symbol	Relationship	Residuals
Short rate	r_t	$u_t = \ln(r_t)$	ϵ_{u_t}
Intermediate rate	s_t	$\theta_t = \ln(s_t)$	ϵ_{θ_t}
Total Return	T_t	$x_t = \exp(\Delta \ln(S_t)) + d_t - 1$	ϵ_{x_t}
Price Return	S_t	$y_t = \Delta \ln(S_t)$	ϵ_{y_t}
Dividend Index	D_t		
Dividend Yield	$d_t = \frac{\sum_{i=t-11}^{t} D_i}{12 S_t}$		ϵ_{d_t}
Inflation Index	P_t	$w_t = \Delta \ln(P_t)$	ϵ_{w_t}

Table 1:

6 Risk Neutral Scenario Generation

The notation of our series is contained in Table 1. All of our studies will be conducted on monthly returns.

We will generalized the DMRP^{TM}model into a multivariate Brownian Motion defined as

$$DY = (b + AY)Dt + G'DZ \tag{1}$$

Here the term $b + AY$ is the deterministic drift term of the multivariable model. The G' is the Choleski decomposition of the covariance of the residuals of the series after the drift relationships have been removed.

First we must construct the drift term part of the model. In appendix B.1 and B.2, we obtain the following two equations

$$w_t = .0038u_t - .0037\theta_t + .4349w_{t-1} + .0034 + \epsilon_{w_t} \tag{2}$$

$$d_t = .0001u_t + .9312d_{t-1} + .0006 + \epsilon_{d_t}. \tag{3}$$

We also determined that the total return of the S & P 500 total return did not have any relationships with the underlying state variables u and θ.

We then determined the residuals of all the variables by the use of the following equations:

$$\epsilon_{u_t} = \Delta u_t - \kappa_u(\theta - u_t - \lambda_u)dt \tag{4}$$

$$\epsilon_{\theta_t} = \Delta \theta_t - \kappa_\theta(\bar{\theta} - \theta_t - \lambda_\theta)dt \tag{5}$$

$$\epsilon_{w_t} = w_t - (.0038u_t - .0037\theta_t + .4349w_{t-1} + .0034) \tag{6}$$

	u	θ	w	x	d
u	0.097515	0.018401	0.000109	-0.007519	0.000015
θ	0.018401	0.042590	0.000169	-0.008271	0.000025
w	0.000109	0.000169	0.000080	-0.000219	0.000001
x	-0.007519	-0.008271	-0.000219	0.023506	-0.000076
d	0.000015	0.000025	0.000001	-0.000076	0.0000003

Table 2: Annualized Covariance Matrix

$$\epsilon_{x_t} = x_t - \overline{x_t} \quad (7)$$
$$\epsilon_{d_t} = d_t - (.0001u_t + .9312d_{t-1} + .0006) \quad (8)$$

The following values of the various parameters were used in actually constructing the residuals:

$$\begin{aligned} dt &= 1/12 \\ \kappa_u &= 0.5 \\ \kappa_\theta &= 0.15 \\ \lambda_u &= 0.1982 \\ \lambda_\theta &= 0.4732 \\ \bar{\theta} &= \ln(0.106821) \\ \overline{x_t} &= .00982 \end{aligned}$$

Once the residuals are obtained, we determine the convariance matrix of the residuals. Table 2 has the annual covariance matrix, which was obtained by multiplying the monthly covariance matrix by 12.

Using the Choleski decomposition algorithm, we factor the annual covariance matrix into a upper matrix G, so that that $G'G$ equals the annual covariance matrix.

Producing the risk adjusted model, we will assume that the b vector has the form in Table 4 and the A matrix the form in Table 5.

