Pricing, Modeling and Managing Physical Power Derivatives

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As the Electricity market deregulates, it is becoming increasingly obvious that the use of naive Black-Scholes for physical power options leads to severe pricing and hedging errors. In what follows, we introduce a powerful new framework that follows in Black and Scholes footsteps but is explicitly tied to the nature of physical electricity. This framework allows us to price physical options with substantially increased accuracy and opens the door for pricing retail electricity services.

I. Introduction

The thrust of this paper is to develop a framework for the pricing and management of physical power options. There are three main difficulties in achieving this goal. First, the dynamics of electricity markets are inherently different from most other commodities due to the lack of physical storage and the strict transmission constraints that exist on the system. Second, as the pace of deregulation increases existing market dynamics are bound to change rapidly: making histories of regulated prices all but useless. Finally, even the best set of market dynamics is worthless without a practical strategy to manage any options that are sold under such a model. In the electricity market in particular, careful consideration needs to be given to the best mix of physical and financial assets needed to hedge obligations and appropriately control any associated risks.

II. Goals

I. Develop a framework for the pricing of physical power options.

   a. The framework must be financially sound, and should use generally accepted financial methods as much as possible given the special nature of electricity.

   b. The framework must not be too heavily conditioned on historical data, since we expect the market dynamics to evolve with deregulation.

   c. The framework must reflect the physical nature of power, including the behavior of the generation stack, and the ability to model plant outages.

II. Develop hedging strategies and systems to control the risks associated with physical options.

   a. Examine and describe what risks are even theoretically hedgeable under the developed framework.

   b. Survey available market contracts and liquidity available to lay off embedded risks.

   c. Determine residual risk, and discuss additional contract terms that might be used to control such events.

III. Market

In analyzing the structure of the physical power market we chose as our starting set of data the FERC filings for both load and system lambda (the marginal cost of generation in $/MWH) from Jan 1, 1993 - Dec 31, 1996. There were two primary reasons for using this data set as a starting point:

I. This is the only real data we have. Historically prices have been regulated and the small amount of spot electricity that was traded does not give an indicative benchmark of market prices.

II. With the deregulation of electricity, spot prices are likely to become increasingly volatile as larger amounts are traded on a more active basis. Two things will change only slowly however:

   a. The supply of available electricity (the fixed capital base of generation and transmission).

   b. The demand for electricity (early pilot studies indicate that most users demand is relatively inelastic and will respond only slowly to consistent price pressures).

Therefore, we feel that by making a careful study for the dynamics of the supply and demand in this market we can develop a framework that will be able to adapt well to changing market conditions.

IV. Approach

In order to capture the dynamics of the physical power market, and yet retain compliance with widely accepted financial techniques, we have chosen as our starting point the dynamics for the HJM extension of Black's model for options on futures:

$$\frac{dF(t,T)}{F(t,T)} = b(t,T)dW(t)$$

where

$F(t,T)$ = Futures price at time t for future expiring at T.

$b(t,T)$ = Volatility at time t of F(t, T)

$W(t)$ = Standard Brownian Motion

In examining the system lambda data, some fundamental observations become quickly apparent.

(See Appendix A)

1. Current hour prices are strongly conditioned on the price in the previous hour.

2. Prices have a strong tendency towards mean reversion. Even if lambda starts out high, it has a marked tendency to revert to a more normal level very quickly as the supply shortage passes and the system operators struggle to bring the system back in balance and shut off expensive peaking units as soon as possible.

3. As the load rises on the system, resources available to meet that load become fewer in number so that the very same event in terms of an outage or a surge in demand has a correspondingly greater impact at a higher system load level than it would at a lower system load level. In financial terms this can be described by saying that as prices rise the level of volatility will also rise. Furthermore, this also makes sense from a physical point of view when we examine a graph of the marginal cost of generation as plotted versus system load. As the system load increases, the marginal cost of generation increases slowly at first, but then jumps sharply as short term gas fired or peaking units need to be employed and the system becomes increasingly constrained in terms of power availability.

4. The graph of system lambda is characterized by patterns which follow regular diurnal cycles but have occasional sharp peaks where lambda surges well beyond these bands. The first possible cause for such jumps would be a sharp change in the supply, as produced by an unscheduled plant or transmission outage: we will address this further in what follows. The second possible cause is a jump in demand: this we must also capture.

