

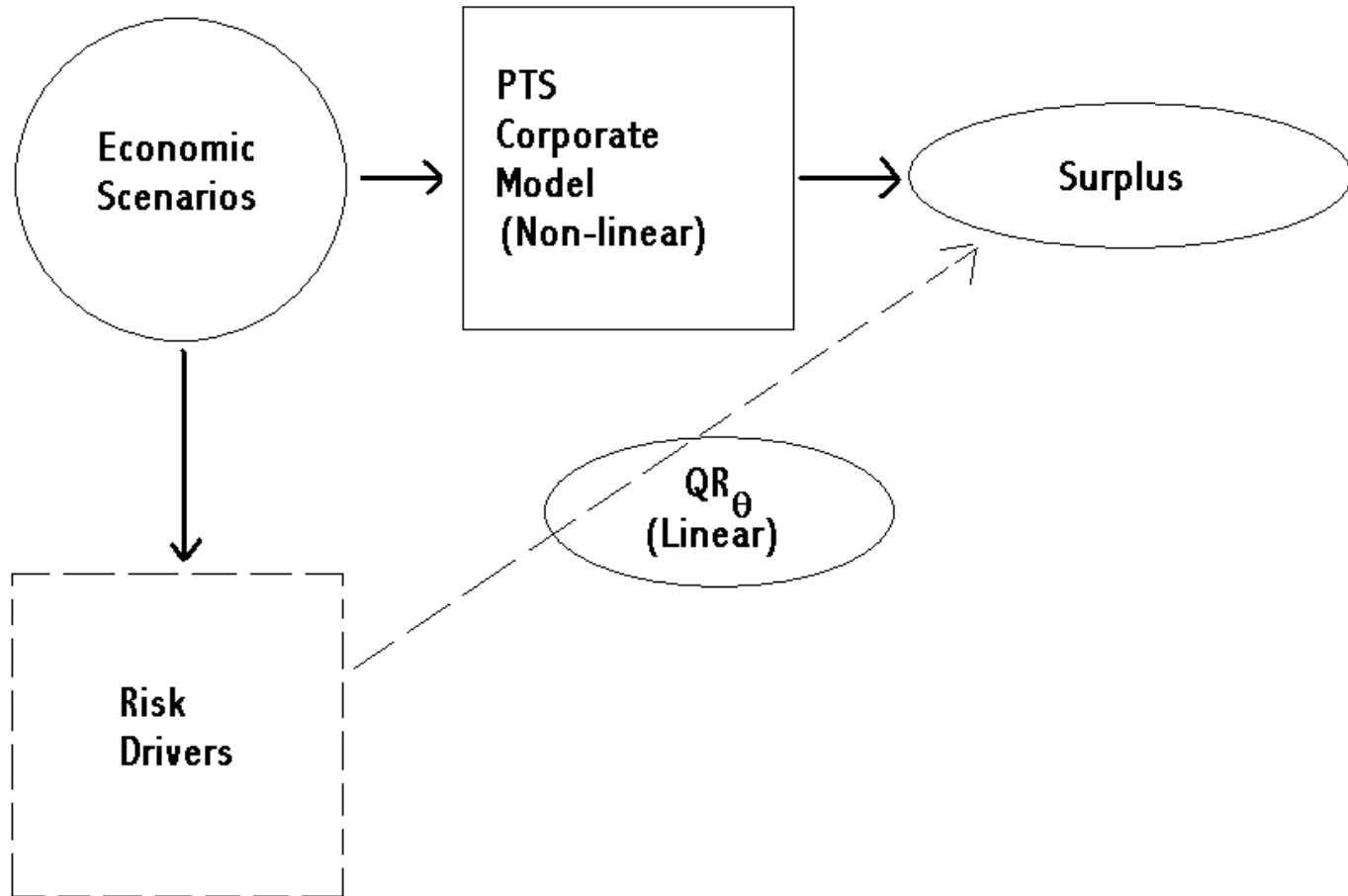
Risk Drivers Revealed: Quantile Regression

Steve Craighead

Don Leggett

Corporate Models

- We have a non-linear computer corporate model which takes economic scenarios as input and produces certain model output, which represents the surplus of the corporate model.
- Next, we define a risk driver to be a function of the economic scenarios through time, which distills the most informative characteristics of the economic scenarios that have an impact on the model output.



