Portfolio Selection and "Floppy Triangles" Optimization

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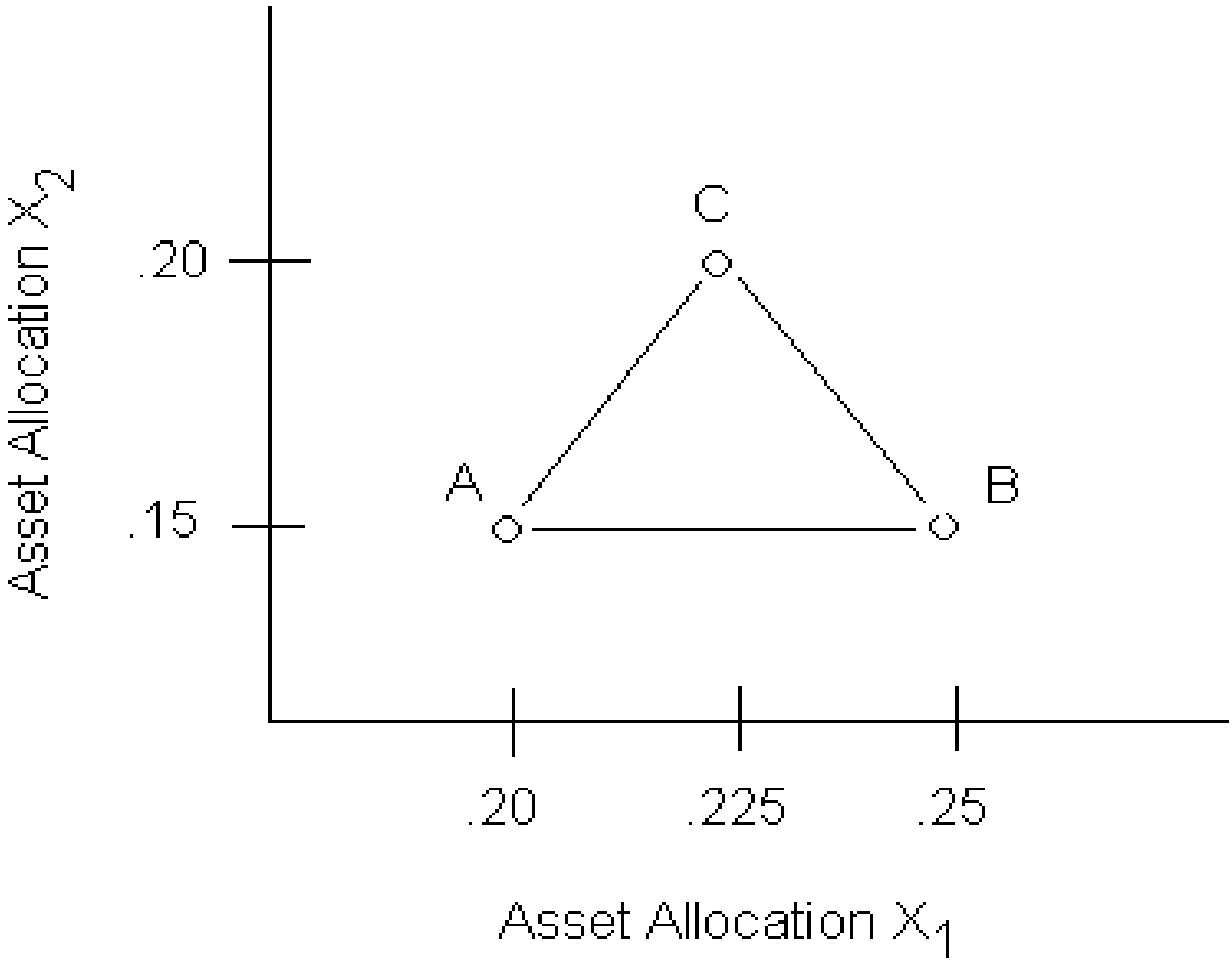
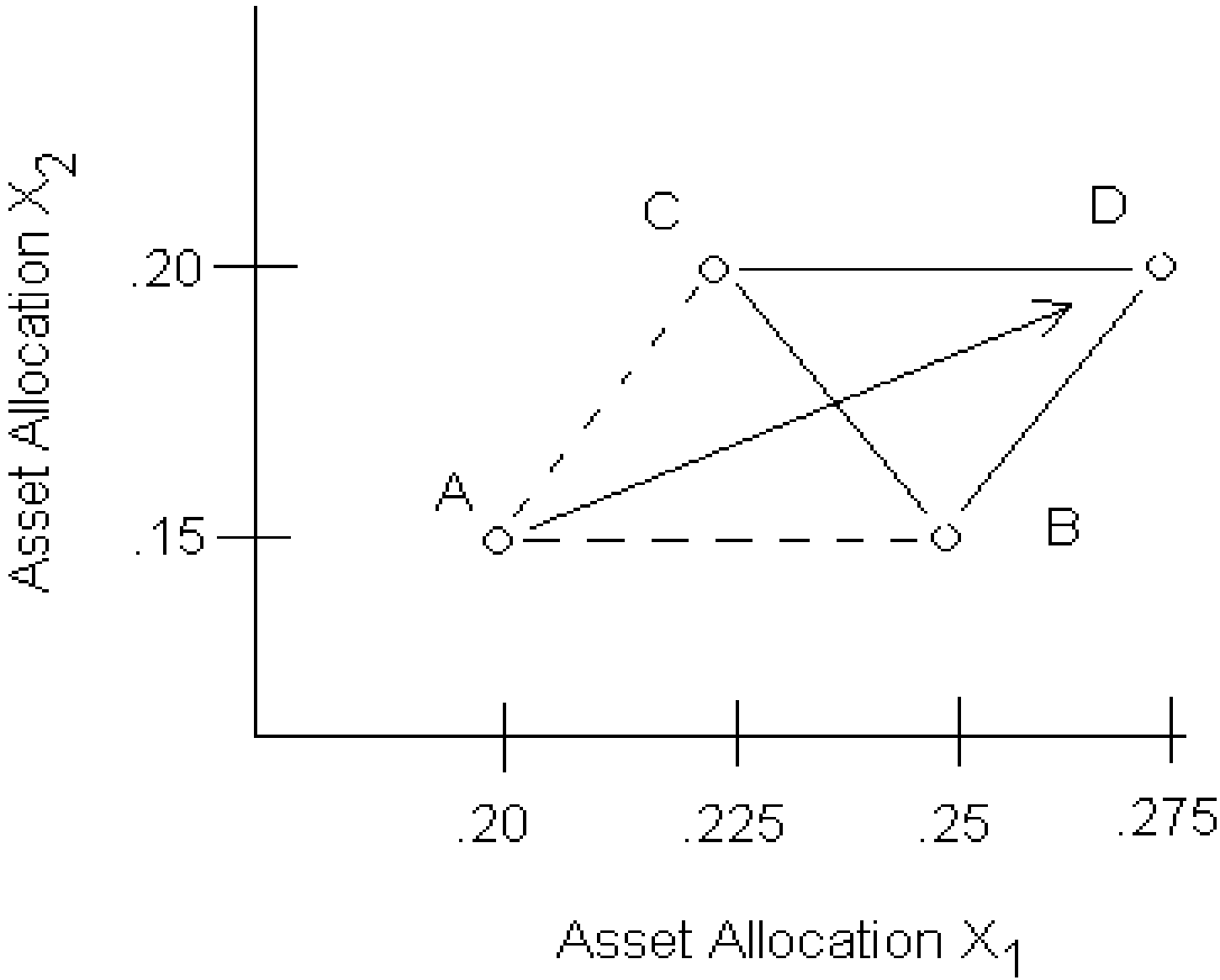
Optimization Problems