However, we will use the following risk adjusted parameters from the DMRP^{TM}model in the b vector and the A matrix.

$$\kappa_u = 0.689$$

	u	θ	w	x	d
u	0.312275	0.000000	0.000000	0.000000	0.000000
θ	0.058927	0.197781	0.000000	0.000000	0.000000
w	0.000350	0.000075	0.008927	0.000000	0.000000
x	-0.024078	-0.034646	-0.020726	0.145931	0.000000
d	0.000048	0.000014	0.000094	-0.000474	0.000194

Table 3: Choleski Decomposition G'

u	$-\kappa_u \lambda_u$
θ	$\kappa_\theta(\bar{\theta} - \lambda_\theta)$
w	0.0
x	$-0.5\sigma_x^2$
d	0.0

Table 4: The b Vector

	u	θ	w	x	d
u	$-\kappa_u$	κ_u	0.0	0.0	0.0
θ	0.0	κ_θ	0.0	0.0	0.0
w	0.0	0.0	0.0	0.0	0.0
x	0.0	0.0	0.0	0.0	0.0
d	0.0	0.0	0.0	0.0	0.0

Table 5: The A Matrix

u_t	log of instantaneous rate
θ_t	log of intermediate target rate
ϵ_{w_t}	inflation residual
ϵ'_{x_t}	Stock residual with variance drift
ϵ_{d_t}	Dividend yield residual

Table 6: Process Variable Names

$$\begin{aligned} \kappa_\theta &= 0.0369 \\ \lambda_u &= 0.0 \\ \lambda_\theta &= 0.0 \\ \bar{\theta} &= \ln(0.106821) \\ -0.5\sigma_x^2 &= -0.00982 \end{aligned}$$

In the production of economic scenarios we then generate the multivariate Brownian motion on the process variables in table 6.

Finally the actual economic scenarios are genenerated from the following equations:

$$d_t = \min(\max(.0001u_t + .9312d_{t-1} + .0006 + \epsilon_{d_t}, .01/12), .08/12) \quad (9)$$

$$x_t = \exp(\sum_{i=0}^{t} r_t) \exp(\epsilon'_{x_t}) \quad (10)$$

$$y_t = \frac{x_t}{(1 + d_t)} \quad (11)$$

7 Generalized Method of Moments and Realistic Scenarios

In the prior section, we determined the risk neutral model. In the following we will discuss our future research in the pursuit of a realistic economic generator using the Generalized Method of Moments (denoted GMM).

GMM is best explained by an example. Suppose you had a series of samples of number of claims of a certain type of policy in a given time period. Say that the mean and standard deviation of the number of claims is μ_s and σ_s^2. Here one expects that the underlying population distribution to be the Poisson distribution. Using the method of moments, you would set λ equal to μ_s. However, λ in the Poisson distribution corresponds both to the population mean and the population variance. So, λ would be best fit from information from both μ_s and σ_s^2. So you could find the λ that minimizes the objective function $(\lambda - \mu_s)^2 + (\lambda - \sigma_s^2)^2$ or something similar. GMM is this process of minimizing an objective function that reflects several multiple moment equations.

We will revise the various elements of the b vector and of the A matrix, so that statistics on the scenarios of the generator will replicate some our various stylized facts and the historical averages and standard deviations. The variance modeling of the risk adjusted model must remain uneffected when moving to realistic modeling, so there will be no revisions to the values of the G' matrix.

Since u and θ have already been calibrated, we will only consider other values in the b vector and other rows in the A matrix in our the calibration.

To properly estimate the unknown population distribution of the economic series we must make our objective target statistics to be at a very high epoch. This is because the initial starting values can have a dramatic influence upon the target statistics if conducted too early in the process. By choosing a high epoch (such as 100 or 1000 years) the resulting distribution will be unaffected by the initial conditions.

Next we must decide what our targets will be, and then we must collect the statistics on these targets from historical data.

Next we must construct the objective function that will be minimized.

Finally, we will will require some simulated annealing algorthim such as Marquardt-Levenburg, to minimize the objective function. See [14].

The program implementing the above must accomplish the following:

1. Take starting values for the b vector and the A matrix.

2. Produce economic scenarios.

3. Collect statistics on those scenarios.

4. Evaluate the objective function on the observed statistics and the target statistics.

5. Modify the values of the b vector and the A matrix, under the direction of the Marquardt-Levenburg algorithm to estimate the gradient of the objective function.

6. Iterate the above while using the gradient to minimize the objective function.

Note: In some of our preliminary pursuits of the above, we have found that by using low discrepancy sequences (see [5, 11, 12, 13, 19]), we can speed up the convergent process by reducing the number of necessary scenarios.

