TRANSPORT AND MOBILITY MODEL FOR SMALL DEVICES

For small devices of sub-micron channel lengths, the drift-diffusion model is not very accurate . The drift-diffusion (DD) model assumes small perturbations around equilibrium . For channel lengths less than 0.25 um the perturbation from equilibrium is large due to the presence of large lateral field and their gradients.

Device Simulator like PISCES employs a local-field model to find the current generation in short channel devices . This type of model assumes that the carrier acquires the steady-state energy corresponding to the local field at each grid point .

In small devices, the mobile carriers do not have enough time to reach equilibrium .With the increased applied electric field ,the velocity overshoot effect becomes apparent . This effects the energy distribution of the electron and the electron energy can depart from its value in thermal equilibrium .

One of the ways to incorporate the above effects is to base a model based on electron transport at microscopic level . But it is too time consuming and therefore inappropriate for device simulators.

The Hydrodynamic Model

The energy transport equation is based on Hydrodynamic Model . This model incorporates electron temperature and energy flux in the device .

The hydrodynamic equations are given as :

$$\frac{\partial n}{\partial t} + \nabla.(nv) = \frac{dn}{dt}$$

$$\frac{\partial v}{\partial t} + (v.\nabla)v = \frac{eE}{m^*} - \frac{n\nabla(nkT)}{m^*} + \frac{dv}{dt}$$

$$\frac{\partial w}{\partial t} + v.\nabla w = ev.E - \frac{\nabla(nvkT)}{n} + \frac{dw}{dt}$$

where

n : carrier concentration
e : electronic charge
v : drift velocity
w : average carrier energy
E : electric field

Certain Assumptions are made for solving the equations :

1) Steady State (which eliminates time derivatives)

2) Very small kinetic energy so that average electron energy can be approximated as thermal energy (w=1.5kT)

Then the equations become :

$$\frac{\partial n}{\partial t} + \nabla.(nv) = 0$$

$$v = \frac{\tau}{m^*}\left[eE - \frac{2\nabla w}{3} - \frac{2}{3n}w\nabla n\right] + \frac{2\tau\nabla(v^2/2)}{3}$$

and τ is energy relaxation time .

Equation (2) can be written as :

$$\frac{\partial w}{\partial t} + v.\nabla w = ev.E_y - \frac{\nabla(nvkT)}{n} + \frac{(w_n - w_0)}{\tau}$$

The carrier energy density is,

$$W_n = n.w_n$$

Therefore the above equation can be written as :

$$n.\frac{\partial w}{\partial t} + nv.\nabla w = envE_y - n\frac{\nabla(nvkT)}{n} + n.\frac{(w_n - w_0)}{\tau}$$

Or,

$$\frac{\partial W_n}{\partial t} = -\nabla(nvw + nvkT) + \frac{(W_n - W_0)}{\tau} - envE_y$$

where carrier energy density in thermal equilibrium

$$W_0 = \frac{3nkT_0}{2}$$

The Energy Relaxation Time is empirically expressed as:

$$\tau = \frac{3kT\mu_n}{2qv_{sat}}$$

When the MOS device is operating in strong inversion Region , the carrier velocity can be expressed as : (neglecting the diffusion current)

$$v = \mu_n\left(\frac{\partial \psi}{\partial y} - \frac{\partial U_n}{\partial y}\right)$$

where U is the thermal voltage of the carriers

i.e $U = \frac{kT}{q}$

Therefore $W = \frac{3nkT}{2} = \frac{3n.q.U}{2}$

At steady state , $\frac{\partial W}{\partial t} = 0$

That is,

$$0 = \frac{-\partial}{\partial y}\left[\{\frac{3nqU}{2} + nqU\}\mu_n\{\frac{\partial \psi}{\partial y} - \frac{\partial U}{\partial y}\}\right]$$

$$-\mu_n qn.\left[\frac{\partial \psi}{\partial y} - \frac{\partial U}{\partial y}\right].\frac{\partial \psi}{\partial y} - \frac{(W_n - W_0)}{\tau}$$

Or,

$$-\frac{7nq\mu}{2}.\frac{\partial U}{\partial y}.\frac{\partial \psi}{\partial y} + \frac{5nq\mu}{2}.(\frac{\partial U}{\partial y})^2 + nq\mu.(.\frac{\partial \psi}{\partial y})^2.$$

$$= \frac{nq(U - U_0)}{2.\tau}$$

This equation will be used further when we develop the Mobility Model .

