CAT

Complementary Associations Theory

An overview

Doron Shadmi 2003

Dear reader,

Since ancient times, there have been two basic approaches to exploration.

The holistic approach looks for one simple principle that exists in the basis of all explored things.

The reductionist approach is looking at the variety of the relations that may or may not exist within and among the explored elements.

For the last 500 years, the preferred approach is reductionism and most achievements in the modern science are based on it.

During the 20th century, some basic paradigms have been changed and scientists are now forced to reexamine their pure and applied methods.

To make this process more balanced, scientists find themselves examining methods that are based on the holistic approach, but this time not in a naive way, but with some insights that are based on the reductionist approach.

The result of this combination can be a very fruitful method for modern science, and its success depends on a very fine-tuning between the two approaches.

Many new insights in a variety of scientific areas were found and developed during the 20th century.

Some of them belong to: Information Theory, Quantum Mechanics, Topology, Complexity Theory and a lot of researches about the connections between computation, neurobiology and cognition.

The common base of all these areas is the Mathematics language, which (I think) may be one of the most powerful tools for mankind’s survival.

If we find some simple principles that can be a natural (not forced) base to a variety of scientific areas through this language, I think that we shall obtain an organic and dynamic structure to our scientific methods that can help us to continue our participation in the "game" of evolution.

My work follows this idea and tries to define an organic structure, which is based on an association between the Continuum and the Discreteness concepts.

A formal solution to Hilbert's 1st and 6th problems

p and ***q*** are real numbers.

If $p < q$ then $[p, q] = \{x : p <= x <= q\}$ or
$(p, q] = \{x : p < x <= q\}$ or
$[p, q) = \{x : p <= x < q\}$ or
$(p, q) = \{x : p < x < q\}$.

A single-simultaneous-connection is any single real number included in ***p***, ***q***
(= ***D*** = Discreteness = a localized element = {.}).

Double-simultaneous-connection is a connection between any two different real numbers included in ***p***, ***q*** , where any connection has exactly 1 ***D*** as a common element with some other connection (= ***C*** = Continuum = a non-localized element = {.____.}).

Therefore, ***x*** is . XOR .____.

In Conventional Math 0^0 is not well defined, because each member is ***D***.

Let us say that power ^{0} is the simplest level of existence of some set's content.

Because there are no ***D***s in ***C***, its base value = 0 , but because it exists (unlike the emptiness), its cardinality = $0^0 = 1$.

There are now 3 kinds of cardinality:

|{}| = 0 = the cardinality of the Empty set.

|{._.}| = 0^0 = 1 = the cardinality of ***C***.

|{.}| = 1^0 = 1 = the cardinality of ***D***.

Any point is a *D* element. Any line is a *C* element.

XOR connective between LINES and POINTS

0(LINE) 0(POINT) -> 0-(No information) -> no conclusion.
0(LINE) 1(POINT) -> 1-(Clear Particle-like information) -> conclusions on points.
1(LINE) 0(POINT) -> 1-(Clear Wave-like information) -> conclusions on lines.
1(LINE) 1(POINT) -> 0-(No clear information) -> no conclusion.

We can break ***C*** infinitely many times, but always we shall find an invariant structural state of {._.}, which is a **connector** between any two ***D***s.

Let power ^{0} be the simplest level of existence of some set's content.

{._. .} = ***D***inf = Infinitely many ***D***s and/or ***C***s (∞^0 = ***C*** XOR ***D*** = 1).

{___} = ***C***inf = Infinitely long ***C*** (0^0 = 1).

$0^0 = \infty^0 = 1$ and we can see that we can't distinguish between ***C*** and ***D*** by their quantitative property.

But by their Structural property ***D***inf is not ***C***inf.

From the above we can learn that the Structure concept has more information than the Quantity concept in Math language.

Any element that is under a definition like "finite or infinitely many ..." cannot be anything but a member of ***D*** or ***D***inf sets, which have the structure of the Discreteness concept.

So, any line's segment is not a container but a **connector** between any two points {.___.}, and you can find this state in any scale that you choose.

C XOR ***D***, and through this approach you don't have any contradiction between the Discreteness and the Continuum concepts, because any point is not in the Continuum, **but an event that breaks the Continuum.**

The Continuum does not exist in this event (because of the XOR between any line to any point), but any two events can be connected by a Continuum.

Take for example, the end of a line is an event that breaks the line and it turns to a Nothingness, so from one side we have the Continuum, from the other side we have the Nothingness, and between them we have a break point, that can be connected to another break point that may exist on the other side of the continuous line.

Another way to look at these concepts is:

Let a Continuum be an infinitely long X-axis.

Let a point be any Y(=0)-axis on the X-axis.

So what we get is a non-localized X-axis and infinitely many Y(=0)-axises points on (not in) the X-axis.

Through this point of view, the X-axis is a connector (not a container) between infinitely many Y(=0)-axises events.

In general, there are two levels of XOR: A) ({} XOR {.}) OR ({} XOR {_})
B) {.} XOR {_}

There are 4 important conclusions from the above:

For example, let n = 3 = 1+1+1

A) 0^0 = Continuous 1

B) 1^0 = 1 Connector

C) $n/1^0$ = n Connectors (.__.__.__. = 1 1 1)

D) $n/0^0$ = Continuous n (.________. = 3)

Through this approach, each natural number is the associations (AND connective) between its continuous side (Continuous n) to its discrete side (n Connectors).

So from my point of view, Mathematics is more than variety of systems, where each system has its consistent universe.

As I see it, through this attitude one of the most important things of the evolution is cut out of today's Math.

Any evolution is based at least on two principles, variety and mutation.

The meaning of a mutation is to redefine existing things or familiar terms.

As I see it, the Modern Math has to look at this kind of approach as if it is a mutation in the Continuum concept and not as another axiomatic system where we can get:

$|Q| < |R|$ = Pointed Continuum < A Pointless Continuum.

Pointed Continuum is simply a contradiction, so as this concept does not exist, we can get a simple solution to the CH problem where:

$|Q| < |R| <$ Continuum.

Through this approach, any association between {._. .} to {__} can't be examined by 1 to 1 correspondence, but by an AND connective between clearly distinguished structural set's contents of {._. .} AND {__} forms.

I think that to look on Math as a variety of consistent (interesting) systems ,without any mutation's possibility, might be a very convenient approach for the mathematicians from a tactical point of view, but it is bad for Mathematics and mathematicians from a strategic point of view.