8 Future Research

As mentioned in Section 7, our next goal is to model the realistic scenario generator, by the use of GMM. We have recieved requests to extend our model to include other series such as Dow Jones Industrial Average stock return, currency exchange rates, LIBOR rates, unemployment rates for the U.S. economy, as well as, for other major industrial economies. We have found in the research of these models that the requests for additional series are only outnumbered by the requests to revise the stylized facts. The addition of other series is not so difficult in the U. S. economy, as that of the calibration of the parameters to replicate historical targets and stylized facts. However, in the pursuit of modeling other economies, the underlying term structure model of that economy must also be modeled. One of the current goals of Mathematical Finance Company is to model these industrial economies and term structures.

Other areas of potential research is to revise the DMRP^{TM}model so that the variance and the drift terms are dynamic as in GARCH modeling. Other areas of research is the revision of the DMRP^{TM}model to allow for dynamic risk premia. Also, we need to research if there is inflation risk premium, and how to model that risk premium. (The above risk adjusted model assumes that the risk premium is zero).

We also need to increase our product and cash flow modeling understanding by using these economic scenarios.

Other technical development is for the scenario generation software to be implemented using OCX control. This will allow the software to be easily included in whatever corporate and/or pricing software currently being used in the actuarial industry.

Acknowledgements

(Steve Craighead) I would like to thank Steve Hodges, Russ Osborn, Uli

Stengele, John Marcsik, and Will Babcock for many hours of insightful discussions on this subject. I also thank Will Babcock for his taking a mass of disorganized thoughts and turning them into a very readable document. I also want to thank my wife Patricia and my children Sam, Michelle, Bradley, Evan, and Carl for their patience while researching and writing this paper.

(Mark Tenney)

A The DMRP^{TM} Interest Rate Model

$$du = \kappa_u(\theta - \lambda_u - u)dt + \sigma_u dz_u$$

$$d\theta = \kappa_\theta(\overline{\theta} - \lambda_\theta - \theta)dt + \sigma_\theta dz_\theta$$

$$\rho = corr(dz_u, dz_\theta)$$

B Statistical Models

Below we will examine the linear models used for inflation, dividend yield and total return for the S & P 500.. These models will be regressed (if possible) against u_t, and θ_t, and autoregressed with previous values of the data series.

We will construct the dividend model before the total return because the total return is built up from the price return of the S & P 500.

We will use the following range for time: t ranges from 1 (January 1970) to 300 (December 1994).

B.1 Inflation

Using the notation of Table 1

$$w_t = \Delta \ln(P_t) \tag{12}$$

for t= 12..300. See Table 7 for basic statistics on w_t.

Min.	-0.004585
1st Qu.	0.002374
Median	0.003751
Mean	0.0046
3rd Qu.	0.006711
Max.	0.0179
Std. Dev.	0.003486597
Geo. Mean	0.004594

Table 7: Summary statistics on w_t

The linear regression for equation 2 is contained in Table 8.

The residual standard error on w_t is 0.002602 on 283 degrees of freedom. The multiple R-squared is 0.4476. The F-statistic is 76.43 on 3 and 283 degrees of freedom, where the associated probability of rejection of H_0 is 0.

Summary statistics on the residuals ϵ_{w_t} are in Table 9.

The lag 1 autocorrelation of the residuals is -0.08371177, where for lag 2 is 0.1193589. Examination of the residuals indicated that we should consider fitting them as an autoregressive process of order 14. However, when comparing the residuals ϵ_{w_t} to the residuals remaining after the the AR(14)

	Values	Std. Error	t value	Pr(> |t value|)
Intercept	0.0034	0.0015	2.2578	0.0247
u_t	0.0038	0.0006	6.2329	0.0000
θ_t	-0.0037	0.0009	-4.3464	0.0000
w_{t-1}	0.4349	0.0524	8.2955	0.0000

Table 8: Regression on w_t

Min.	-0.01023
1st Qu.	-0.001565
Median	-0.0000857
Mean	1.36 10¯20
3rd Qu.	0.001626
Max.	0.01169
Std. Dev.	0.002588018

Table 9: Summary statistics on ϵ_{w_t}

process is removed, showed little or no improvement. See Figure 7. Because of the insignificant contribution of the autoregressive model on the residuals we will not complicate the model with this additional structure. See [6] for their justification of elimination of complicated stuctures when there is not significant difference in the model residuals.