- The corporate model software have long runtimes, and hence an experiment is very costly.
- Most complex search algorithms require many different experiments.
- Most simple search algorithms only influence one control variable at a time.
- If the researcher runs out of time, an optimal answer may have not been obtained.

“Floppy Triangles”

- Used by chemical engineers in the 1960’s to design high yielding chemical processes.
- Consists of defining initial experiments
- Conducting these initial experiments.
- Using the algorithm to design the next set of experiments.


Asset Allocation X_2
.20
.15
C
A
B
.20
.225
.25
Asset Allocation X_1


Asset Allocation X_2
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C
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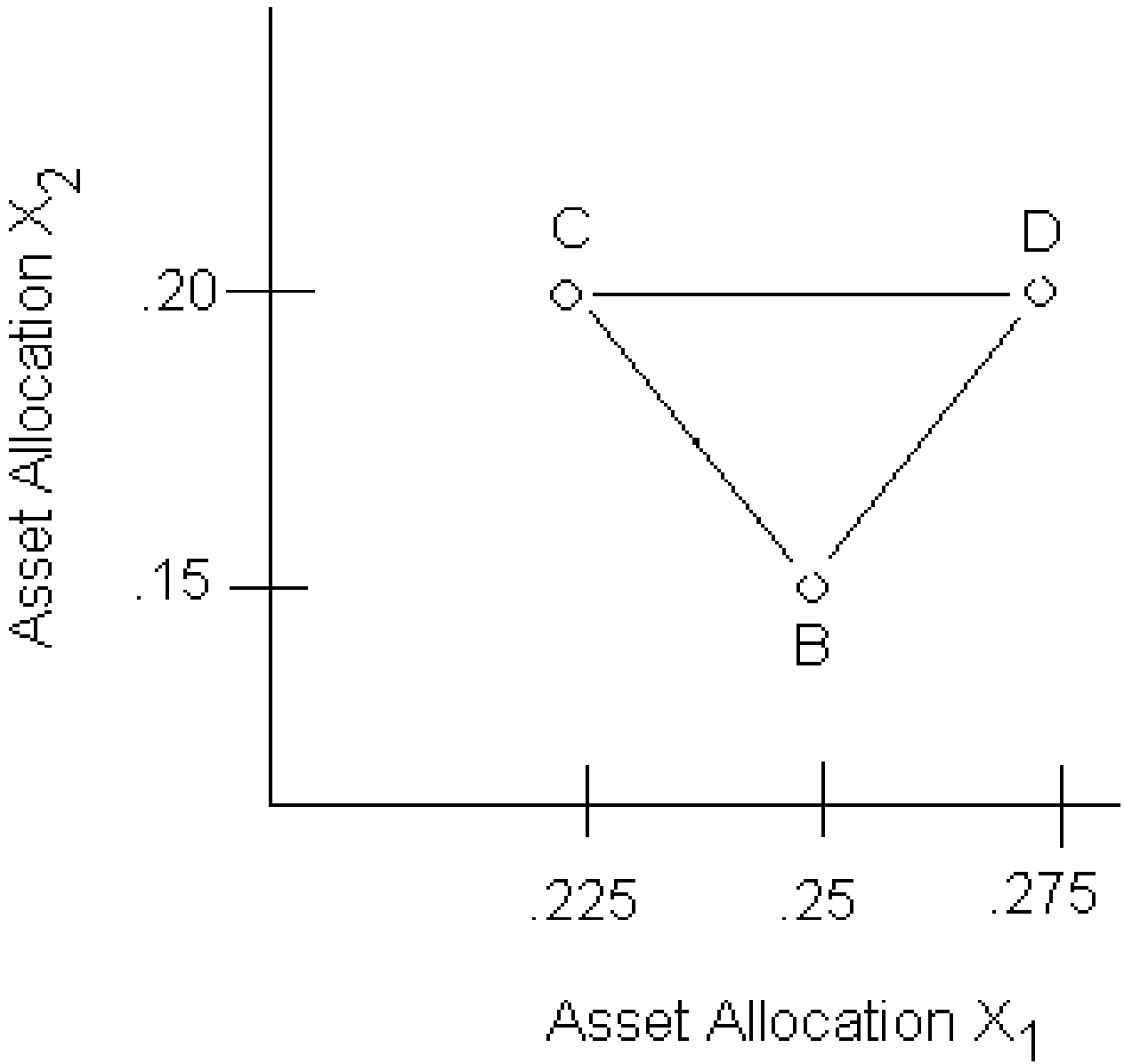
Asset Allocation X_1