To apply these properties in our model for electricity, we generalize the Black-Scholes framework by adding dependence of volatility on price level and the addition of jumps (see See R. Merton, 1990, "Continuous Time Finance," Blackwell Publishers, p. 313 for a discussion of general jump diffusion processes). Next, we model the jumps as Poisson processes since this is the classical distribution used in the literature to model failure events. Finally, we make all jumps of limited duration to reflect their transient nature in the power system. This leads to the generalized dynamic:

$$\frac{dF(t,T)}{F(t,T)} = b(t,T,F)dW(t) + \Sigma dq_i \tag{1}$$

where

$F(t,T)$ = Price at time t for future delivery of power at time T.

$b(t,T,F)$ = Volatility at time t of F(t, T), conditioned on price level F(t, T), and conditional on no arrivals of "jump" information.

dq_i = Independent Poisson processes, i, representing jumps in system supply or demand.

l_i = mean number of jumps per unit time for jump process i.

k_i = $E[Y_i - 1]$ where $Y_i - 1$ is the random variable percentage change in the futures price if the Poisson event i occurs and E is the expectation operator over the random variable Y_i

d_i = Random variable representing the duration of the jump due to Poisson process i, if a jump occurs

In other words, (1) could be written in the more cumbersome fashion

$$\frac{dF(t,T)}{F(t,T)} = b(t,T,F)dW(t) \qquad \text{if no Poisson events occur}$$

$$\frac{dF(t,T)}{F(t,T)} = b(t,T,F)dW(t) + (Y_i - 1) \qquad \text{if one Poisson event, i, occurs}$$

$$\frac{dF(t,T)}{F(t,T)} = b(t,T,F)dW(t) + (Y_i - 1) + (Y_j - 1) \qquad \text{if two Poisson events, i and j, occur}$$

… etc.

This equation can be converted to the risk neutral measure and discretized using the standard first order Euler method as

$$\frac{F(t+\Delta t,T)}{F(t,T)} = \exp[(-b^2/2)\Delta t + b(t,T,F)e\sqrt{t}]Y(n) \qquad (2)$$

where

ε = N(0, 1) iid r.v.

Δt = Simulation time step

Y(n) = 1 if n = 0 , Y(n) = $\prod_{\substack{j=1,n \\ i=1,n}} Y(i, j)$ for n >= 1 where the Y(i, j) are iid independent Poisson processes distributed with parameter l_i *t with random duration d(i, j) in the event that a jump occurs. Specifically, if a jump of magnitude Y(i, j) occurs at time t of duration d, then at time t+d we require a jump of magnitude 1/Y(i,j) to represent the end of the event.

V. Benefits and Features of the Model

1. Extension of Black-Scholes process as applied to electricity markets and is familiar to traders and auditors.

2. Volatility dependence on price level . b(. , ., F), reflects real world constraints of load stack & observed data where price volatility tends to increase as we move up the stack.

3. Volatility dependence on forward maturity date, b(t, T, .) allows us to capture term structure of volatility and ensures that mean-reversion takes place through the decay of the volatility curve (Samuelson's hypothesis). This is allows us to reflect the long term mean reversion properties of the system discussed earlier.

4. Jump process provides for explicit modeling of outage events. This is more realistic from a physical point of view since plant outages happen in discrete steps and thus gives us a more accurate dynamic. In addition, having a separate process for outages helps us to price physical options where the payoff is tied to an outage event.

VI. Estimation of Parameters

Because of the many interacting pieces, estimation of these parameters is somewhat complex.

Volatility:

To estimate the term structure of the volatility, b, we proceed in two stages. First, the dependency of b on price, F, is estimated by comparing the percentage change in volatility versus price level. This gives us a term structure for the volatility smile b(. , ., F) that is independent of the absolute level of volatility. Next, b(t, T) is fitted to the observed market implied volatilities by examining the traded prices for options on forward contracts and backing out the implied volatility as per the standard method for HJM.

