Author's Refutation of Reviewer's Objections

1. … The author finds an equation for a term *b* in a (rational) cycle, given in equations [1] and [2] (depending on whether the **last element of the cycle** is even or odd). The author compares equation [1] and equation [2], and tries to infer that a solution *b* to [1] is smaller than a corresponding solution to [2], giving a contradiction [except for the cycle (1, 2)]. Unfortunately the (value of the) quantity S in equation [1] will be different than the value of the quantity S in equation [2] (it should be denoted S' to avoid this **confusion**). **The value of S depends on the exact sequence of even and odd iterates in the cycle.** So no inequality can be derived, and the author cannot infer there are no other cycles. …

The "*confusion*" alluded to by the reviewer in his objection is only his own. It is true that **the value of *S* depends on the exact sequence of even and odd iterates in the cycle**. However, it is very clear that I was talking about **the cycle** where b_{min} (*the minimum-valued cycle term — it is not known whether* b_{min} *is odd or even but by comparing cycle-equations* [1] *and* [2]*, it could be concluded that* b_{min} *must be even and so cannot be a positive integer*) is the "*last element of the cycle*" — there is only one definite sequence of even and odd iterates in this b_{min}-last-element-cycle. For example, consider the length-11 cycle **(-17, -25, -37, -55, -82, -41, -61, -91, -136, -68, -34)** that I depicted in Table 1 on page 15 of my *first-submitted* paper — while there is truly a different value for *S* for each different value for the "*last element of the cycle*" *b* (*as delineated in the* 11 *rows of* Table 1), there is only one value for S (*which is* 9452) that yields $b = b_{min}$ = **-136** (*which is an even integer*) as the "*last element of the cycle*" **(-68, -34, -17, -25, -37, -55, -82, -41, -61, -91, -136)** [*that is, row* 9 *of Table* 1].

Therefore, I have indubitably proved that any preferred Collatz 3x+1 sequence in the positive integers domain can only have, at most, a subsequence <(2,1)>.

I do not know how to explain this issue any simpler — considering the very notorious "*unsolvability*" hype hurled at the Collatz 3x+1 problem, I genuinely believed that I wrote my manuscript so that even a good high school student could understand it easily.

2. … The author asserts (without proof) that **<u>if a function on the positive integers to itself has a single cycle (1, 2) and no other finite cycles, then it cannot have a divergent trajectory. This statement is not true</u>**: Consider for example the function f(x) = x - 1 if x is even, f(x) = 3x - 1 if x is odd. This is a function of the type proposed by the author. It has (1, 2) as a cycle, and for starting values x ≥ 3 all orbits diverge to +∞.

Again, this objection is just another of the reviewer's own confusion of my true argument — his concern, presumed "*my assertion*" (*without proof*), and counterexample function *f* are indeed very far removed from my actual contentions in my proffered proof of the Collatz 3x+1 conjecture in the positive integers domain:

From the very definition of the Collatz 3x+1 iteration function T

$$T(n) = \begin{cases} \dfrac{3n+1}{2} & \textit{if} \quad n \equiv 1 \pmod{2} \\[2ex] \dfrac{n}{2} & \textit{if} \quad n \equiv 0 \pmod{2} \end{cases}$$

every Collatz 3x+1 sequence has the form C_n = <*n*, T(*n*), $T^2(n)$, …, $T^k(n)$, …>.

Let *b* = $T^k(n)$ for arbitrary positive integers *n* and *k* — that is, *b* could be the 2nd, 3rd, 4th, *etc.* term of C_n. What I have really asserted is that the following trivially holds:

$T^{-1}(b)$ = T(*b*) always yield the valid *solution-values* *b* = 1 and *b* = 2

where

$$T^{-1}(b) = \begin{cases} \dfrac{2b-1}{3} & \text{(whenever this is a positive integer)} \\[2ex] 2b & \end{cases}$$

— or,

$$\frac{2b-1}{3} = \frac{3b+1}{2} \quad \textit{yields} \quad b = -1 \quad \textit{which is not a valid solution};$$

$$\frac{2b-1}{3} = \frac{b}{2} \quad \textit{yields} \quad b = 2 \quad \textit{which is a valid solution};$$

$$2b = \frac{3b+1}{2} \quad \textit{yields} \quad b = 1 \quad \textit{which is a valid solution};$$

$$2b = \frac{b}{2} \quad \textit{yields} \quad b = 0 \quad \textit{which is not a valid solution}.$$

I am claiming that, just like the fact that any sequence of binary or decimal digits prefixed by a *fractional expansion point* trivially lies between 0 and 1 requires no proof since this is a matter of *mathematical convention* (*that is, a definition*), the fact that $T^{k-1}(n) = T^{k+1}(n)$ trivially always hold for every starting positive integer *n* and some corresponding positive integer *k*, or for each positive integer *k* and some corresponding positive integer starting number *n*, from the very definition of the Collatz 3x+1 function T need not be proved. In other words, the very form of any Collatz 3x+1 sequence warrants that it includes, at least, the periodic subsequence <(2,1)>. Combining this with my assertion in (1) above, then I have properly proved that every Collatz 3x+1 sequence exactly includes the cyclic subsequence <(2,1)>.