Any localized element cannot be anything but ***D***.

Any association between localized elements cannot be anything but ***C***.

Yours,

Doron Shadmi, June 2003.

Abstract

This overview has two parts, an introduction of an axiom and definitions of a new system, and a short example of a new number system representation, compared to the base value expansion method and based on said number system.

We define a new axiomatic system, which is based on associations between the redundancy AND uncertainty concepts.

By doing this we can show that the number system that is based on ZF or Peano's axioms, is a proper subset of the new number system which is based on the new axiomatic system.

The number system's new elements, exist between the Discrete set's form = {._. .} and the Continuous set's form = {___}.

It means that we have an original solution to the CH problem (from a new point of view that gives us new insight of the problem) and on the other hand we are enriching the common base value expansion representation method.

The CH problem is: $|Q| < ? < |R|$ = Continuum .

We show that by using the Continuum concept not in terms of its opposite concept (which is the Discreteness Concept) but by using the Redundancy and the Uncertainty concepts we have:

$|Q|$ = Infinitely many elements with finite or infinitely many digits, where those with infinitely many digits, are built on repetitions over scales.

$|R|$ = Infinitely many elements with infinitely many digits, without repetitions over scales.

And we get: $|Q| < |R| <$ Continuum .

Introduction

If we examine the content of a set in terms of the symmetry concept, we can find at least two levels of symmetries that can be ordered by their simplicity degrees.

The most symmetrical and simplest content is Emptiness, which is represented by the empty set notation { } = content does not exist.

On top of this simplicity we can define two opposite types of symmetry contents,
{__} and {._. .} .

Let power ^{0} be the simplest level of existence of some set's content.

{__} is an infinite non-localized element that notated as $0^0 = 1$ (1 continuum)

{._. .} is infinitely many elements that are notated as $\infty^0 = 1$ (connector XOR point)

So, what we get is this basic information structure: {._. .} { } {___}

Let { } be ***E*** simplicity or ***E***sim (***E*** for Emptiness).

Let {___} be ***C***sim (***C*** for Continuum).

Let {._. .} be ***D***sim (***D*** for Discreteness).

~ = NOT

Any transformation from {} to {__} or {._. .} is based on phase transition, because we have |{}|(=0) to ~|{}|(=~0) transition.

A ***C***sim and a ***D***sim are opposites because ***C***sim is a one continuum and ***D***sim is finite or infinitely many elements.

The above identification is based on the structure property, and it can not be done by the quantity property, because ***C***sim XOR ***D***sim are exclusively = 1 .

So, in the case of the symmetry concept, the structure property is more sensitive than the quantity property, when we examine them by the information concept.

If we want to go beyond the information about the existence of ***C***sim XOR ***D***sim, we have to associate between them, by changing XOR to AND connective.

By doing this we can define elements that have properties, which are combinations of ***E***sim, ***C***sim and ***D***sim.

Let us find a definition for existence under Complementary Associations Theory (CAT) :

<u>Existence</u>

Definition AA: Un-explorable Existence is a state of some opposite concepts, before there is any mutual influence on each opposite's property.

Example: Light before turning into darkness, darkness before turning into light.

Definition BB: Explorable Existence is a state of some opposite concepts, where there is a mutual influence between their opposite properties.

Example: Light turning into darkness, darkness turning into light.

Let us write the CAT's axiom and definitions.

<u>The Axiom of exploration:</u>
Explorable is any association between ***C***sim AND ***D***sim.

<u>Definition A:</u>
Explorable Product (***EP***) exists iff it is an association between Continuum (***C***sim) and Discreteness (***D***sim) concepts, so ***CD*** is ***C***sim AND ***D***sim .

Now, let us answer some questions:

Q: What is an "association"?
A: Association is any possible mutual influence between opposite concepts.

Q: What is an "explorable product"?
A: According to definition BB, it is the element coming from association, and it can be explored.

<u>Definition B:</u>
Association-Level (***AL***) is an invariant quantity, being kept through ***CD*** associations.

<u>Definition C:</u>
Computational Root (***CR***) is ***EP*** in ***AL***.

(An example of definitions B and C:

```
.   .   .      .   .   .      .   .   .
|   |   |      |   |   |      |   |   |
|   |   |      |___|_  |      |___|   |
|   |   |      |       |      |       |
|___|___|_     |_______|      |_______|
|              |              |
```

CR quantity is being kept through ***CD*** associations)

<u>Definition D:</u>
Redundancy and Uncertainty (***RU***) concepts, are used as invariant structural degree of ***CR***, determining its exact position in ***AL*** (there is an algorithm for this).

<u>Definition E:</u>
Full ***RU*** (***FRU***) is the first ***CR*** in ***AL***.

<u>Definition F:</u>
Not ***RU*** (~***RU***) is the last ***CR*** in ***AL***.

<u>Definition G:</u>
Partial ***RU*** (***PRU***) is any ***CR*** which is not ***FRU*** and not ~***RU***.

An example of definitions E, F and G:

```
.   .   .      .   .   .      .   .   .
|   |   |      |   |   |      |   |   |
|   |   |      |___|_  |      |___|   |
|   |   |      |       |      |       |
|___|___|_     |_______|      |_______|
|              |              |
FRU CR         PRU CR         ~RU CR
```

<u>A general graphic description of a ***CR***</u>

```
.     .     .<------ D (Discreteness)
|     |     |
|     |     |
|     |     |<------ The association between CD
|     |     |
|     |     |
|_____|_____|__<---- RU marker
|  ^
|   \____ C (Continuum)
|
|<---- Next-AL marker
```

If we connect ideas coming from Information Theory and Topology, then we can use a concept like symmetry, to describe a connection between structure and information's clarity-degree, for example:

```
<-Redundancy->
c   c   c  ^<----Uncertainty
b   b   b  |    b   b
a   a   a  |    a   a   c       a   b   c
.   .   .  v    .   .   .       .   .   .
|   |   |       |   |   |       |   |   |
|   |   |       |___|_  |       |___|   |
|   |   |       |       |       |       |
|___|___|_      |_______|       |_______|
|               |               |
FRU CR          PRU CR          ~RU CR
```

The uncertainty is based on XOR connective between a,b,c,...