Jumps - Traditional Estimation:

Traditionally, the jump parameters for a Poisson jump-diffusion process are estimated by using moment matching techniques. Following the outline in [Ball & Torus, "On Jumps in Common Stock Prices and Their Impact on Call Option Pricing, *Journal of Finance*, 40(1) - March 1985, p. 155-73] we could proceed as follows. Suppose that $\ln Y \sim N(m,\ d^2)$. Then, since the drift for futures prices is zero, we obtain the density for the return on futures prices as

$$p(x) = \sum_{n=0}^{\infty} \frac{e^{-l}\, l^{n}}{n!} f(x; nm, b^2 + n\, d^2)$$

where

$$f(x; m, v^2) = (2p\ \ v^2)^{-1/2} \exp(-(x-m)^2 / 2v^2)\ .$$

We then obtain sample moments, m_s , from the historical price data via

$$m_s = 1/T \sum_{t=1}^{T} [\Delta Z(t)]^s$$

where $\Delta Z(t)$ is the change in the natural log of the security price during time t, and T is the number of days in the estimation period. Note that because jumps are transient on electricity systems, we must filter the above sample data by excluding down jumps back to a "normal" state that occur after an up jump so as not to double count any jumps that occur. Next, using the sample moments, sample cumulants K_s can be determined e.g.: [Kremer and Roenfeldt, *Warrant Pricing: Jump-Diffusion vs. Black-Scholes*, JFQA 28(2), 255-271]

$$K_1 = m_1$$
$$K_2 = m_2 - m_1^2$$
$$K_3 = m_3 - 3m_1 m_2 + 2m_1^3$$
$$K_4 = m_4 - 4m_3 m_1 - 3m_2^2 + 12 m_2 m_1^2 - 6m_1^4$$

Such estimates are subject to severe numerical roundoff problems, however, and so the preferred estimation method is the more careful scheme outlined in Press, Teukolsky, Vetterling, Flannery, *Numerical Recipes in C 2nd ed.*, Cambridge University Press, p. 612.

From these cumulants, we then solve equations for the parameter estimates as [Press, *A Compound Events Model for Security Prices,* Journal of Business 40 (July 1967), 317-35]

$$m^4 - \frac{K_3}{K_1} m^2 + \frac{3K_4}{2K_1} m - \frac{{K_3}^2}{2{K_1}^2} = 0$$

$$l = \frac{K_1}{m}$$

$$d^2 = \frac{K_3 - m^2\ K_1}{3\,K_1}$$

$$s^2 = K_2 - \frac{K_1}{m}\left(m^2 + \frac{K_3 - m^2\ K_1}{3K_1}\right)$$

In solving for m, care must be taken since there are usually two possible solutions in the interval of interest. Press shows, however, that the equation for m has only two real roots of opposite sign. So, we simply choose the root which gives $l > 0$.

Jumps - Estimation for the Electricity Market:
The key observation that makes this model extremely powerful for the physical power market is that unlike the classical financial case shown above, we can calibrate the jumps to actual physical events on the system. Since we have hour by hour system lambda, simultaneous data for plant outages, and concurrent system load data we can calibrate these events in a rather precise way. Thus, for example, the set of individual plants at a location can be modeled as a log-binomial collection with the impact of each plant's outage on lambda calibrated to its MWH output. This gives a nice asymptotic tie-in to the single jump process specified above since, via the Central Limit Theorem, for a large set of log-binomially distributed jump events the distribution converges to $\ln Y \sim N(m, d^2)$.

VII. Hedging Issues

Even in theory it is impossible to be perfectly hedged versus jumps. The reason for this is that no replicating position can protect you against such a discontinuity and the best that you can hope for is that on average you are able to cover your expected cost for these jumps. In practice, the problem is even worse because one cannot easily rebalance in the physical asset and indeed there are cutoff dates for both pre-schedules and monthly schedules that determine how much power you must block out as your best estimate for these reserves. However, the key finding to note here is that the presence of jumps can lead to significantly different deltas than what we would obtain with a plain vanilla Brownian motion model. In addition, our calibration of events also provides a powerful portfolio balancing tool that lets us know how many times we can oversell our plant to cover a group of options we have sold at any desired level of confidence. This can be a key competitive advantage for the aggressive pricing of options in the deregulating marketplace.

