

# CHOQUET AND SIPOS INTEGRAL FOR AGGREGATION OF INTERACTIVE CRITERIA IN SORTING PROBLEM

*Massimo Riccardo Costanzo, Salvatore Greco, Benedetto Matarazzo*

*Dipartimento di Economia e Metodi Quantitativi,  
Facoltà di Economia, Università di Catania,  
Corso Italia, 55, 95129 Catania, Italy.*

## EXTENDED ABSTRACT

We consider the use of integrals defined for non-additive measures (or capacities) as the Choquet and the Sipoš integral in multiple criteria sorting problem.

Sorting problem consists in assigning a set of actions  $A = \{a_1, a_2, \dots, a_p\}$  evaluated with respect to criteria  $g_1, g_2, \dots, g_n$  to one of some categories which are pre-defined and preferentially ordered decision classes (e.g. bad, medium, good). The assignment of an alternative  $a_h$  to a specific category results from a comparison of its evaluation on all criteria with some profiles corresponding to “boundary” actions separating different contiguous classes.

To represent the importance of criteria we use a non-additive measure  $\mu$  and we aggregate the differences with respect to criteria  $g_1, g_2, \dots, g_n$  between the evaluations of each actions to be classified with the considered profile by an integral relative to measure

The non-additivity of measure  $\mu$  is used to represent interaction among considered criteria.

Let  $\mathbf{CI} = \{C_t, t \in T\}$ ,  $T = \{1, \dots, m\}$ , be a set of classes of  $A$ , such that each  $a_h \in A$  belongs to one and only one class  $C_t \in \mathbf{CI}$ . The classes of  $\mathbf{CI}$  are

increasingly ordered, i.e. for  $a_h, a_k \in A$  we have  $a_h \in Cl_r$  and  $a_k \in Cl_s$  with  $r, s \in T$ , such that  $r > s$ , then  $a_h$  is classified better than  $a_k$ . Let also consider the set of profiles  $p_t = [p_{t1}, \dots, p_{tn}]$ ,  $t=2, \dots, m$ , such that  $\forall r, s \in T$  for which  $r > s$  we have  $p_{ri} > p_{si}$ ,  $i=1, \dots, n$ .

We consider the following procedure to assign  $a_h \in A$  to one class  $Cl_t \in Cl$ :

$$a_h \in Cl_t \Leftrightarrow t = \max \{s \in T: F(\Delta_1(a_h, p_s), \dots, \Delta_n(a_h, p_s)) \geq 0\}$$

where  $\Delta_i(a_h, p_s) = g_i(a_h) - p_{si}$ ,  $i=1, \dots, n$  and  $s=2, \dots, m$ , and  $F: \mathbf{R}^n \rightarrow \mathbf{R}$  is a function not decreasing in each argument.

Specifically, we consider the above function  $F$  being an integral relative to a non-additive measure  $\mu$  modelling the importance of criteria at hand. Since the values  $\Delta_i(a_h, p_t)$  can be negative we discuss the extension to negative values of an integral relative to a non-additive measure. Two possibilities are analysed: a symmetric and an asymmetric extension of the Choquet integral. The symmetric extension has been proposed by Sipos and it is defined Sipos integral.

**Key-words** : multicriteria decision making, Choquet integral, Sipos integral, capacity, interactive criteria, sorting problem.

## REFERENCES

- G. Choquet, Theory of capacities, *Ann. L ' Institut Fourier* 5 (1953/54) 131-295
- D. Denneberg, “*Non-Additive Measure and Integral*”. Kluwer Academic Publishers, Dordrecht/Boston/London, 1994.
- M. Grabisch, Ch. Labreuche, “The Sipos integral for the aggregation of interacting bipolar criteria”, IPMU, 2000.
- M. Grabisch, Ch. Labreuche, J.-C. Vansnick, “On the extension of Pseudo-Boolean Functions for the Aggregation of interacting Criteria”, Working Paper, LIP6 2000/032: Rapport de Recherche, 2001.
- S. Greco, B. Matarazzo, R. Slowinski, “The axiomatic approach to multicriteria sorting” in *Atti del Ventiquattresimo Convegno A.M.A.S.E.S.*, Padenghe sul Garda, 6-9 Settembre 2000, 359-366.
- J.-L. Marichal, “Aggregation operators for multicriteria decision aid”, Ph.D. thesis, Institute of Mathematics, University of Liège, Liège Belgium, 1998.

J.-L. Marichal & M. Roubens, "Determination of Weights of Interactive Criteria from a Reference Set", Working notes, University of Liège, Liège Belgium, 1998.

T. Murofushi & M. Sugeno, "Choquet integral models and independence concepts in multiattribute utility theory", *Int. J. Uncertainty, Fuzziness and Knowledge-Based Systems*, **5** 563-585, 1997.

M. Roubens, "Ordinal multiattribute sorting and ordering in the presence of interacting points of view", Working Paper, 2000.

B. Roy & D. Bouyssou, "*Aide multicritère à la décision: Méthodes et Cas*", Economica, 1993.

J. Sipos, "Integral with respect to a pre-measure", *Math. Slovaca*, **29 (2)** 141-155, 1979.

J. Sipos, "Non linear integral", *Math. Slovaca*, **29 (3)** 257-270, 1979.

M. Sugeno, "Theory of fuzzy integrals and its applications", Ph.D. thesis, Tokyo Institute of Technology, Tokyo Japan, 1974.

P.P. Wakker, "*Additive representations of preferences*", Kluwer Academic Publishers, Dordrecht/Boston/London, 1989.

P.P. Wakker, "The Choquet integral versus the Sipos integral, Choquet expected utility versus prospect theory, and concave/convex versus convex measures: three logically independent, but practically related, debates", 22<sup>nd</sup> Linz Seminar on Fuzzy Set Theory, 2001.