The Saturation Region

In the Saturation mode , the drain voltage is greater than Vdsat.
In this region , the lateral electric field cannot be neglected ,that is,

$$V(y) = \frac{\lambda^2 E_{sat}}{L-l}[\cosh(P) - 1] + \lambda E_{sat}\sinh(P) + V_{dSAT}$$

where

$$P = (y - L + l)/\lambda$$

The net electric field is constituted of both longitudinal(along channel) As well as the lateral(down) field .
The equation of potential and electric field in this region is found By the quasi 2-D solution of the poisson's equation and is given by,

To find the Saturation Drain Voltage :

When the MOSFET just enters saturation the drain voltage is Vdsat . The electric field at y=L is assumed to be equal to Ec , the critical Electric field .

Using the same equation as in linear region ,

At y=L

$$I_{ds} = \frac{[V_{gst} - (1+\delta)V_{dSAT}]}{1 + \alpha E_c} E_c$$

Also from equation (1)

$$I_{ds} = WC_{ox}\mu^0{}_n \frac{[V_{gst}V_{dSAT} - 0.5(1+\delta)V^2{}_{dSAT}]}{\alpha V_{dSAT} + L^{'1}}$$

Where

$$L^1 = L + E_y(0)[\beta L^2{}_v - \alpha L_v]$$

Equating the two expressions we can write ,

$$\frac{\alpha E_c}{2} V^2{}_{dSAT} + V_{dSAT} \left[\frac{V_{gst}}{(1+\delta)} + E_c L^1 \right] = V_{gst} E_c \frac{L^1}{(1+\delta)}$$

Neglecting the first term ,

$$V_{dSAT} = \frac{\frac{V_{gst}}{(1+\delta)} E_c L^1}{\frac{V_{gst}}{(1+\delta)} + E_c L^1}$$

The Current in the Saturation region is given as

$$I_{ds} = \frac{-A_2 + \sqrt{A_2{}^2 - 4A_1 A_3}}{2A_1}$$

Where

$$A_1 = \frac{\{\beta \; L^2{}_v - \alpha L_v\}}{W\mu^0{}_n C_{ox} V_{gst}}$$

$$A_2 = L_{eff} + \alpha V_{ds}$$

$$A_3 = -W\mu_n{}^0 C_{ox} [V_{gst} V_{ds} - \frac{(1+\delta) V_{ds}{}^2}{2}]$$

To Find the Effective Length

To find the Effective Length we start with the potential Equation in the saturation (drain side) region where L>Leff

$$V(y) = \frac{\lambda^2 E_{sat}}{L - l}[\cosh(P) - 1] + \lambda E_{sat} \sinh(P) + V_{dSAT}$$

Where

$$P = (y - L + l)/\lambda$$

And

$$y = L - l$$

At y=L ,

$$V_{ds} = V_{dSAT}$$

Therefore at y=L the potential is

$$\frac{V_{ds} - V_{dSAT}}{\lambda E_{sat}} = \lambda \frac{[\cosh(\frac{l}{\lambda}) - 1]}{L - l} + \sinh(\frac{l}{\lambda})$$

Or,

$$\frac{V_{ds} - V_{dSAT}}{\lambda E_{sat}} = \lambda \frac{[0.5\exp(\frac{l}{\lambda}) + 0.5\exp(\frac{-l}{\lambda}) - 1]}{L - l} + 0.5\exp(\frac{l}{\lambda}) - 0.5\exp(\frac{-l}{\lambda})$$

Let

$$a = \frac{\lambda}{L - l}$$

$$c = \exp(\frac{l}{\lambda})$$

$$b = \frac{V_{ds} - V_{dSAT}}{\lambda E_{sat}}$$

Or,

$$2b = a(c + 1/c - 2) + (c - 1/c)$$

Or,

$$c = \exp(\frac{l}{\lambda}) = \frac{b + a + \sqrt{b^2 + 2ab + 1}}{a + 1}$$

Or,

$$l = \lambda \ln\left[\frac{b + a + \sqrt{b^2 + 2ab + 1}}{a + 1}\right]$$

Since a depends on λ as well , the solution of above is not closed form but can be calculated iteratively .

Finally the **effective length** is calculated as ,

$$L_{eff} = L - l$$

I-V plots generated from the physical model for L=0.2 um and 5 um

NOTE : The curve is not smooth because linear and saturation regions are modeled Differently. One can use a smoothing function (I used a linear function) in the region of convergence between linear and saturation.