VIII. Customer Behavior

In the financial world, most of the options that have been developed assume exercise for a particular volume. In contrast, the gas and electricity markets allow customers to "swing" in the amount of volume that they take. As an example, we might sell a customer a physical power contract to take 100 MWH/H at COB with the right to take up to 20 additional MWH in any hour for the month of June 1998 at the price of $25/MWH. How much is this option worth? The first questions that arise in this context are as follows:

1. To what degree is the customer exercising the option economically versus they just need the power?

2. More generally, is the customer's exercise behavior correlated with price levels?

3. How elastic is the customer's demand: would higher price levels or a pass through of prices affect their behavior?

4. How will the American style features of the above contract affect its value and optimal exercise time?

In addition, most contracts like the one above have additional American exercise features and boundaries which limit either the daily, weekly or monthly swings that can be exercised. As a first attempt to answer these questions, we note that several studies have been done throughout the US by utilities attempting determine customers' price elasticity. The result of these studies is clear: without strong, consistent and long term pricing incentives that punish expensive customer behavior, there is almost no impact of price on the amount of electricity taken by most customer classes [K.H. Tiedemann, *Time-Of-Use Rates, Demand Charges and Residential Peak Energy Demand*, 1996 EPRI Conference on Innovative Approaches to Electricity Pricing, p. 16-1]. Only the largest customers are really price sensitive and even they have a limited ability to shift their load. A simplifying assumption then, is that customers are inelastic: they do not or cannot exercise these options economically and have a simple correlation of load with price level. What is more, correlating customer takes with price levels helps us to cherry pick the most profitable customers. As our starting point, we take the famous "Quanto" option in finance whose name came from an abbreviation for Option Quantity Unknown. The idea behind a Quanto option, originally developed for foreign stocks, is that the payoff is given by

$$C = \overline{X}\max[S'^{*} - K',0] = \max[S'^{*}\overline{X} - K,0]$$

where

$\overline{X}$ = A fixed foreign currency conversion rate.

S'^{*} = The final price of the foreign stock, in foreign currency

K' = The strike price in foreign currency.

K = The strike price in domestic terms.

For example, a US investor buying a British stock Quanto option with a strike of K' = £20, a fixed conversion rate of $\overline{X}$ = 1.50 (\$/£) and a final stock price of $S'*$ = £28 would receive a payoff of

$$C = \overline{X}\max[S'* - K',0] = 1.5*(28-20)$$

Notice here that the option writer's exposure is determined by two unknowns. The first unknown is the final value of the stock price. The second unknown is how much foreign currency they will need to convert into domestic currency at the fixed exchange rate $\overline{X}$. Hence the "Quantity Unknown" is the amount of foreign currency to convert at option payoff. The analogy for the energy market is now obvious: think of K' as the strike price for the power swing option, $S'*$ as the spot price for power in a particular hour, and the "unknown amount of foreign currency" as the unknown amount of power that the user will take. (Note that we are ignoring the American features of this option for the moment.)

Under the Black-Scholes type framework, we can obtain a closed form formula that gives a price for a single swing option under the assumption that the customer's load is log-normally distributed and correlated with price levels. This leads to the standard Quanto formula for the call price [Eric Reiner, 1992, "Quanto Mechanics," published in "From Black Scholes to Black Holes," Risk Magazine Press (1992), p. 152]

$$C = \overline{X}\left\{ S'\left(\frac{r_d}{r_f}\right)^{-t} \exp(-r_{S'X}\, s_{S'} s_X t) N(x_3) - K' r^{-t} N(x_3 - s_{S'}\sqrt{t}) \right\}$$

where

$$x_3 = \frac{\log(S' d^{-t} / K' r_f^{-t}) - r_{S'X}\, s_{S'} s_X t}{s_{S'}\sqrt{t}} + \frac{s_{S'}\sqrt{t}}{2}$$

The above formula provides a particularly simple model for pricing retail options where the customer swing is driven by price levels. In practice, much more sophisticated models of customer behavior can be developed. We would advocate this formula as a way of calibrating numerical results obtained from the more realistic jump diffusion process and an in house model of customer behavior.