Location, Location, Location

- The internal company standards and the external regulatory requirements force the actuary to determine when (and how often) model results fall in “bad” locations.
- This knowledge is very important to understand the potential magnitude of the company’s risk exposure.

Corporate Models

- When using stochastic corporate models most actuaries only qualitatively link the input scenario information to the output results
- They sort the output into increasing order and only examine the economic environment implied by the scenarios generating the worst (or best) n results.

Corporate Models

- Using these n economic scenarios, the actuary makes an attempt to understand the risk drivers that most affect the product or line of business.
- The remaining quantitative analysis then consists of a statistical analysis of the surplus or dividend output only.

Standard Corporate Analysis

Strengths & Weaknesses

- At least there is some linkage between the input and output.
- This linkage is tentative. One only has a relative relationship between specific risk drivers and the output.

Strengths & Weaknesses, Cont.

- It is difficult to determine if some derivative series of the economic scenarios could contribute risk.
 - Examples: How does the spread between the 10 year treasury rate and the 90 day T-bill rate at different times affect the results? The change in spread?
- In the past, one would have to generate specialized economic scenarios and reprocess the model.

Linkage

- There have been few attempts to scientifically link the input scenario information to the output results.
 - Regression
 - Neural Nets
 - Single representative scenario (John Manistre)
 - Proprietary methods-Numerix and LDS generators.
 - Stratified or importance sampling (Lord and Vanderhoof)

Regression

- Linear Regression
 - Creates a model that links the mean of the responses to that of the design matrix
 - The mean is usually not near the extreme location where the actuary wants to study.
 - Minimizes the least squares sum of errors.
 - The results of the regression is very sensitive to outliers.

Quantile Regression

- Introduced in 1978 by Roger Koenker and G. Basset
- Allows one to create a regression emphasizing specific quantiles (i.e. percentiles).
- Minimizes the sum of the absolute value of the errors.

Quantile Regression

- Originally used the simplex algorithm to obtain the solution.
 - Becomes very slow when the number of variables or the number of observations exceed 10,000
- Current methods by Koenker and Portnoy use an interior point method and the high dimensional limitation is altogether eliminated.

Implementation

- Their interior point implementation is in Unix based Splus or in the Oxford C-like matrix language Ox. The algorithm is discussed in “The Gaussian Hare and the Laplacian Tortoise...” in Statistical Science Vol. 12, No. 4. (1997).

Implementation

- The older implementation (using the simplex algorithm) of Quantile Regression is available in Fortran as algorithm AS 229 in Applied Statistics Vol. 36, No 3. (1987) with enhancements in Vol. 43, No 2. (1994).

Quantile Regression Formula

Let the $\{X_i\}$ be the risk driver, where X_i is the value of the risk driver at time i .

QR

- In Quantile Regression, the actual value of the coefficients are not as critical to our understanding, as their relative magnitude is when compared to all of the coefficients.
- We will use the absolute magnitude of the coefficients to locate the year of a specific risk exposure as defined in the design matrix of the regression.

QR Risk Reporting

- Note: the constant coefficient in the RQ formula is not used in the significance formula
 - The constant term can swamp the relative significance formula.

Other Statistics

- R^1 statistic. This corresponds to the R^2 statistic used in standard regression to measure the goodness of fit of the model to the actual data observed.
- Wald Estimator. In standard regression a coefficient is considered significant (different from 0) if the absolute value of the coefficient divided by its standard error exceeds a certain Student t value. The correspondent test in QR is to use the Wald Estimator, and use the chi-square distribution. In the following analysis if the Wald Estimator exceeds 4.00 the S_i is significant.

Other Statistics

- See Koenker article in December 1999 JASA for a further discussion of the above two estimators.
- Discussion of the attached report.