This example describes ***AL*** 3 and we can see the connection between structure's symmetry-degree and information's clarity-degree, determining ***CR***'s exact position in ***AL*** 3.

Because ***RU*** concepts are not used in ZF or Peano's axioms, all their number system is limited to ~***RU CR***, therefore their number system is a proper subset of CAT's number system.

Some claim that using sequences instead of sets can do it, but we use the ***RU*** concepts as the fundamentals of all information structures, where ~***RU*** is a private case of some information structure, that used as the information structure of ZF or Peano's axioms.

More than that, because we clearly distinguish between the Continuum and the Discreteness concepts, we can represent a number by using the familiar notations but with a new meaning, for example let us represent the number four:

```
    1
    +
    1
4 = +
    1
    +
    1
```

```
      1
  2 = +
      1
4 = +
      1
      +
      1
```

```
      1
  2 = +
      1
4 = +
      1
  2 = +
      1
```

```
        1
        +
    3 = 1
4 = +   +
        1

        1
```

```
            1
        2 = +
    3 = +   1
4 = +
            1

            1
```

In the above example we can see fades between the continuous side of number four, which is represented by a single notation (4), and the discrete side of number four, which is represented by four notations (1 1 1 1), where the quantity in each information structure of number four is being kept, and we can learn that the Structure concept is more sensitive than the Quantity concept in Math language.

<u>CAT's number system representation</u>

Now let us examine the number system representation of ***AL***s 0 to 4:

```
0 =  __   .  = {} (Before Association)
                *

      0
      .
1 =   |   (First Association between Continuum and Discreteness)
      *


    1   1
    0   0     0   1
    .   .     .   .
    |   |     |   |
2 = |___|_    |___|
    |         | *


    2   2   2
    1   1   1       1   1
    0   0   0       0   0   2       0   1   2
    .   .   .       .   .   .       .   .   .
    |   |   |       |   |   |       |   |   |
3 = |   |   |       |___|_  |       |___|   |
    |   |   |       |       |       |       |
    |___|___|_      |_______|       |_______|
    |               |               |   *


    -------------->>>
    3  3  3  3           3  3           3  3
    2  2  2  2           2  2           2  2
    1  1  1  1     1  1  1  1           1  1     1  1  1  1           1  1
    0  0  0  0     0  0  0  0     0  1  0  0     0  0  0  0     0  1  0  0
    .  .  .  .     .  .  .  .     .  .  .  .     .  .  .  .     .  .  .  .
    |  |  |  |     |  |  |  |     |  |  |  |     |  |  |  |     |  |  |  |
    |  |  |  |     |__|_ |  |     |__|  |  |     |__|_ |__|_    |__|  |__|_
    |  |  |  |     |     |  |     |     |  |     |     |        |     |
    |  |  |  |     |     |  |     |     |  |     |     |        |     |
    |  |  |  |     |     |  |     |     |  |     |     |        |     |
    |__|__|__|_    |_____|__|_    |_____|__|_    |_____|____    |_____|____
    |              |              |              |              |
4 =                2  2  2
                   1  1  1        1  1
    0  1  0  1     0  0  0  3     0  0  2  3     0  1  2  3
    .  .  .  .     .  .  .  .     .  .  .  .     .  .  .  .
    |  |  |  |     |  |  |  |     |  |  |  |     |  |  |  |
    |__|  |__|     |  |  |  |     |__|_ |  |     |__|  |  |
    |     |        |  |  |  |     |     |  |     |     |  |
    |     |        |__|__|_ |     |_____|  |     |_____|  |
    |     |        |        |     |        |     |        |
    |_____|____    |________|     |________|     |________|
    |              |              |              |   *
```

The ***CR***s marked by `*` are number system representations, based on Peano's axioms.

Let us examine the structure of `*` representations, through CAT's eyes:

```
0 = { } = __  . (Before CD associations)


1 = {{ }} = {0}
      .
      |


2 = {{ },{{ }}} = {0,1}
      .    .
      |    |
      |____|
      |


3 = {{ },{{ }},{{ },{{ }}}} = {0,1,2}
      .    .     .    .
      |    |     |    |
      |____|     |____|
      |          |
      |__________|
      |


4 = {{ },{{ }},{{ },{{ }}},{{ },{{ }},{{ },{{ }}}}} = {0,1,2,3}
      .    .     .    .      .    .     .    .
      |    |     |    |      |    |     |    |
      |____|     |____|      |____|     |____|
      |          |           |          |
      |__________|           |__________|
      |                      |
      |______________________|
      |
      |
```

This Fractalic-Information-Structure (*FIS*) is based on

```
      .    .
      |    |
      |____|
      |
```

which is ~*RU CR* in *AL* 2 .

If we build the *FIS* by using FRU CR in AL 2, we get *FIS*

```
      .    .     .    .      .    .     .    .
      |    |     |    |      |    |     |    |
      |____|_    |____|_     |____|_    |____|_
      |          |           |          |
      |__________|______     |__________|______
      |                      |
      |______________________|__________________
      |
      |
```

which has a different structure of number system representation.

Let us examine how a number system representation, based on Peano's axioms, uses ~***RU CR*** to represent a number system which is based on base value expansion representation, and for example we shall use number 53 represented by base 10 and base 3:

Number 53 represented by base 10:

```
               ^0- 0123456789
                   ||||||||||
                   |_||||||||
                   |__|||||||
                   |___||||||
~RU CR in AL 10 =  |____|||||
                   |_____||||
                   |______|||
                   |_______||
                   |________|
               ^1- |
                   |                             1      0
                                           ( 5*10 + 3*10 )
^0- 012345678901234567890123456789012345678901234567890123
    ||||||||||||||||||||||||||||||||||||||||||||||||||||||
    |_|||||||||_|||||||||_|||||||||_|||||||||_|||||||||_||
    |__||||||||__||||||||__||||||||__||||||||__||||||||__|
    |___|||||||___|||||||___|||||||___|||||||___||||||||__...
    |____||||||____||||||____||||||____||||||____|||||||__...
    |_____|||||_____|||||_____|||||_____|||||_____||||||__...
    |______||||______||||______||||______||||______|||||__...
    |_______|||_______|||_______|||_______|||_______||||__...
    |________||________||________||________||________|||__...
^1- |    0    |    1    |    2    |    3    |    4    | 5
    |_________|         |         |         |         |
    |___________________|         |         |         |
    |_____________________________|         |         |
    |_______________________________________|         |
    |_________________________________________________|
    |_ ...
    |
```