Asset Allocation X_2
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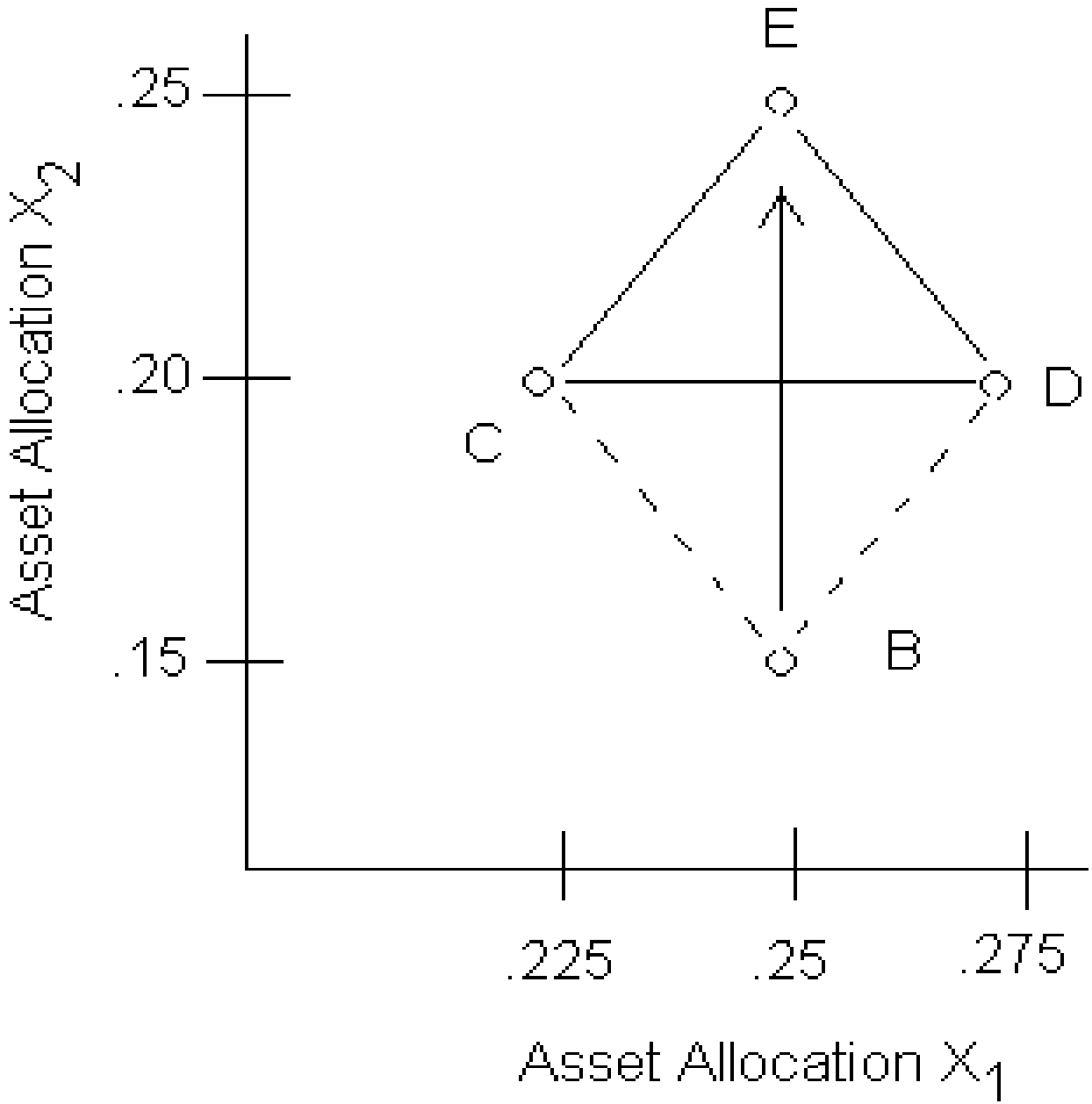
Asset Allocation X_1


E
.25
.20
.15
C
D
B
Asset Allocation X_2
.225
.25
.275
Asset Allocation X_1

Asset Portfolios

- Designing and future maintenance of an asset portfolio backing a new line of business is critical to proper AL/M for that line of business.

Considerations

- Economic Scenarios
- Asset Universe
- Liability Models
- Profit Measures
- Asset strategies
- Optimization

Economic Scenarios

- Historical Models
- Risk premium issues

Asset Universe

- Types
- Quality
- Defaults
- Models

Liability Models

- An accurate model of the line of business is assumed.

Profit Measures

There must be some type of consideration of trade off between return and risk.