IX. Additional Issues

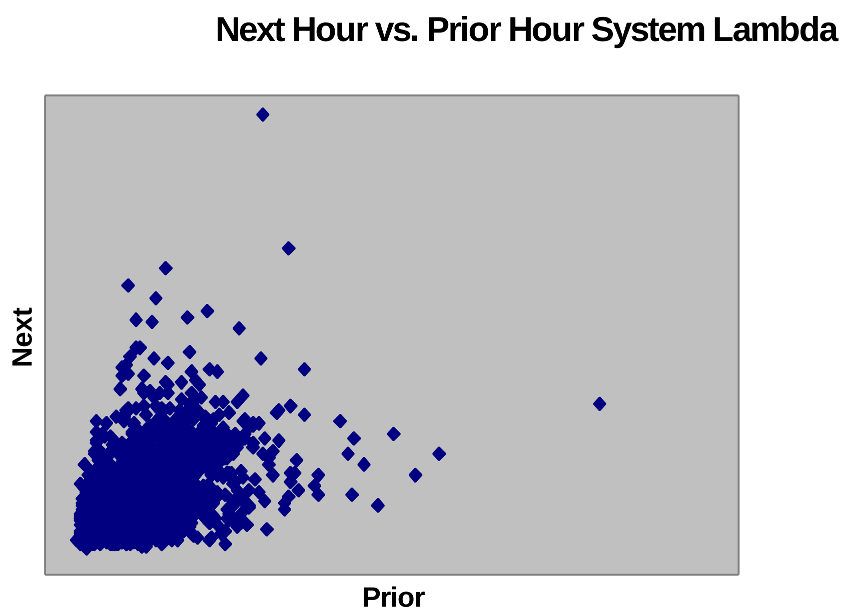
Although the described jump diffusion model is very powerful, significant practical caveats remain. First, how liquid is the physical pool at which the option has been sold? This will dictate to what degree sellers need to have physical assets on hand and how easily they can adjust their positions by purchasing spot power. Second, what are the liabilities and obligations if the seller over/under delivers and how can these be managed from both a risk and reputation point of view? Finally, what kind of financial system is needed to properly track and manage the risks in these types of deals? While vital, such questions tend to be addressed differently within each trading organization and are best dealt with on a case by case basis.

X. Summary and Conclusions

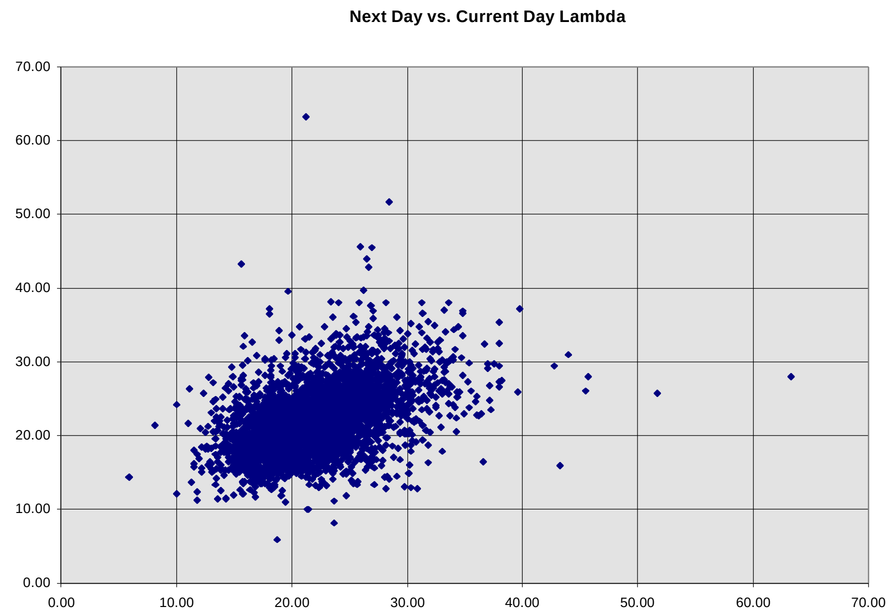
This paper gives an introduction to some of the fundamental considerations in pricing physical power options at the retail level and outlines a simple framework to address these issues. To make the discussion concrete, a particular jump diffusion process was built to capture a number of the special features in the power market. In addition, we showed how properly understanding and modeling these features leads to significant advantages in hedging, trading and pricing both retail and wholesale deals.