Number 53 represented by base 3:

```
               ^0- 012
~RU CR in AL 3 =   |||
                   |_|
               ^1- |                  3     2     1     0
                                 ( 1*3 + 2*3 + 2*3 + 2*3 )
^0- 012012012012012012012012012012012012012012012012012012
    ||||||||||||||||||||||||||||||||||||||||||||||||||||||
    |_||_||_||_||_||_||_||_||_||_||_||_||_||_||_||_||_||_|
^1- |0 |1 |2 |0 |1 |2 |0 |1 |2 |0 |1 |2 |0 |1 |2 |0 |1 |2
    |__|  |  |__|  |  |__|  |  |__|  |  |__|  |  |__|  |
    |_____|  |_____|  |_____|  |_____|  |_____|  |_____|
^2- |  0     |  1     |  2     |  0     |  1     |  2
    |________|        |        |________|        |
    |_________________|        |_________________|
^3- |        0                 |        1
    |__________________________|
    |_ ...
    |
```

Now let us represent Continuum XOR Discreteness before they are explorable:

```
                                                                  0
<--''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''--> D = ∞  = 1 = Exists
                                                               (un-explorable)
                      No Association
                                                                 0
<--____________________________________________________________--> C = 0  = 1 = Exists
                                                               (un-explorable)
```

And after association we get the elementary information structure, which is the base of any representation of number 53:

```
                                           0          0
FRU CR in AL 54 = Association between  54*0  and  54*1
                                                                   0
'''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''-> D = 54*1  = difference
||||||||||||||||||||||||||||||||||||||||||||||||||||||
||||||||||||||||||||||||||||||||||||||||||||||||||||||<----------- Association
||||||||||||||||||||||||||||||||||||||||||||||||||||||             0
______________________________________________________-> C = 54*0  = sum
```

By writing "Computational Root" we mean that a ***CR*** is the building block of any ***FIS*** (Fractalic-Information-Structure) that can be systematically explored.

Here are some ***FIS***es which are not ~***RU CR***:

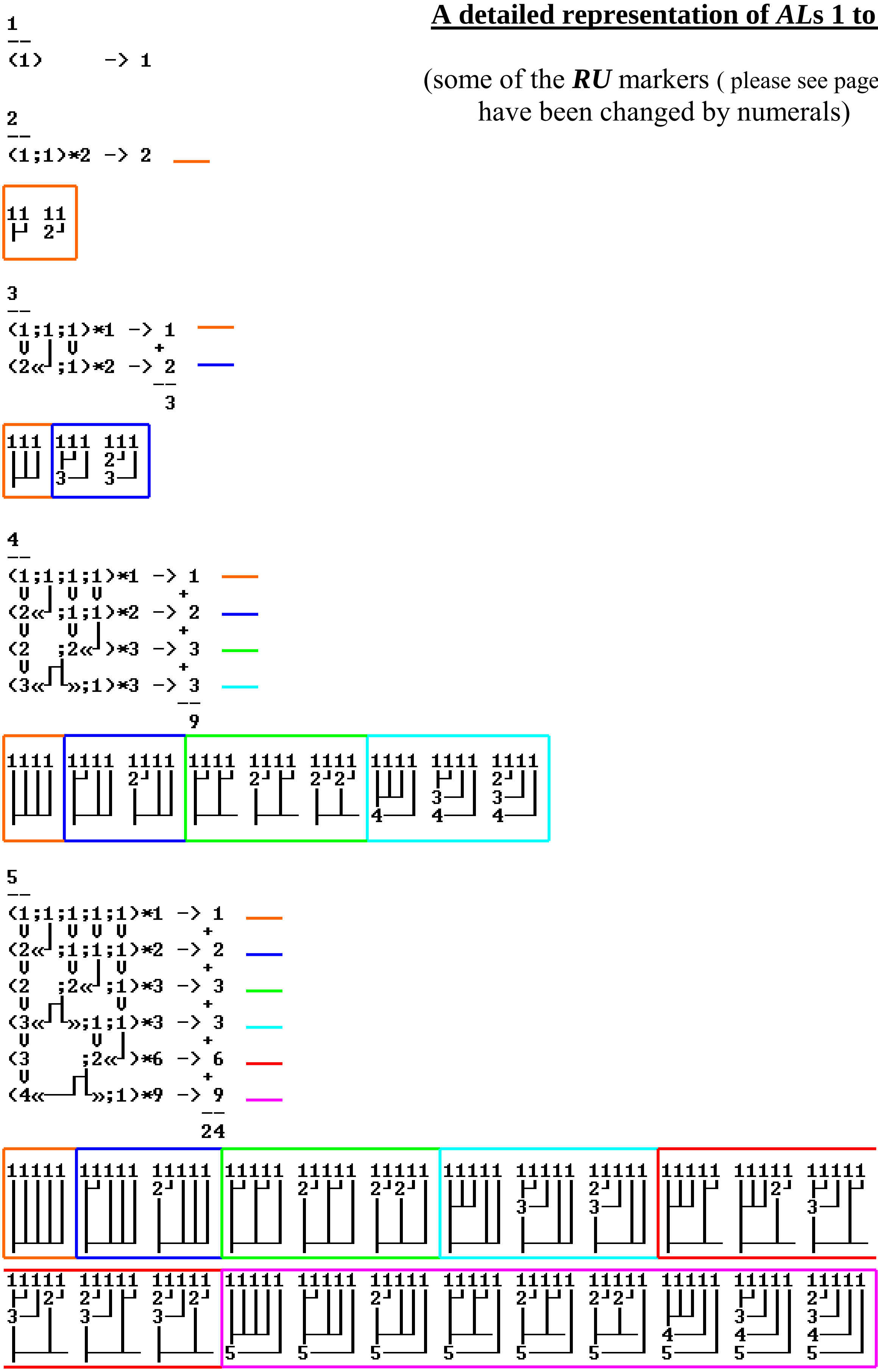
```
c   c   c    c   c   c    c   c   c
b   b   b    b   b   b    b   b   b        b   b        b   b        b   b
a   a   a    a   a   a    a   a   a        a   a   c    a   a   c    a   a   c
.   .   .    .   .   .    .   .   .        .   .   .    .   .   .    .   .   .
|   |   |    |   |   |    |   |   |        |   |   |    |   |   |    |   |   |
|   |   |    |   |   |    |   |   |        |___|_  |    |___|_  |    |___|_  |
|   |   |    |   |   |    |   |   |        |       |    |       |    |       |
|___|___|_   |___|___|_   |___|___|_       |_______|    |_______|    |_______|
|   c        |   c        |   c            |   b        |   b        |   c
|   b        |   b        |   b            |   a        |   a        |
|   a        |   a        |   a            |____________|________    |
|            |            |                |                         |
|            |            |                |                         |
|____________|____________|__________      |_________________________|
|                  c                       |            b
|                  b                       |            a
                   a
```