- Utility
- Global Roe and Lower Second Partial Moments (or partial standard deviation)
  Parstd =(($\sum$min(x-ave(x),0)^2)/(n-1))$^{1/2}$

Asset Strategies

- Initial Investment Strategy
- Renewal Investment Strategy
- Disinvestment Strategy
  - Borrowing
  - Selling off assets

Asset Strategies Continued

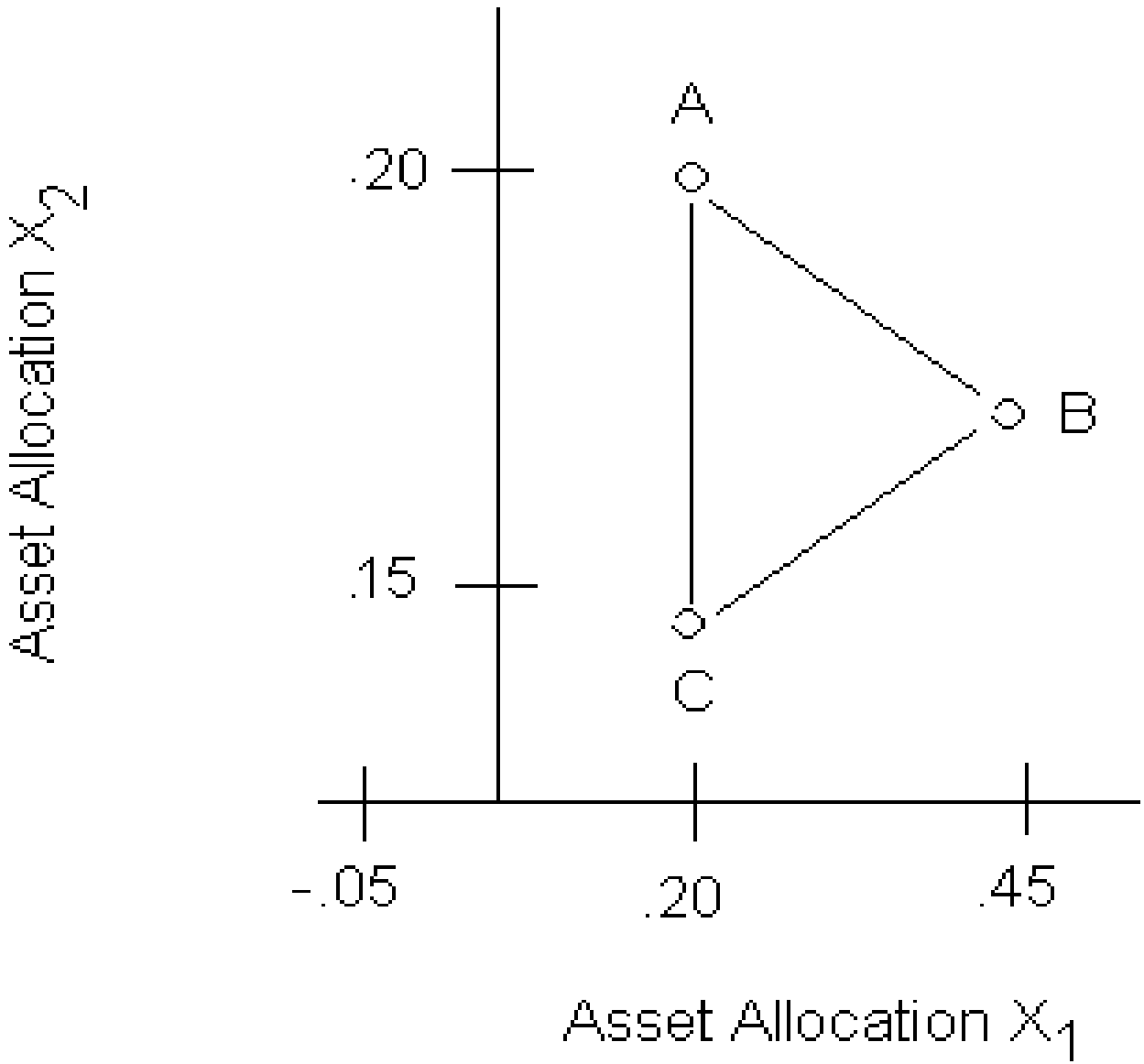
- Static
  - Static mix of assets that maximize the profit measures for all scenarios processed.
- Dynamic or Interactive
  - Determination of optimal asset mixes in different economic scenarios.

