Reviewer's report on: "$3X + 1$ Conjecture Proved!" by B. Cawaling

This manuscript asserts a proof of the $3x + 1$ conjecture. If true, the result would certainly merit publication.

The paper asserts two main results on this question : (1) a proof that there is only one cycle of the ("preferred") $3x+1$ map on the positive integers. (2) a proof that there are no divergent trajectories on the positive integers

The proofs of both statements above are faulty, as given below. In this reviewer's opinion, the proofs are unfixable, without significant new ideas being added.

Detailed Comments

1. (mistake, p. 11 top) An argument is presented that there can be no cycle on the positive integers except $(1, 2)$,. It proceeds by analyzing the form of a solution to a (rational) cycle and assuming it is a cycle of positive integers, and viewing the smallest element in the cycle, which is necessarily an odd integer.

   The author finds an equation for a term b in a (rational) cycle, given in equations [1] and [2] (depending on whether the last element of the cycle is even or odd.) The author compares equation [1] and equation [2], and tries to infer that a solution b to [1] is smaller than a corresponding solution to [2], giving a contradiction (except for the cycle $(1, 2)$.) Unfortunately the (value of the) quantity S in equation [1] will be different than the value of the quantity S in equation [2] (it should be denoted S' to avoid this confusion.) The value of S depends on the exact sequence of even and odd iterates in the cycle. So no inequality can be derived, and the author cannot infer there are no other cycles.

   The author might examine reference [15], where rational cycles are considered.

2. (mistake p. 12, line 8) The author asserts (without proof) that if a function on the positive integers to itself has a single cycle (1, 2) and no other finite cycles, then it cannot have a divergent trajectory.

   This statement is not true: Consider for example the function $f(x) = x - 1$ if x is even, $f(x) = 3x - 1$ if x is odd. This is a function of the type proposed by the author. It has $(1, 2)$ as a cycle, and for starting values $x \geq 3$ all orbits diverge to $+\infty$.

3. (unlocated mistake) The author claims to have a general decision procedure for orbits reaching 1 of a general decision process.

   However, there exists an paper by J. H. Conway, "Unpredictable Iterations", proving such a decision procedure is undecidable (non-computable) for general functions (not necessarily the $3x + 1$ function.)