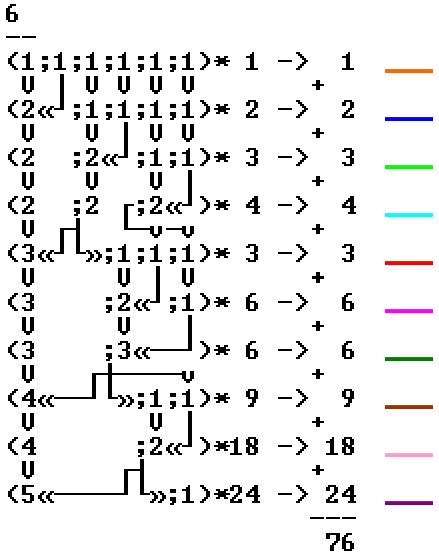
by using ***RU*** concepts when defining a number system, we are enriching the ways of using a number system representation.

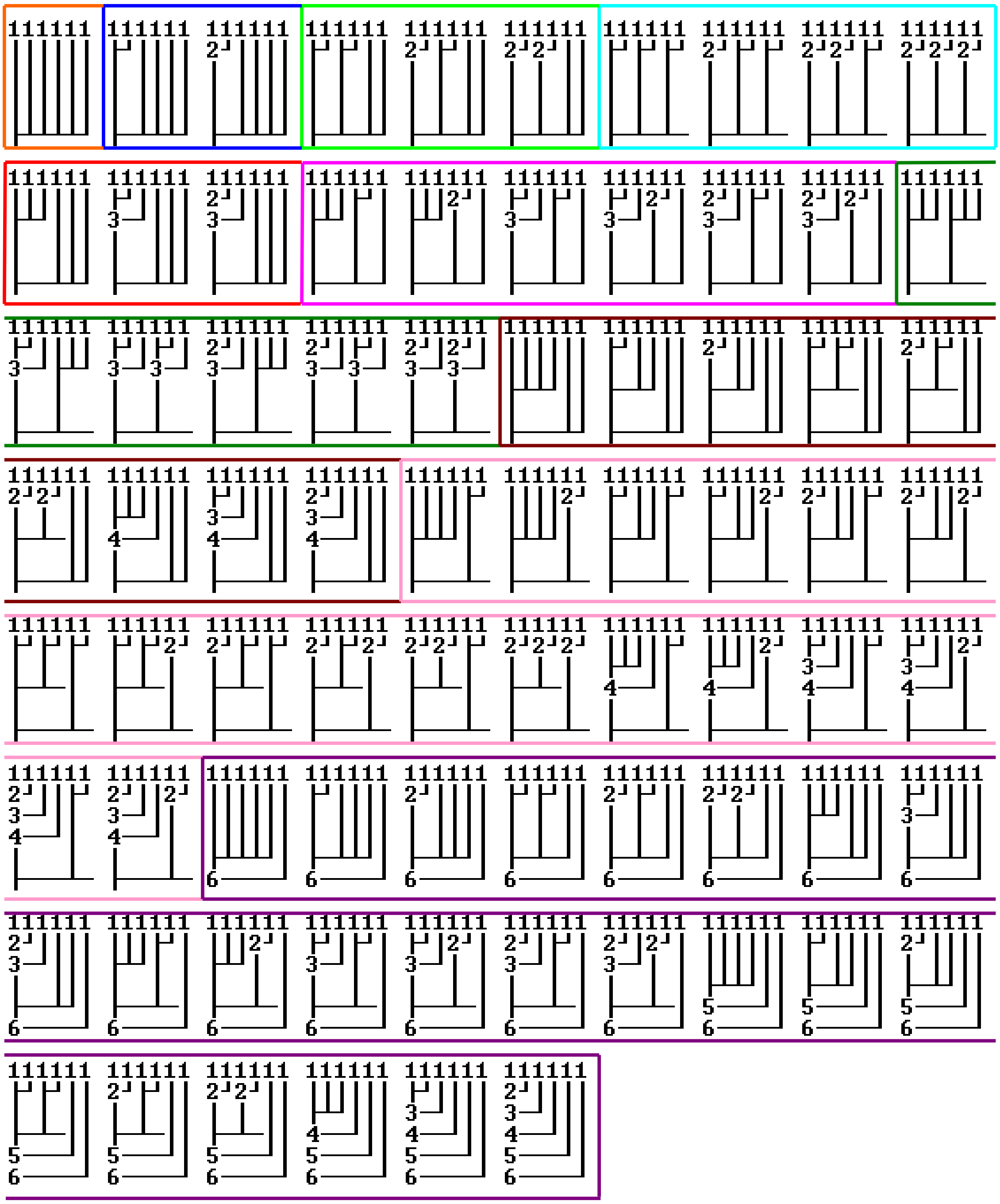
CAT has a ***C***-bound 0^0 and a ***D***-bound ∞^0, giving us a mathematical universe which has infinite levels of structure's symmetry-degrees connected to infinite levels of information's clarity-degrees, over infinite scales (where some representations of it can be found in the next pages).