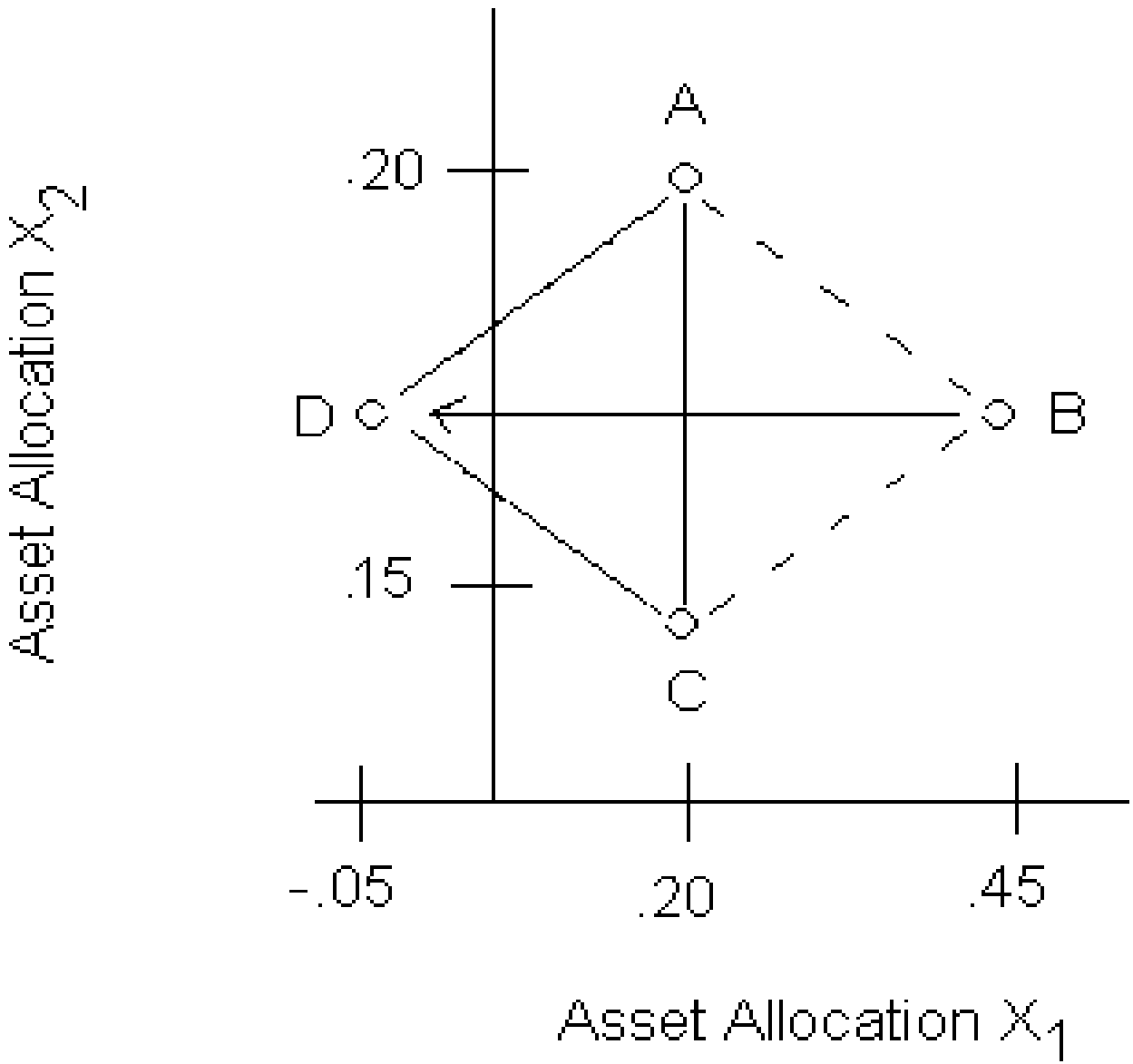
Optimization

- Based on the given profit measure(s), you can obtain an optimal asset strategy by using various search algorithms.

Using the standard method

- The standard FT can create negative asset allocations in the portfolio allocations.