Our Study: Data Source

- We modeled a variable annuity with the PTS actuarial modeling software
- We used 249 economic scenarios
- The model consists of a single block of issues with 10% of the premium allocated to the fixed account and the remaining 90% allocated to the variable account.

Our Study: Data Source

- The equity return for the variable account is based on a blend of equity, bond, and money market returns, and the fixed account is credited the portfolio rate on the underlying assets less a 150 basis point spread. The credited rate is reset annually.
- The business model does not reflect dynamic policyholder behavior, and transfers between accounts are not modeled.

Our Study: Data Source

- The product modeled has a seven-year surrender charge period.
- The surplus held in the business model is 250% of Risk Based Capital (RBC) requirements.
- Output: OAVDE at 9.75% for each scenario.

Risk Drivers Used

- D1-The change in the 90-day Treasury bill rates in the input scenarios.
- D2-The change in the 5-year Treasury bond rates in the input scenarios.
- D3-The spread between the 5-year Treasury bond rates and the 90-day Treasury bill rates.
- D4-The equity returns from the stock index in the input scenarios.
- D5-The change in the 5-year Treasury bond rates from the rolling average 5-year Treasury Bond rates.
- D6-The change between the equity returns and an assumed compound return of 8%.
- D7-The change in the current 12-month rolling equity returns from the 12-month equity returns from the prior year.

Quantiles Covered by the Same QR Regression Formula

Driver	D1	D2	D3	D4	D5	D6	D7
Low	3.75	8.75	5	8.75	5	5	8.75
High	92.5	91.25	93.75	92.5	92.5	91.25	93.75

General Observations

- Note that the linear QR formula is repeated from 1.25% to 8.75% handouts. However, the Wald estimator changes and hence the S_i will differ between various quantiles.
- The identical QR formula for the extreme quantiles is an result of using a simplex solution methodology.

R¹ Ranges for Drivers

Driver	D1	D2	D3	D4	D5	D6	D7
Left	0.68	0.68	0.64	0.94	0.66	0.95	0.69
Minimum	0.26	0.18	0.26	0.86	0.20	0.91	0.22
Right	0.72	0.71	0.68	0.95	0.71	0.98	0.71

QR -- General Observations

- Good and Bad
 - Good: Better fit at the tails . This may be due to the fact that we land in the tail for a select few scenarios. Validates the sorting of results and determining the business behavior
 - Bad: Shows the limitation of not having enough scenarios. Larger scenario runs, produce more accurate linear models.