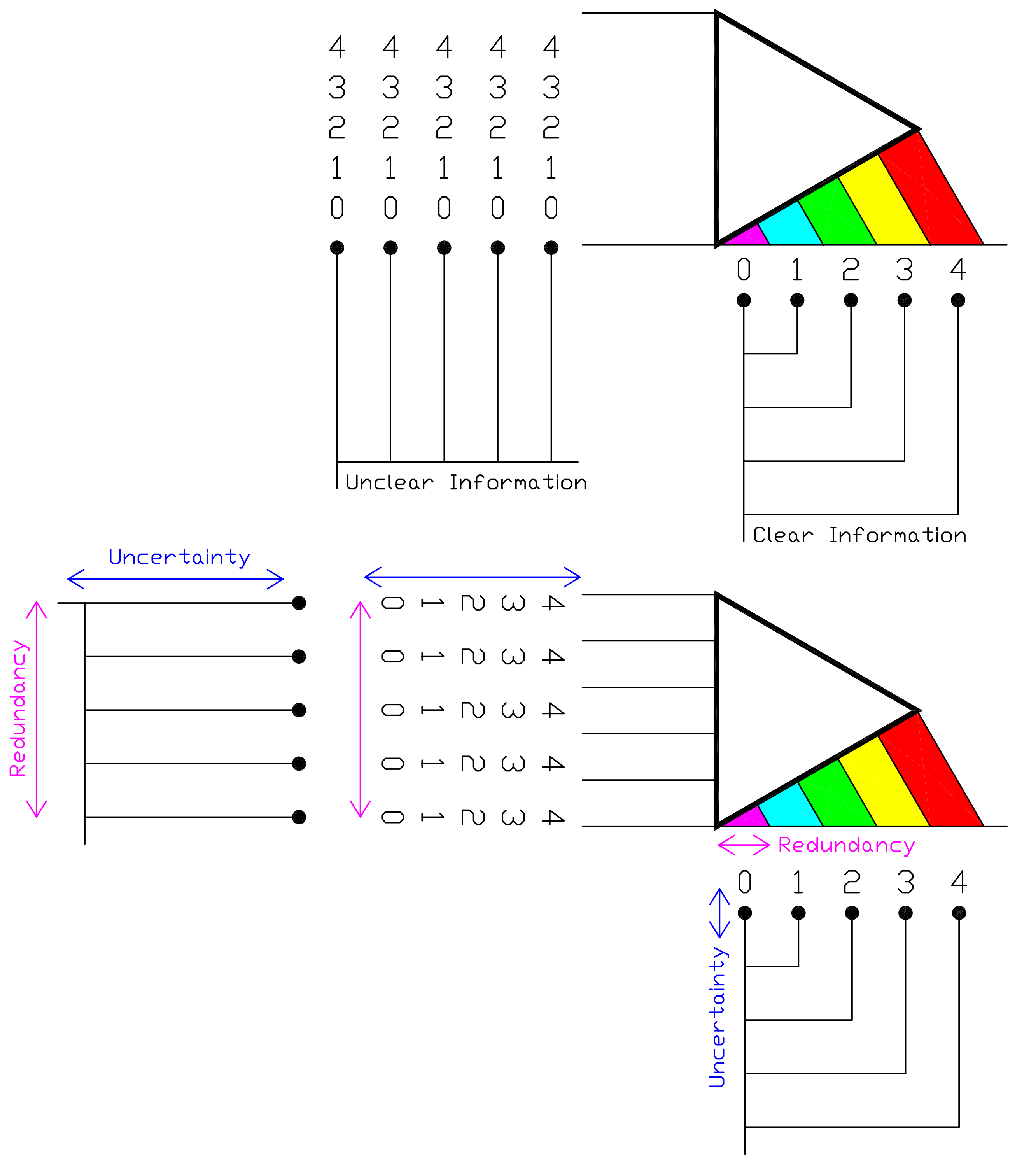
A detailed representation of *AL*s 1 to 6

(some of the ***RU*** markers (please see page 9) have been changed by numerals)