B.2 Dividend Yield

Using the notation of Table 1

$$d_t = \frac{\sum_{i=t-11}^{t} D_i}{12 S_t} \tag{13}$$

for t= 13..300. See Table 10 for basic statistics on d_t. Notice that we calculate a monthly dividend yield by first determining an annual model and divide it by 12.

The linear regression for equation 3 is contained in Table 11.

The residual standard error on d_t is 0.0001551 on 284 degrees of freedom. The multiple R-squared is 0.9596. The F-statistic is 3375 on 2 and 284 degrees of freedom, where the associated probability of rejection of H_0 is 0.

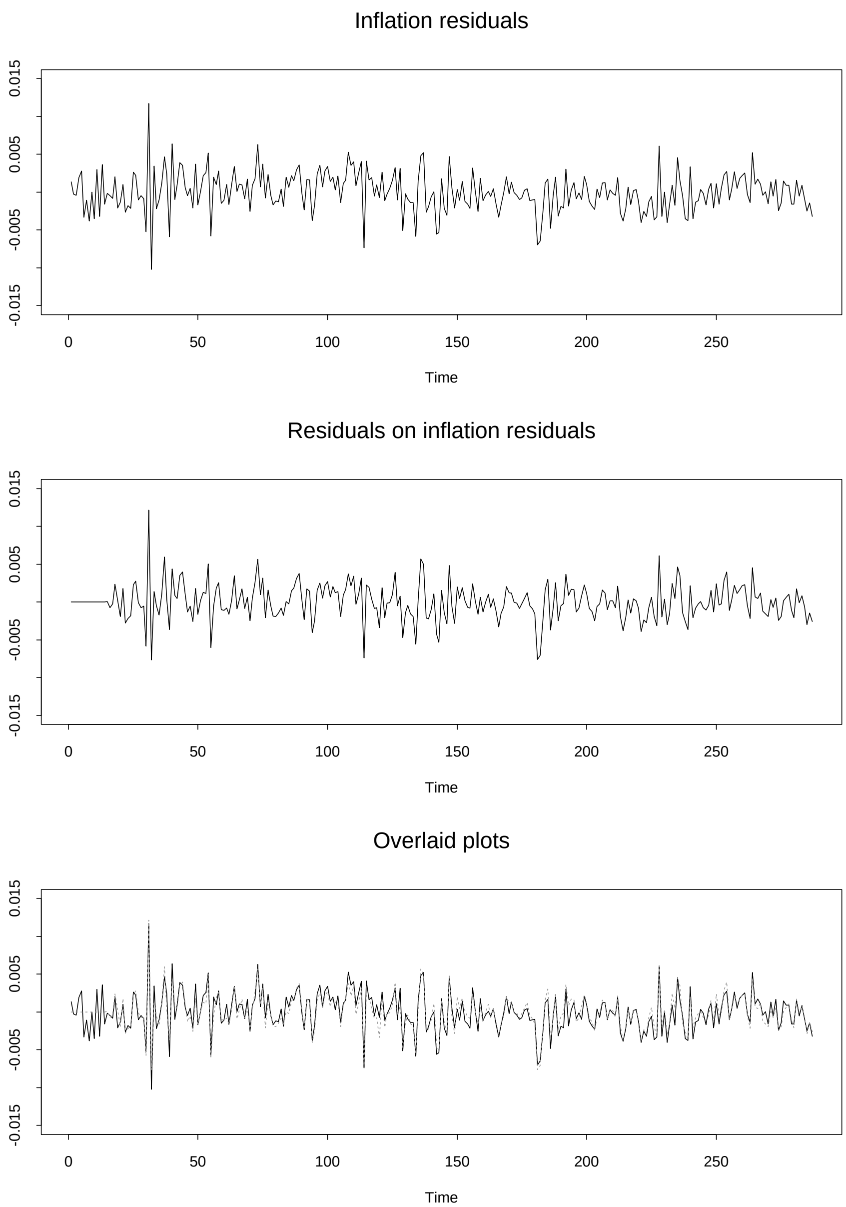

Figure 7: Inflation residuals

Min.	0.002178
1st Qu.	0.002571
Median	0.003049
Mean	0.003239
3rd Qu.	0.003857
Max.	0.005307
Std. Dev.	0.000768518