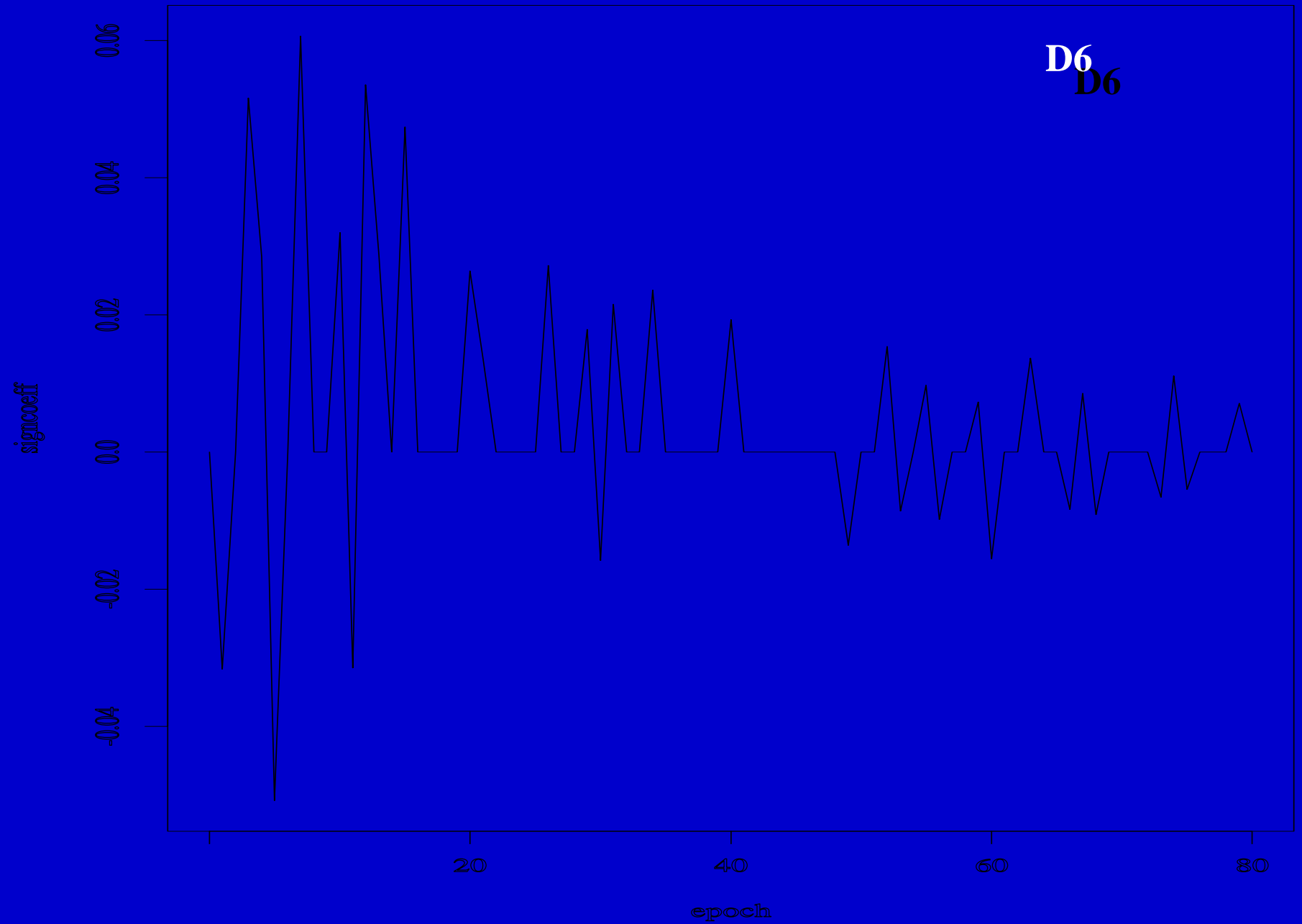
Business Analysis

- Basic statistics on OAVDE values are:
- Minimum \$-2,531,000,
- First Quartile \$260,000
- Median \$685,100
- Mean \$636,600,
- Third Quartile \$1,115,000
- Maximum \$3,990,000
- Standard Deviation \$823,268.

Business Analysis

- 40 of the 249 scenarios result with OAVDE values below zero, which states that the probability of ruin is approximately 16%.

Drift Sensitivity at 1.25%



Business Analysis

Drift Sensitivity

- As expected, D6, the change between the equity return and compound growth of 8%, has the greatest impact on the surplus model, with an R^1 ranging from 0.90 to 0.98.
- 90% of the premium is initially allocated to the variable account
- Above average equity returns imply higher asset fees and thus higher profits
- Below average equity returns imply lower profits
- Positive D6 implies above average equity returns

Business Analysis

- Graphs show D6 is most significant in the first 4 to 5 years of the projection
- Partially due to the fact that the product has a 7-year surrender charge period with relatively low surrender charges in years 6 and 7.
- As the surrender charge decreases, lapses increase and units in force decrease, which means the magnitude of profits or losses are lower in later years in the projection period, lessening their impact on OAVDE.
- Discounting the results at interest also reduces the impact of later projection years on OAVDE.

Business Analysis

- There are some large negative coefficients in the second, sixth and twelfth quarters in the graph for the lowest quantile.
- We believe this is statistical noise.
- We used only 249 scenarios in our model. We have observed in other studies that as the number of scenarios increase the volatility between the coefficients decrease.