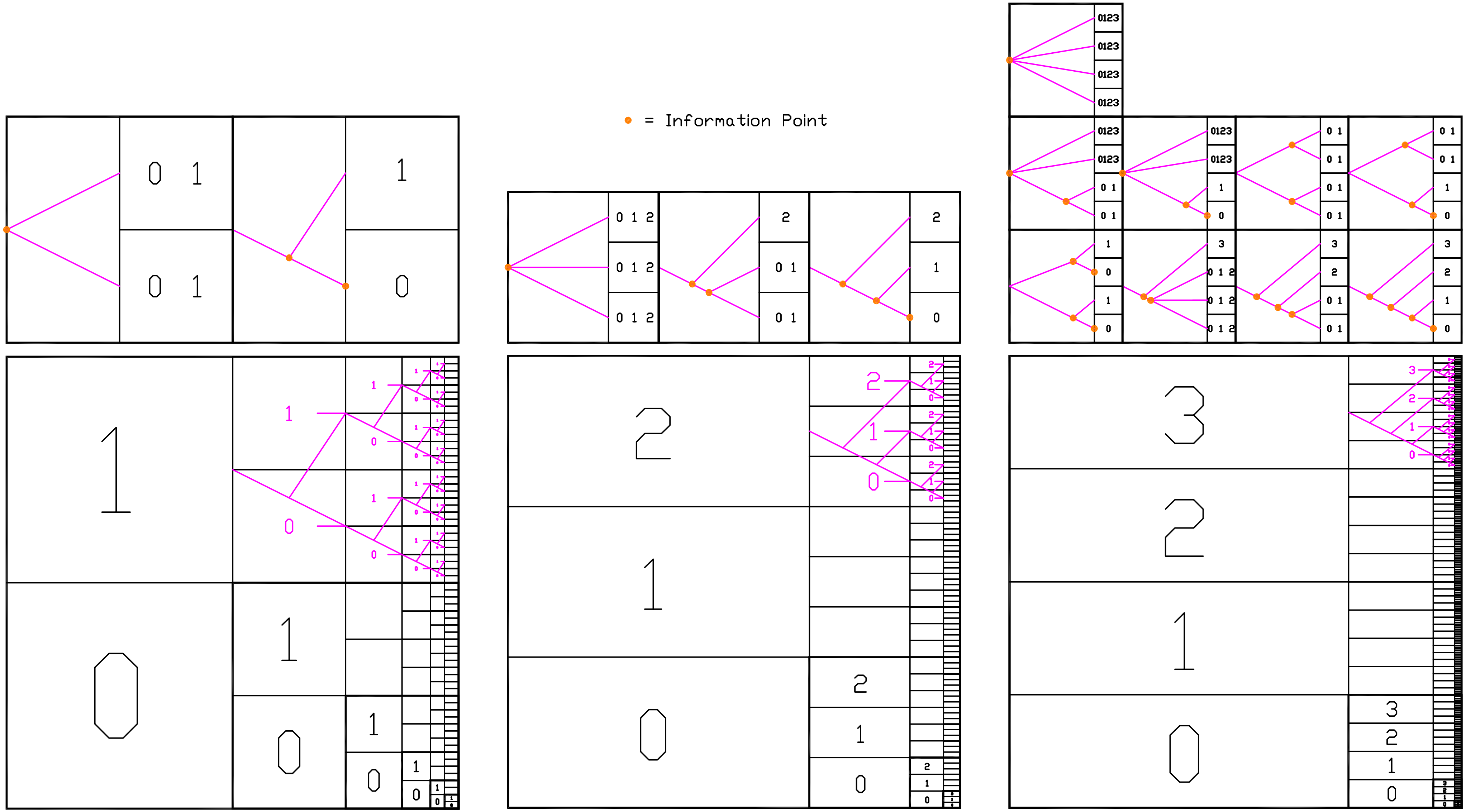


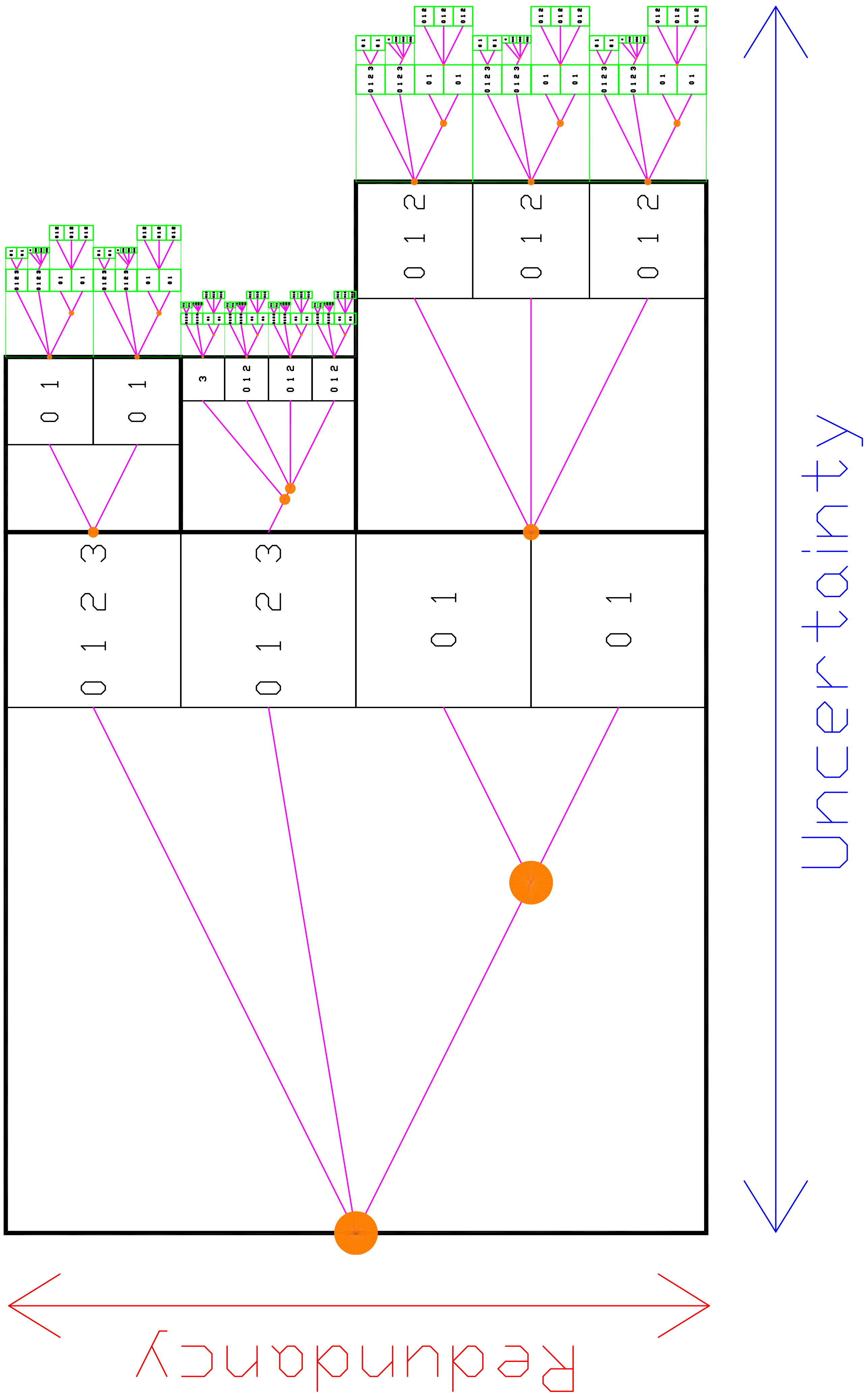

6
(1;1;1;1;1;1)* 1 -> 1
(2«;1;1;1;1)* 2 -> 2
(2 ;2«;1;1)* 3 -> 3
(2 ;2 ;2«)* 4 -> 4
(3«»;1;1;1)* 3 -> 3
(3 ;2«;1)* 6 -> 6
(3 ;3«)* 6 -> 6
(4«»;1;1)* 9 -> 9
(4 ;2«)*18 -> 18
(5«»;1)*24 -> 24
76