Appendix A

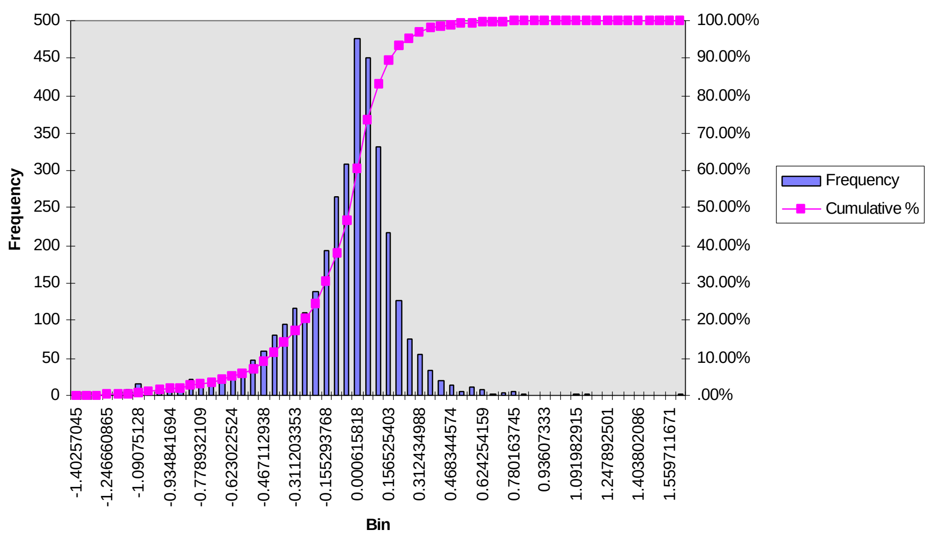
1. Graph of system lambda Current Hour vs. Prior Hour at a typical power pool. Note that as the price in the Prior Hour rises the volatility of the lambda for the next hour also increases.



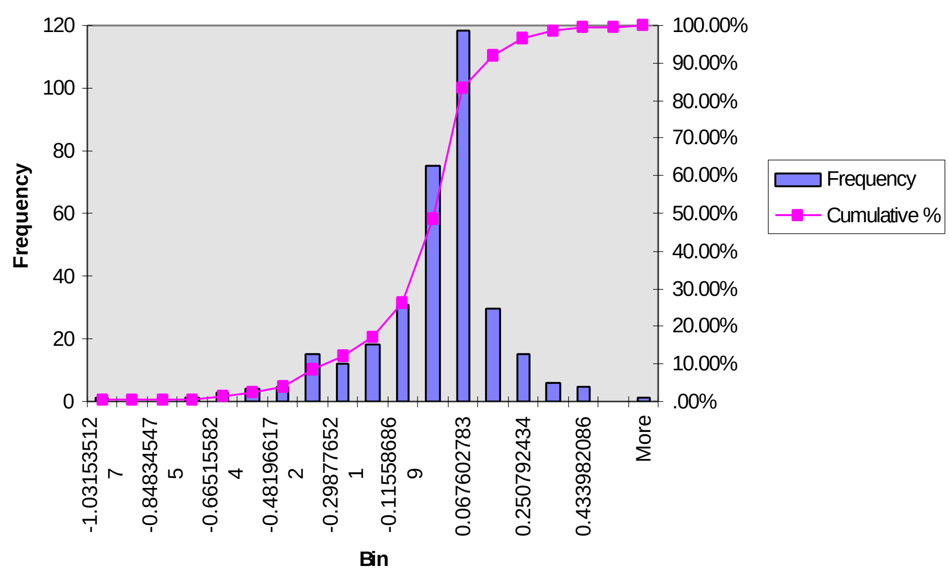
2. Plot of Next Day vs. Prior Day system lambda at the same hour. Note the much weaker correlation, hinting at a reversion effect. This is confirmed by plots of Hour on Hour system lambda for prior hour prices at the 70th percentile and 90th percentile. Note that the 90th percentile has some expected reversion due to sample bias since the percentiles and next/prior hours are all chosen from the same sample data.