Business Analysis

Variable Equity Return

- For D4, the quarterly equity returns, R^1 ranged from 0.85 to 0.95.
- The graphs for D4 appear to exhibit exponential decay, with the most significant coefficients appearing in the first few quarters of the projection.
- This is consistent with our expectation that the results from the first five to seven years of the projection have the greatest impact on OAVDE.

Business Analysis

- These graphs indicate that we are very likely to experience losses in our business model if we have negative equity returns in the first 20 quarters of the projection.
- Consistent with our earlier results, we find our surplus is at risk when we experience declining equity returns.

Business Analysis

Rolling Variable Return

- The R^1 for D7 ranged from 0.22 to 0.71.
- Although our our results are heavily dependent on the equity returns, the R^1 for D7 is lower than the R^1 's for D6 and D4.
- D7 also has more negative coefficients.

Business Analysis

- D7 does not contain as much information as D6.
- If D7 is positive, it means that our return over the last 12 months was higher than the prior year 12-month equity return.
- If the prior year 12-month return was low, the return over the last 12 months could still be below average.
- Even if the current 12-month return is high, we still may have had a low return to date for the scenario, so we may not earn high profits for that scenario.

Business Analysis

- However, if D6 is positive, then we know that we have had an above average return since the beginning of the projection -- it is much more likely that we will turn a profit.
- As a result, D6 shows a better correlation of profits to equity return than D7.
- This shows the importance of testing various drivers in QR to determine the risk drivers of your model, as we cannot be sure in advance which drivers will prove to have a significant impact on OAVDE.

Business Analysis

Delta 5-Year Treasuries

- We expect the D2 in the graphs to have less impact on profitability since our model had only 10% of the premium deposited in the fixed account.
- D2 is a leading indicator for our portfolio return, as we would expect our portfolio return to increase in the future if the 5-year Treasury bond rate is currently increasing.

Business Analysis

- Since the credited rate is set annually in the model, an increasing portfolio return indicates that we will earn extra spread during the next year, whereas a decreasing portfolio rate indicates we will earn less spread than expected during the next year.

Business Analysis

- There is some indication of this in the graph for D2, as it shows more positive coefficients than negative coefficients, which generally means OAVDE is positively affected if we have a positive change in D2.
- However, the magnitude of the contribution for the coefficients stays fairly level throughout the study period.

Business Analysis

- Due to lapses and interest discounting, we would expect the magnitude of the contribution to OAVDE for later years to be smaller than the magnitude of the contribution for early years in the study.
- The fact that the magnitude stays relatively constant throughout the study period may indicate that the D2 risk driver does not have a significant impact on profits, or on our risk of loss.

Business Analysis

- This might change if:
 - our business model included dynamic policyholder behavior, so that our credited rate would have an impact on lapses and transfers between accounts
 - we allocated a higher percentage of the initial premium to the fixed account.

Advantages of QR

- One can examine various drivers, such as:
 - the level of rates
 - spreads between different levels of rates
 - the change in spreads.
- One must only rerun the QR algorithm, which is dramatically faster than rerunning the corporate model.

Advantages of QR

- The QR method reveals what year (or epoch) a specific risk driver has the most influence upon a specific quantile over all scenarios, not just a specific small set.

QR uses

- The pricing actuary can use this approach to determine design flaws when examining low quantiles and positive upside design features in high quantiles.

QR uses

- The valuation actuary can use this type of report to locate various risks and locations of those risks in existing lines of business.
- This also allows the actuary and the financial engineer to determine risk exposure from embedded options in the business.

QR uses

- The financial engineer can also use these methods to improve his or her derivative hedge. This method also examines the effect from all scenarios on the specific R_q .
- Note: The same methodology can be used in risk neutral pricing with standard LS regression, since the risk neutral price is the mean.

Problems

- Sample sizes.
 - The coefficients tend to stabilize from the earliest to latest. The value of the coefficients associated with later years settle down as the number of scenarios increase.
 - Possibly, scenario sizes reduce if the quantile is not too extreme.