Unclear Information
Clear Information
Uncertainty
Redundancy


= Information Point

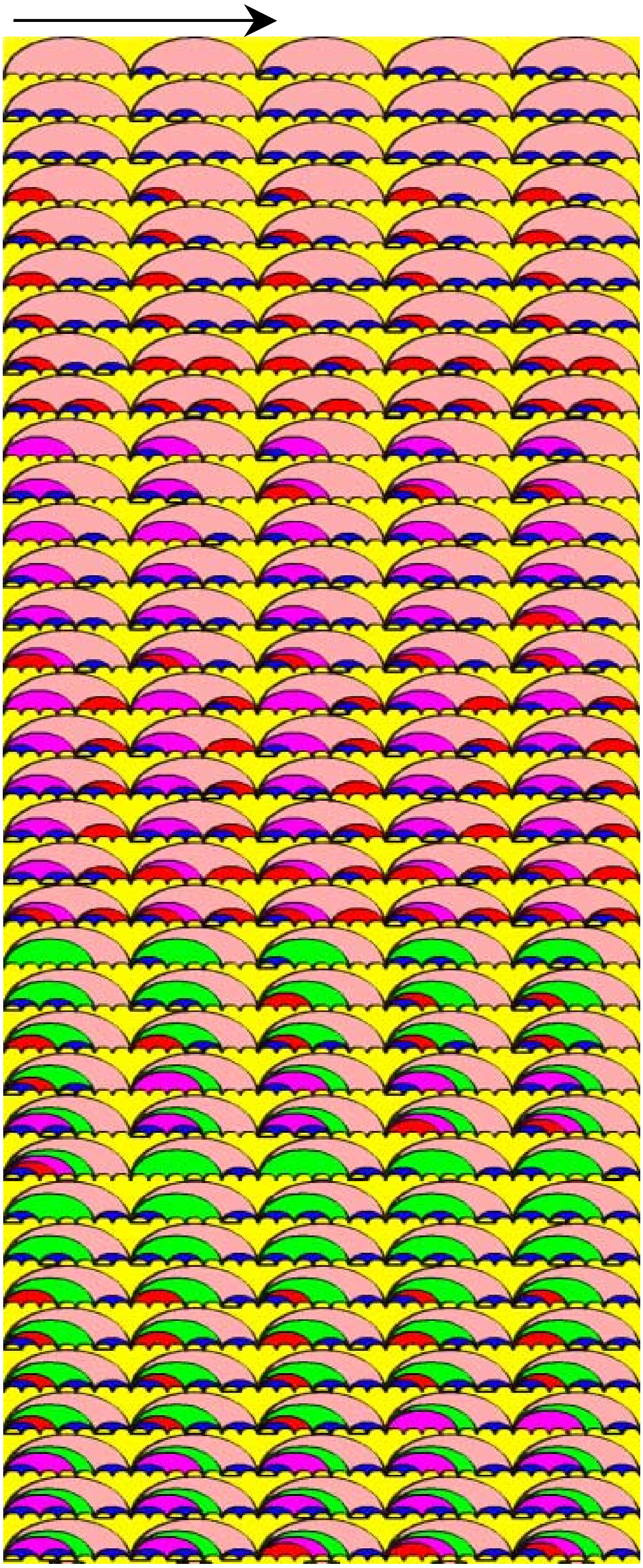

Uncertainty
Redundancy

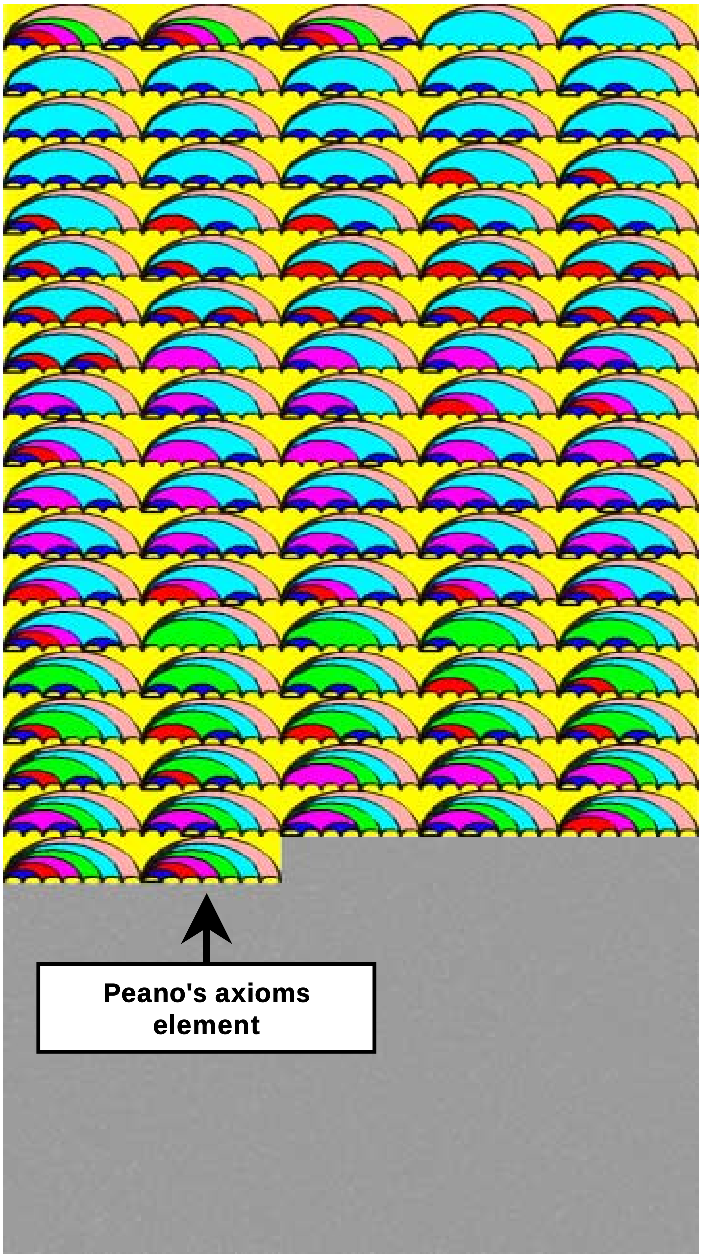
A graphic presentation of AL 7

Algorithm & programming by

Yaron Perlman

(The left-right permutations must be ignored)




Peano's axioms
element

A quick reference of CAT's acronyms

CAT = Complementary Associations Theory.

Association = Any possible mutual influence between opposite concepts (under CAT its between Continuum and Discreteness).

CD = Continuum AND Discreteness associations

EP = Explorable Product (exists iff it is an Association between CD).

AL = Association Level is an invariant quantity, being kept through CD associations.

CR = Computational Root is EP in AL.

RU = Redundancy and Uncertainty concepts, are used as invariant structural degree of CR, determining its exact position in AL.

FRU = Full RU is the first CR in AL.

~RU = Not RU is the last CR in AL.

PRU = Partial RU is any CR which is not FRU and not ~RU.

FIS = Fractalic Information Structure, used to represent numbers that are based on CRs.

Some Graphics of my number system representation of ALs 1 to 7, can be found at the following address:

http://cyborg2000.xpert.com/ctheory/

It was written using java by **Yaron Perlman**, and is based on Cartesian Product, resulting in some left-right permutations that can be ignored.

Doron Shadmi

Email: *complementarytheory@yahoo.com*

Web site:

http://www.geocities.com/complementarytheory/CATpage.html