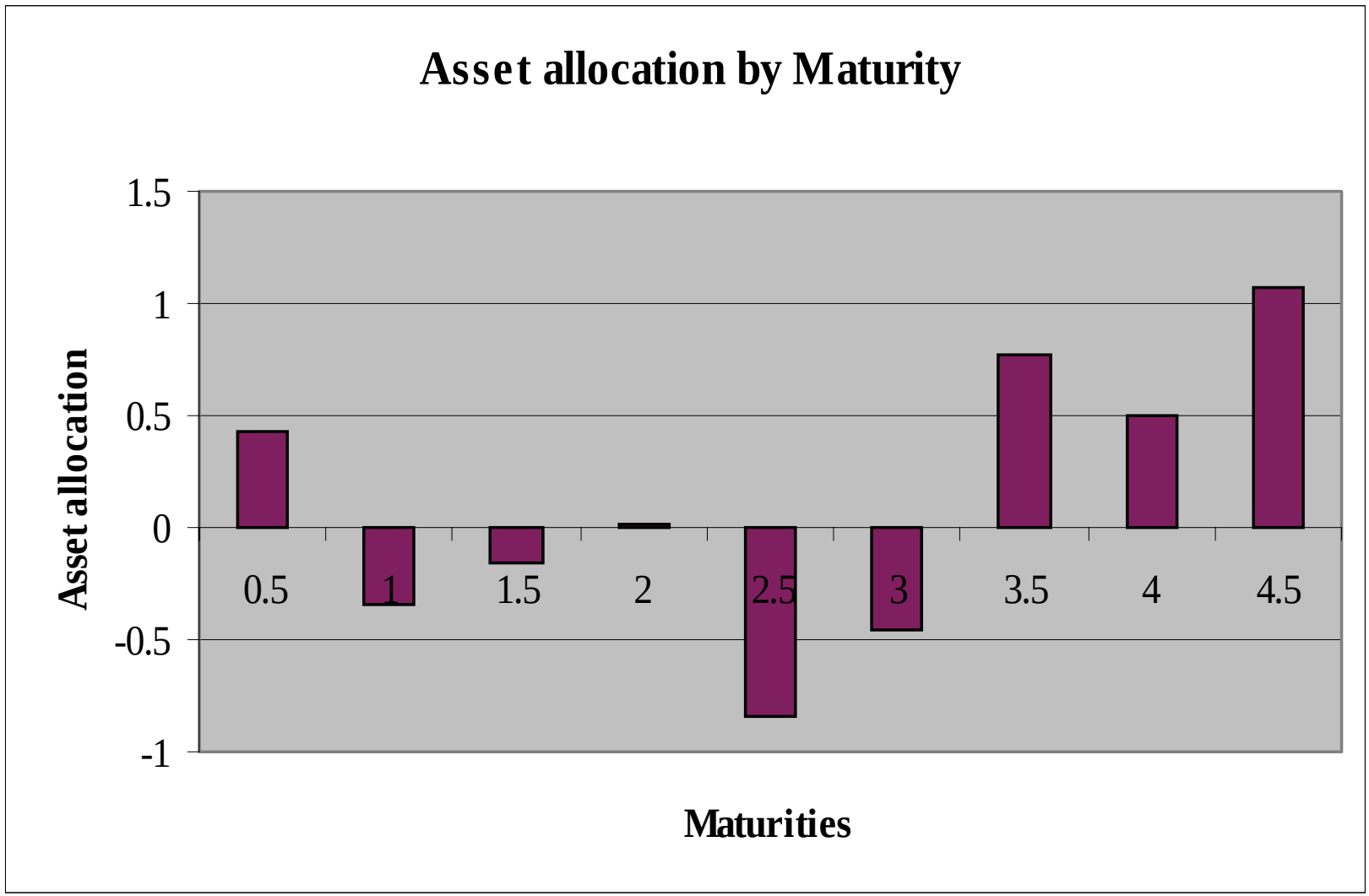

A
.20
B
.15
C
-.05
.20
.45
Asset Allocation X_2
Asset Allocation X_1


A
.20
D
C
B
.15
C
-.05
.20
.45
Asset Allocation X_2
Asset Allocation X_1

Negative Asset Allocations

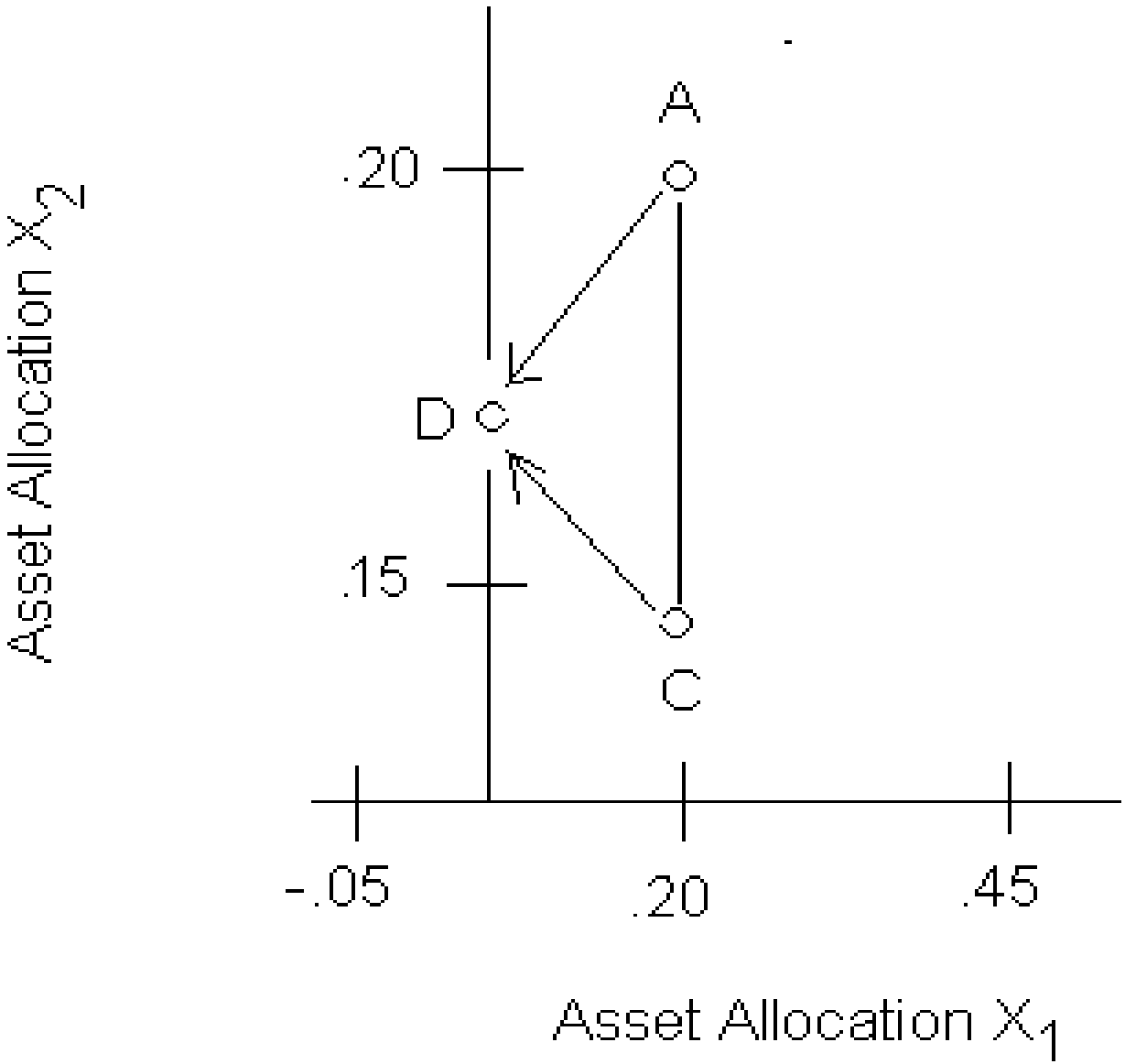
- Maximize economic value.
- Negative allocation may be counterbalancing underlying options in the liability.
- Derivatives can be used to obtain an equivalency to the negative asset allocation.
- If negative asset allocations represent embedded liability options, then an economic price of the options could be calculated.

GIC example


Asset allocation by Maturity
Asset allocation
1.5
1
0.5
0
-0.5
-1
0.5
1
1.5
2
2.5
3
3.5
4
4.5
Maturities

Negative Asset Allocations not allowed

- Statutory requirement do not allow the insurance company to short assets in their portfolios.