Table 10: Summary statistics on d_t

	Values	Std. Error	t value	Pr(> |t value|)
Intercept	0.0006	0.0001	4.4800	0.0000
u_t	0.0001	0.0000	4.1787	0.0000
d_{t-1}	0.9312	0.0166	55.9433	0.0000

Table 11: Regression of d_t

Summary statistics on the residuals ϵ_{d_t} are in Table 12.

Min.	-0.0005847
1st Qu.	-0.00009191
Median	-7.372 10^-6
Mean	-3.376 10^-21
3rd Qu.	0.00007321
Max.	0.0006458
Std. Dev.	0.0001545687

Table 12: Summary statistics on ϵ_{d_t}

The lag 1 autocorrelation of the residuals is 0.01351288, where for lag 2 is -0.01995331. Examination of the residuals indicated that we do not have an AR process. See 8 for a graph of these residuals.

B.3 Total Return

Using the notation of Table 1

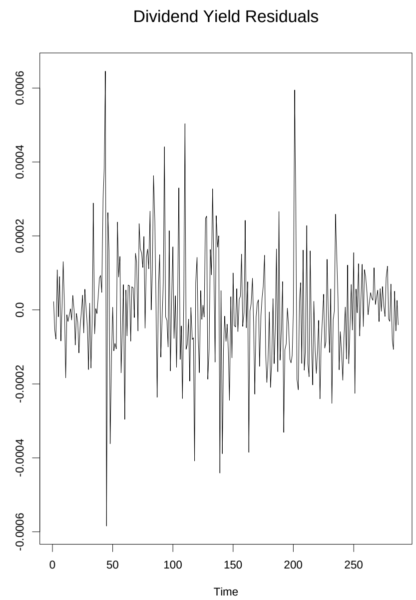

Figure 8: Dividend Yield residuals

$$x_t = \exp(\Delta \ln(S_t)) + d_t - 1 \tag{14}$$

for t= 12..300. See Table 13 for basic statistics on x_t.

Min.	-0.2147
1st Qu.	-0.01581
Median	0.009683
Mean	0.00982
3rd Qu.	0.03783
Max.	0.1671
Std. Dev.	0.04422
Geo. Ret.	0.008843

Table 13: Summary statistics on x_t

We find that there is no significant relation of total return to that of u_t or θ_t. We define the residuals as $\epsilon_{x_t} = x_t - \overline{x_t}$. See 9 for a graph of the residuals.

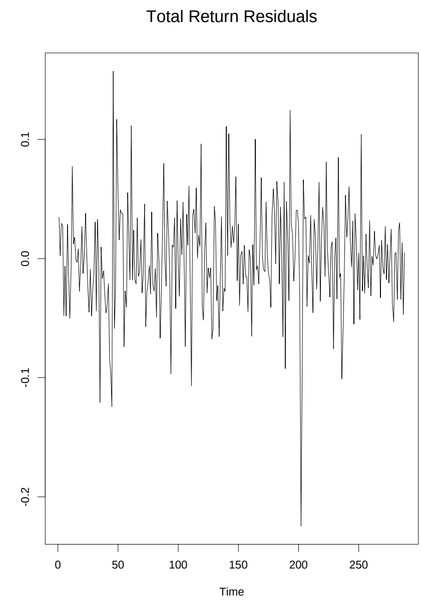

Figure 9: Total Return residuals

B.4 Price Return

Using the notation of Table 1

$$y_t = \exp(\Delta \ln(S_t)) - 1 \tag{15}$$

for t= 12..300. See Table 14 for basic statistics on y_t.

Min.	-0.2176
1st Qu.	-0.01895
Median	0.00654
Mean	0.006581
3rd Qu.	0.03516
Max.	0.163
Std. Dev.	0.04428602
Geo. Ret.	0.005598

Table 14: Summary statistics on y_t

We only need the above information for Generalized Method of Moments targets, so there will not be any regression studies on y_t.

References

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