THE UNIVERSITY OF HONG KONG
DEPARTMENT OF STATISTICS AND ACTUARIAL SCIENCE
STAT 1301 Probability and Statistics 1
Assignment 3

(Due 11/11/2005. Hand in Q 1, 3, 4, 8)

1. The joint probability mass function $p(x, y)$ of two discrete random variables, X and Y, is given by

$$p(x,y) = \begin{cases} \left(\frac{1}{2}\right)^x 10^y \frac{e^{-15}}{x!(y-x)!} & \text{if } x \text{ and } y \text{ are nonnegative integers and } x < y, \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Find the marginal probability mass functions of X and Y.
   (b) Are X and Y independent? Specify the reason.

2. A point is chosen randomly in the interior of an ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

   Find the marginal densities of the x and y coordinates of the point.

3. Find the joint and marginal densities corresponding to the cdf

$$F(x,y) = (1 - e^{-\alpha x})(1 - e^{-\beta y}), \qquad x \geq 0, \qquad y \geq 0, \qquad \alpha > 0, \qquad \beta > 0.$$

4. Let X and Y have the joint probability density function

$$f(x,y) = c(x^2 - y^2)e^{-x}, \qquad 0 \leq x < \infty, \qquad -x \leq y < x$$

   (a) Find c.
   (b) Find the marginal densities.
   (c) Are X and Y independent? Specify the reason.

5. Suppose that X and Y have the joint density function

$$f(x,y) = c\sqrt{1 - x^2 - y^2}, \qquad x^2 + y^2 \leq 1$$

   (a) Find c.
   (b) Sketch the joint density.
   (c) Find $P(X^2 + Y^2 \leq \frac{1}{2})$.
   (d) Find the marginal densities of X and Y. Are X and Y independent random variables? Specify the reason.

6. Let N_1 and N_2 be independent random variables following Poisson distributions with parameters λ_1 and λ_2, respectively. Show that the distribution of $N = N_1 + N_2$ is Poisson with parameter $\lambda_1 + \lambda_2$.

7. Let T_1 and T_2 be independent exponentials with parameters λ_1 and λ_2, respectively. Find the density function of $T_1 + T_2$.

8. Suppose that X and Y are independent discrete random variables and each assumes the values 0, 1, and 2 with probability $\frac{1}{3}$ each. Find the probability mass function of $X + Y$.