A
.20
D
C
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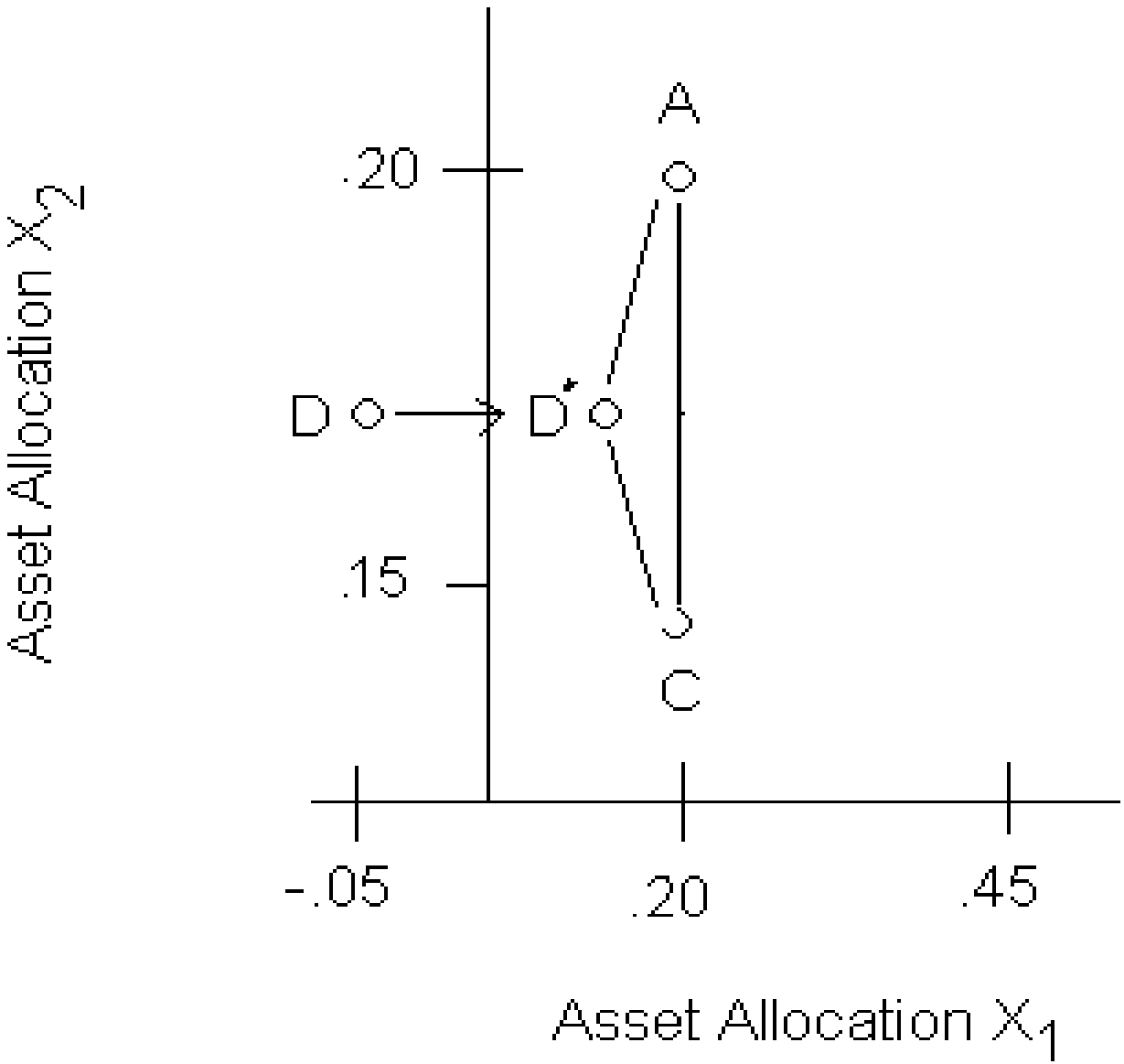
C
-.05
.20
.45
Asset Allocation X_2
Asset Allocation X_1

Simplification of asset universe

If you want to eliminate the asset that gives the negative allocation, you may do so, but you must reallocate the other assets so that you have a 100% allocation. This method does reduce the asset universe and does improve model run times.

Asset universe, Continued

However, if you always want to retain all the assets in the asset universe, but you can’t have negative allocations you should do the following:


A
.20
D
D'
.15
C
-.05
.20
.45
Asset Allocation X_2
Asset Allocation X_1

Negative asset allocations not allowed- Modification 2.

- We use an exponential transformation to exclude the possibility of asset allocations becoming negative.
- Replace the arithmetic mean by a geometric mean.
- Rescale all results to make sure the sum of the asset allocation is equal to 1.

FT's Strengths

- 1. Easy to understand.
- 2. Redesigns the next experiment by modifying all variables.
- 3. Does not require gradient calculation at each new point which reduces the number of experiments.
- 4. If you run out of time, you can stop with the best result that you have obtained so far.

FT's Strengths, Continued

- 5. Per personal communication by Dr. Hongfei Zhang, FT is a form of LDS on the input space with modifications from the output space.
- 6. Experiments are progressive, and designed by the FT process.
- 7. When recommended experiments begin to repeat former experiments, you stop the process.

FT's Weaknesses

- Initial experimental design requires a good representation set. See our paper or

  C. D. Hendrix's paper on this.
- Another possibility is to use a LDS process to design the initial experiments.

FT's Weaknesses, Continued

- No guarantee that you will find the global maximum. This is related to the initial experimental design. However, in ALM an improved answer in a timely manner is preferred to no answer.
- Sometimes multiple experiments are required. Occasionally when most experiments have close optimization target values, FT will require all new experiments except the last. This might increase the required run time to accomplish the next round of experiments.