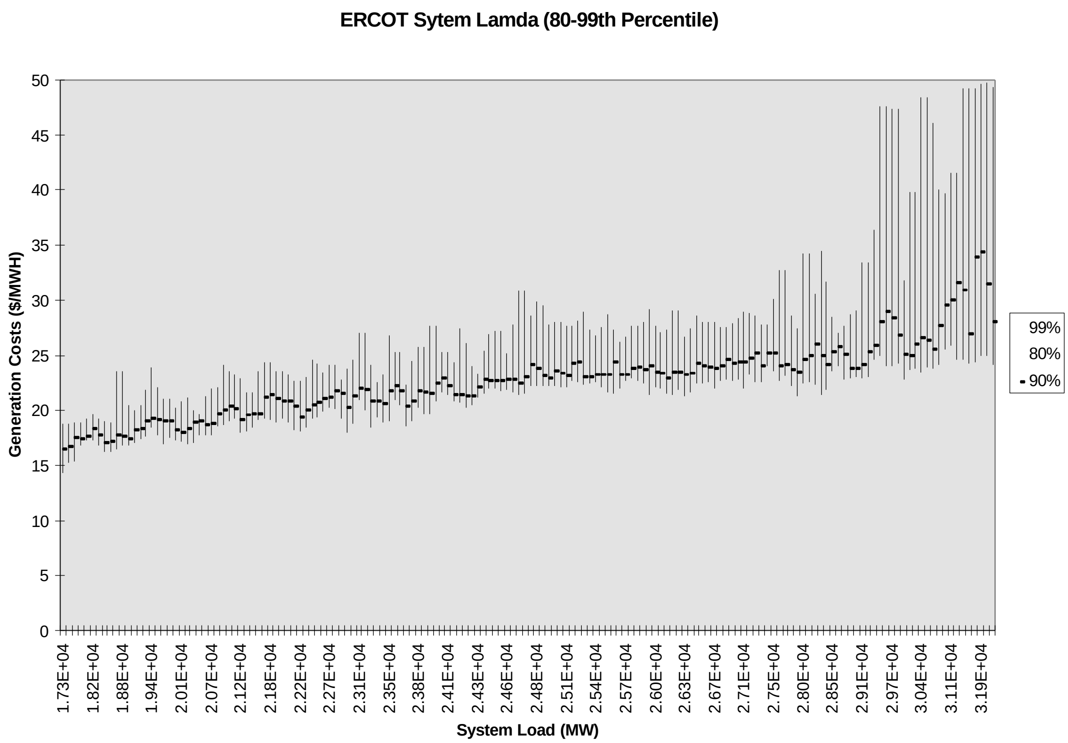
Histogram of Hourly Changes Given Prior Hour at the 70th Percentile