It is plain that my above contention is very different from the reviewer's presumed "my assertion" — which is indeed false — and his counterexample function f does not possess my *bone-of-contention* trivial property of T. Because, for $x \geq 3$, every sequence $F_x = <x, f(x), f^2(x), f^3(x), \ldots>$ is respectively monotonic increasing on the alternating "*even powers*"-of-f and "*odd powers*"-of-f terms then, as a matter of computable generalization, it indeed trivially follows that F_x is divergent. Thus, the cycle (2,1) of f is limited only to F_1 and F_2 which are not subsequences of any sequence F_x with x > 2 inasmuch as neither 1 nor 2 is a *portal cycle-term*. That the reviewer's objection is indeed without merit is manifestly covered in the first paragraph of the third case scenario of my "fundamental logic of our very simple and general approach" summary on page 6 of my *first-submitted* paper.

I resolutely believe that I have truly established that there is no nontrivial [*that is, other than* <(2,1)>] cyclic subsequence for any Collatz 3x+1 sequence — thus, the reviewer's objection on this issue is indeed without merit. On the other hand, I would acquiesce to the reviewer's objection to my somewhat "*heuristic argument*" and that I may not have "*rigorously proved*" the *non-existence* of a "*divergent*" Collatz 3x+1 sequence. In this case, a longer version (*with specific arguments supporting the claim that there are no divergent Collatz 3x+1 sequences in the positive integers domain*) of my manuscript "Collatz 3x+1 Conjecture Proved!" is downloadable from the Internet at http://www.geocities.com/bencawaling/collatz_conjecture_proved_long_version.pdf.

Therefore, I move for reconsideration of the rejection by LMS- JCM for publication of my *first-submitted* paper and I most respectfully submit for further peer review or refereeing my aforementioned revised long version paper "Collatz 3x+1 Conjecture Proved!".

3. … The author claims to have a general decision procedure for orbits reaching 1 of a general decision process.

Once the reviewer comprehends my refutations above to his first 2 objections, then he would also immediately realize that I am simply asserting that the very definition of the Collatz 3x+1 iteration function guarantees that every Collatz 3x+1 sequence C_n always includes exactly the subsequence <(2, 1)>.

> However, there exists a paper by J. H. Conway, ”Unpredictable Iterations”, proving such a decision procedure is undecidable (non-computable) for general functions (not necessarily the 3x + 1 function).

A clarification of these "*entscheidungsproblem*", "*computability*" and "*computational complexity*" issues are covered in my *work-in-progress* appendix (*downloadable from* http://www.geocities.com/bencawaling/collatz_conjecture_proved_long_version_appendix.pdf) to the long version of my "Collatz 3x+1 Conjecture Proved!" manuscript.

When viewed from their beginning *non-periodic* iterates, all the *Collatz 3x+1 sequences* in the positive integers domain defied every attempt of *computable generalization* — hence, the unjustified clamors of "*unsolvability*" of the very simply stated *Collatz 3x+1 conjecture*. However, a straightforward look at the ending periodic terms provides some *clear-cut computable generalization* that readily rules out the *existential-possibility* of an *all-positive-integer-terms Collatz 3x+1 sequence* with *cyclic-subsequence* other than C_2 = <(2, 1)> or which does not converge to C_2 — hence, quickly proving the *Collatz 3x+1 conjecture*. Also, by transforming the *branching Collatz 3x+1 iteration function* into its equivalent streamlined *non-branching* iteration function in the domain of all odd natural numbers, an equally tenable *constructive-inductive solution-approach* is found.

The existence of a very simple proof of the *Collatz 3x+1 conjecture* has truly pervasive ramifications in the *theory of computation* — that is, in *computability theory* as well as in *computational complexity theory*. The *computability* issue and the "*computational complexity classes*" categorization do not appropriately apply to the mathematical problem but distinctly to each of its diverse proposed *solution-methods* in their own appropriate domains — for instance, the *Collatz 3x+1 problem* in the positive integers domain is readily solvable (*by the simple and general solution-technique that I have aptly demonstrated in my paper*) in spite of its having numerous unsuccessful proposed *solution-approaches* (*please refer back to those cited in the introductory section of my paper*) or that the *brute-force solution-method* of evaluating individually its 2^{u+1} ($u \to \infty$) *cycle-equations* to find all its valid cyclic subsequences and to rule out any "*divergent*" sequence is truly "*exponentially computationally complex*" or "*undecidable*".

As explained in details in the a*ppendix* the solvability of a mathematical problem is determined by the *computable genaralizability of its specified domain* — it is as simple as that and this fact (*that is, only problems with computably generalizable domain are suitable subjects of mathematical discourse*) is already generally known which is why *mathematical induction* is explicitly included as one of the *Peano arithmetic axioms*. Indeed, any given mathematical problem is solved by establishing the complete elements of the *solution-set for its unknown variables*. It is purely the absence of some known suitable *computable generalization* (*which trivially yields the solution-set*) of its specified *domain* that exposes any *well-posed non-self-contradictory* mathematical problem to hasty misbranding as "*computationally complex*" or "*undecidable*".

The very simple and general *solution-approach* involving *computable generalization of their respective suitable domains* that we just established cannot be encapsulated in some computer program (*Turing-machine computability or any other future models of computation including hypercomputation*) or it does not require any computing space or time resource at all. Whither *entscheidungsproblem*? *Church-Turing thesis*? $\boldsymbol{P} \neq \boldsymbol{NP}$? *Artificial Intelligence*?

- ► Using the reviewer's counterexample function f, I claim that my above argument for the truth of his assertion that other than the fully cyclic sequences F_1 and F_2 there are only divergent sequences F_x for all starting numbers x could not be captured by any model of computation because no *abstract machine* could discern that it is working with an infinite domain and make some *computable generalization* from them. This is the very same issue with the inherent "*undecidability*" of *Turing's computer-program-halting-problem*.