MOBILITY MODEL

Starting with the equation (From the Hydrodynamic model)

$$\frac{-7}{2}(\frac{\partial U}{\partial y})(\frac{\partial \psi}{\partial y})+\frac{5}{2}(\frac{\partial U}{\partial y})^2+(\frac{\partial \psi}{\partial y})^2=\frac{3(U_n-U_0)}{2\tau_e\mu^0}$$

We also assume the **non-local** mobility equation as :

$$\mu_n=\mu^0\frac{T_0}{T}=\mu^0\frac{U_0}{U_n}$$

We use the following approximation ,

$$\frac{\partial U}{\partial y}=\frac{(U-U_0)}{y}$$

The above equation can thus be written as

$$\frac{-7}{2}(\frac{U_n-U_0}{y})E_y+\frac{5}{2}(\frac{U_n-U_0}{y})^2+E_y{}^2=\frac{3(U_n-U_0)}{2\tau_e\mu^0}$$

Or,

$$\{\frac{5}{2}-\frac{3y^2}{2\tau_e\mu_0}\}U^2{}_n+U_n\{\frac{7}{2}y.E_y+\frac{3y}{2\tau_e\mu_0}\}+E_y{}^2=0$$

(where E is the magnitude of electric field)

From the equation , U can be expressed as

$$U_n=U_0+\frac{A_2-\sqrt{A_2{}^2-4A_1A_3}}{2A_1}$$

Where

$$A_1=\frac{5}{2}-3\frac{y^2}{2\tau_e\mu_0U_0}$$

$$A_3=E^2{}_y$$

$$A_2=\frac{-7}{2}E_y+3\frac{y}{2\tau_e\mu_0}$$

Approximate expression for mobility

Let us define a parameter as $L_v = \frac{7\tau_e \mu_0 E_y}{3}$

Within the device y varies from 0 to L .

1) For y >> L_v the expression can be further approximated as (See Appendix B)

$$U = U_0.[\frac{1+\sqrt{1+\frac{8\tau\mu_0 E^2{}_y}{3}}}{2}]$$

Or,

$$\frac{\mu_0}{\mu_n} = .[\frac{1+\sqrt{1+\frac{8\tau\mu_0 E^2{}_y}{3}}}{2}]$$

Or,

$$\mu_n = \mu._{0.}[\frac{2}{1+\sqrt{1+\frac{8\tau\mu_0 E^2{}_y}{3}}}]$$

Or,

$$\mu_n = \mu._{0.}[\frac{1}{1+\alpha E_y}]$$

Where α is defined to be

$$\alpha = \frac{-1+\sqrt{1+\frac{8\tau\mu^0{}_n E^2{}_y}{3U_l}}}{2E_y}$$

The significance of this parameter will be explained later

2) For $y < L_v$ the mobility expression can be found by approximating,

$$A_1 = \frac{5}{2} \qquad A_2 = \frac{7}{2} \qquad A_3 = E_y^{\,2}$$

The expression for U becomes as

$$U_n = U_0 + y.\frac{\frac{7E_y}{2} - \sqrt{\frac{49E_y^{\,2}}{4} - 10E_y^{\,2}}}{5}$$

Or,

$$U_n = U_0 + y.\frac{\frac{7E_y}{2} - \frac{3E_y}{2}}{5} = U_0 + \frac{2U_0}{5U_0}.y.E_y$$

Or,

$$\mu = \mu_0 \frac{1}{1+\beta y E_y} \qquad \text{where } \beta = \frac{2}{5U_0}$$

The second step to finding an approximate expression for Mobility is phenomenological

$$\mu = \frac{\mu_0}{1+\beta.y.E_y.\exp(\frac{-y}{L_v}) + \alpha.E_y.\{1-\exp(\frac{-y}{L_v})\}}$$

The Denominator D is $1 + \beta.y.E_y.$ for small y

and for y large $1+\alpha.E_y.\{1-0\} = 1+\alpha.E_y$

Fig 1 : Mobility Curve for L=0.67 microns for constant electric field

Fig 2 : Electric field variation along the MOSFET channel for different drain voltages