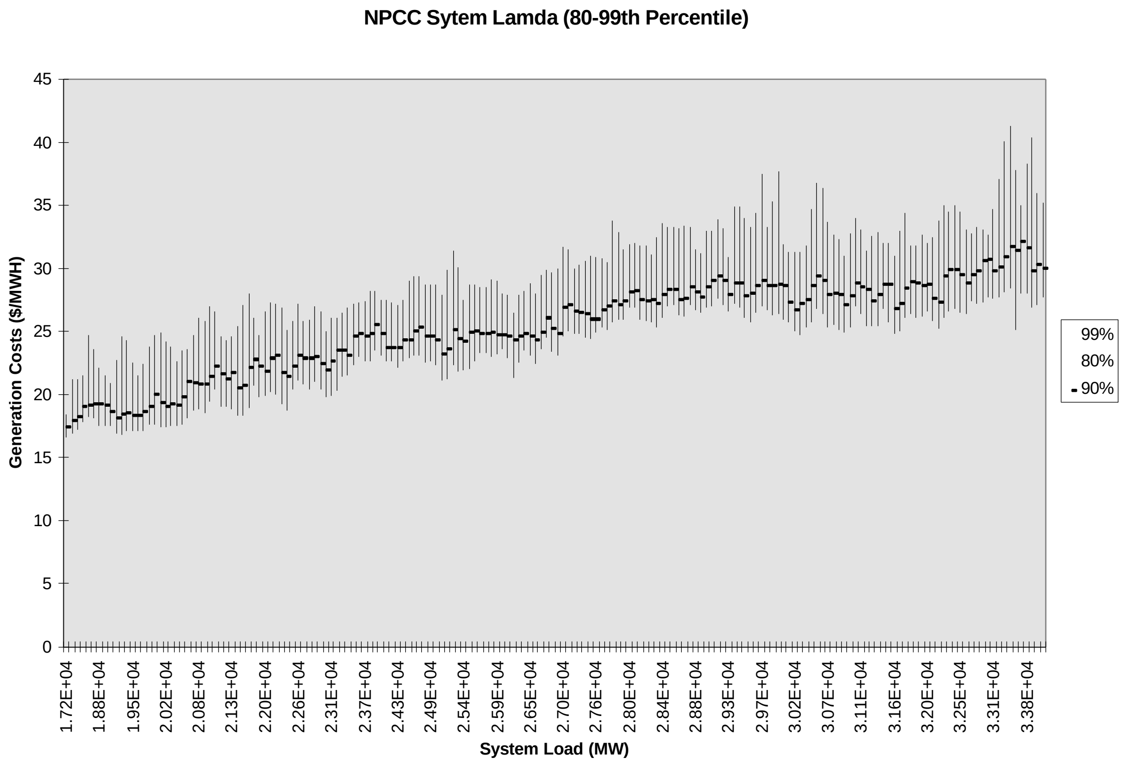


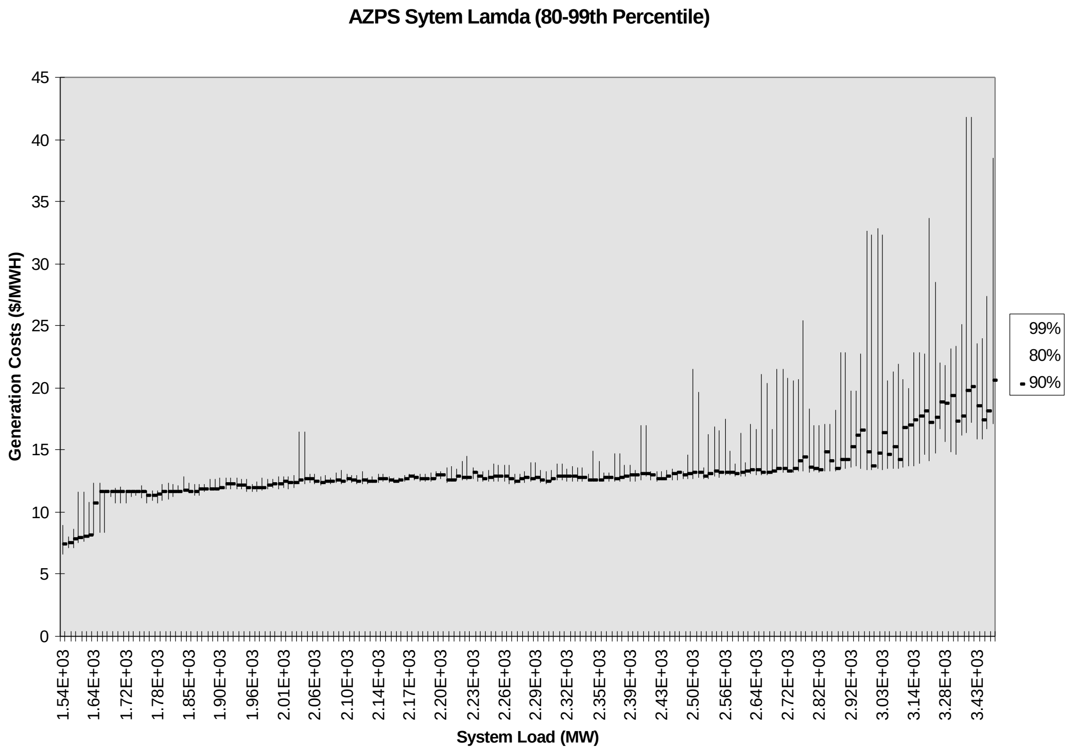
Histogram of Hourly Changes Given Prior Hour at the 90th Percentile



3. Plots of system lambda versus Load.




NPCC Sytem Lamda (80-99th Percentile)
Generation Costs ($/MWH)
45
40
35
30
25
20
15
10
5
0
1.72E+04
1.88E+04
1.95E+04
2.02E+04
2.08E+04
2.13E+04
2.20E+04
2.26E+04
2.31E+04
2.37E+04
2.43E+04
2.49E+04
2.54E+04
2.59E+04
2.65E+04
2.70E+04
2.76E+04
2.80E+04
2.84E+04
2.88E+04
2.93E+04
2.97E+04
3.02E+04
3.07E+04
3.11E+04
3.16E+04
3.20E+04
3.25E+04
3.31E+04
3.38E+04
System Load (MW)
99%
80%
90%



AZPS Sytem Lamda (80-99th Percentile)
Generation Costs ($/MWH)
45
40
35
30
25
20
15
10
5
0
1.54E+03
1.64E+03
1.72E+03
1.78E+03
1.85E+03
1.90E+03
1.96E+03
2.01E+03
2.06E+03
2.10E+03
2.14E+03
2.17E+03
2.20E+03
2.23E+03
2.26E+03
2.29E+03
2.32E+03
2.35E+03
2.39E+03
2.43E+03
2.50E+03
2.56E+03
2.64E+03
2.72E+03
2.82E+03
2.92E+03
3.03E+03
3.14E+03
3.28E+03
3.43E+03
System Load (MW)
99%
80%
90%

4. Plot of typical hourly system lambda.