FT's Weaknesses, Continued

- For any one experiment FT requires a single optimization target. If you have multiple profit measures these must be manipulated until you obtain the single target.

Optimization Targets

- In our example we structured our optimization targets as follows:
- We used the relationship

RS_ROE = (ROE -LBnd)/(UBnd - LBnd)

to adjust the Global Roe to be between 0 and 1.

Optimization Target, Cont.

- We used the relationship

  RS_Parstd=(Parstd- UBnd)/(LBnd - UBnd)

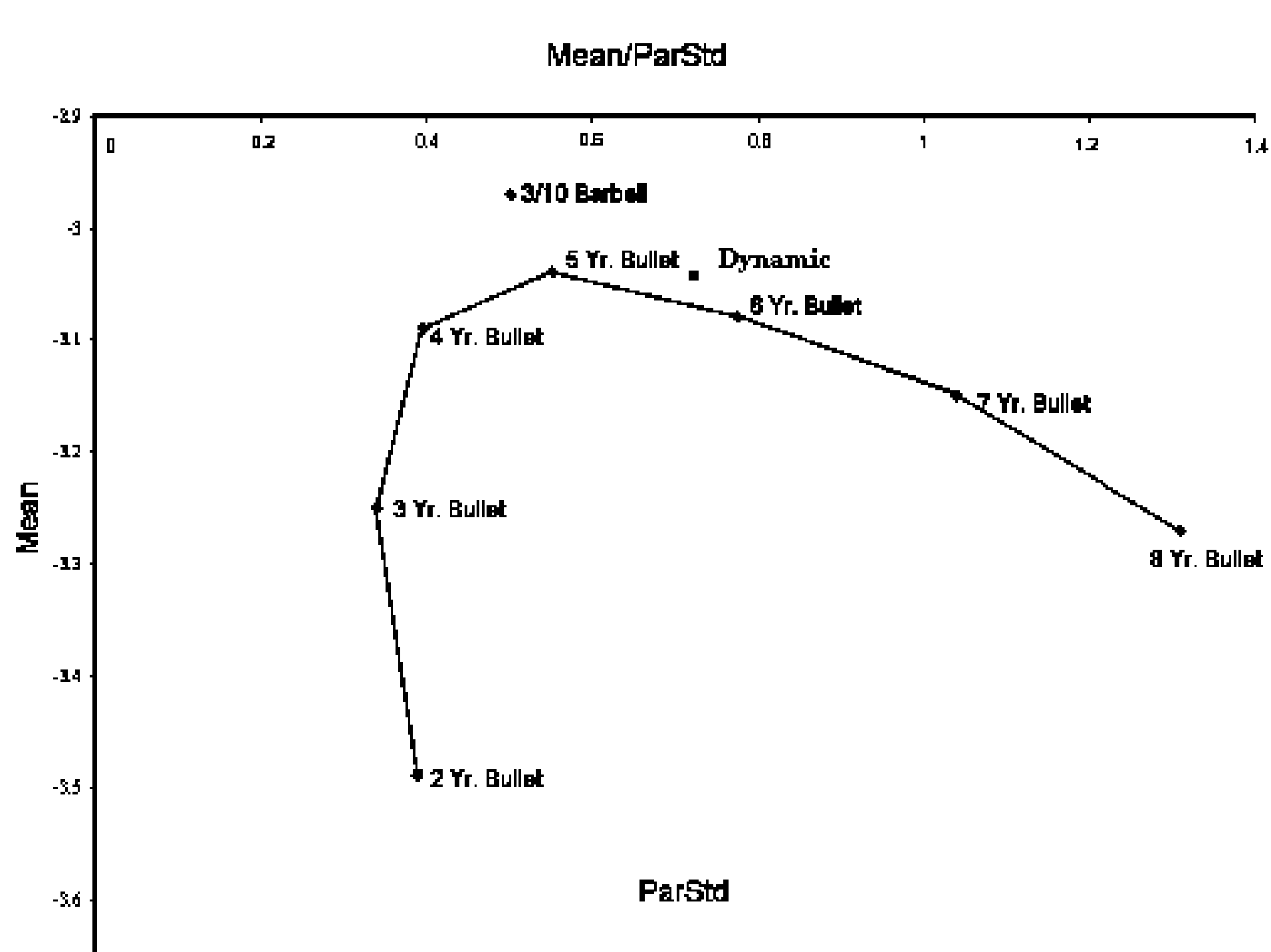
  to adjust the Partial Lower Moment (PARSTD).
- The reversal of the position of the Upper and Lower bounds, in this formula allows for a minimization problem to be converted to a maximization problem.

Optimization Targets Combined

- We placed twice the emphasis on Global ROE using (1+RS_ROE)^2 in the following:

  Y= ((1+RS_ROE)^2)*(1+RS_Parstd))^(1/3)-1

- This rescaled Y will allow it to remain between 0 and 1 and place twice the emphasis on ROE. If this Y is maximized the original ROE is maximized and original PARSTD is minimized.


Mean/ParStd
-2.9
-3
-3.1
-3.2
-3.3
-3.4
-3.5
-3.6
Mean
0
0.2
0.4
0.6
0.8
1
1.2
1.4
ParStd
3/10 Barbell
Dynamic
5 Yr. Bullet
6 Yr. Bullet
4 Yr. Bullet
7 Yr. Bullet
3 Yr. Bullet
8 Yr. Bullet
2 Yr. Bullet