8ª LISTA - VOLUME DE SÓLIDOS DE REVOLUÇÃO

1) $f(x) = -0{,}0944x^2 + 3{,}4 \qquad -5{,}5 \leq x \leq 5{,}5$

$$\left(-\frac{59}{625}x^2+\frac{17}{5}\right)^2=\frac{3481}{390625}x^4-\frac{2006x^2}{3125}+\frac{289}{25}$$

$$V=\pi\int_{-\frac{11}{2}}^{\frac{11}{2}}(f(x))^2\,dx$$

$$V=\pi\int_{-\frac{11}{2}}^{\frac{11}{2}}\left(\frac{3481x^4}{390625}-\frac{2006x^2}{3125}+\frac{289}{25}\right)dx$$

$$V=\pi\left[\left(\frac{3481x^5}{1953125}-\frac{2006x^3}{9375}+\frac{289x}{25}\right)\Bigg|_{-\frac{11}{2}}^{\frac{11}{2}}\right]$$

$$V=\pi\left[\left(\frac{3481\left(\frac{11}{2}\right)^5}{1953125}-\frac{2006\left(\frac{11}{2}\right)^3}{9375}+\frac{289.\frac{11}{2}}{25}\right)-\left(\frac{3481\left(-\frac{11}{2}\right)^5}{1953125}-\frac{2006\left(-\frac{11}{2}\right)^3}{9375}+\frac{289.\left(-\frac{11}{2}\right)}{25}\right)\right]$$

$V = \pi\,(36{,}88 + 36{,}88) =$

$V = 3{,}14 \,.\, 73{,}76$

$V \cong 231{,}62$ polegadas cúbicas

a) $f(x) = \dfrac{1}{\cos x}$ **entre** $x = 0$ **e** $x = \dfrac{\pi}{4}$

$$[f(x)]^2 = \left(\frac{1}{\cos x}\right)^2 = \frac{1}{\cos^2 x}$$

$$\mathrm{V} = \pi \int_0^{\frac{\pi}{4}} [f(x)]^2 dx \qquad V = \pi \int_0^{\frac{\pi}{4}} \frac{dx}{\cos^2 x}$$

$$V = \pi \left(tg.x \;\Big]_0^{\frac{\pi}{4}} \right) \qquad V = \pi \left(tg\frac{\pi}{4} - tg0 \right) \qquad \mathrm{V} = \pi \,\mathrm{uv}$$

2b) $f(x) = \mathbf{sen}\, x$ $\qquad x = 0$ **e** $x = \dfrac{\pi}{4}$

$$[f(x)]^2 = (\mathrm{sen}.x)^2 = \mathrm{sen}^2 x$$

$$V = \pi \int_0^{\frac{\pi}{4}} \mathrm{sen}^2 x.dx \qquad V = \pi \left(\frac{x - \mathrm{sen}.x.\cos.x}{2} \Big]_0^{\frac{\pi}{4}} \right)$$

$$V = \pi \left[\frac{\frac{\pi}{4} - \mathrm{sen}\frac{\pi}{4}.\cos\frac{\pi}{4}}{2} - \left(-\frac{\mathrm{sen}\,0.\cos 0}{2} \right) \right]$$

$$V = \pi \left(\frac{\frac{\pi}{4} - \frac{\sqrt{2}}{2}.\frac{\sqrt{2}}{2}}{2} \right) \qquad V = \pi \left(\frac{\frac{\pi}{4} - \frac{1}{2}}{2} \right) \qquad V = \frac{\pi^2 - 2\pi}{8}$$

2c) $f(x) = \cos x$ **x = 0 e x =** $\frac{\pi}{4}$

$$[f(x)]^2 = \cos^2 x$$

$$V = \pi \int_0^{\frac{\pi}{4}} \cos^2 x.dx \qquad V = \pi \left(\frac{x + \text{sen } x.\cos x}{2} \Big]_0^{\frac{\pi}{4}} \right)$$

$$V = \pi \left[\frac{\frac{\pi}{4} + \text{sen}\frac{\pi}{4}.\cos\frac{\pi}{4}}{2} - \frac{0 + \text{sen}\,0.\cos 0}{2} \right]$$

$$V = \pi \left(\frac{\frac{\pi}{4} + \frac{\sqrt{2}}{2}.\frac{\sqrt{2}}{2}}{2} \right) \qquad V = \frac{\pi^2 + 2\pi}{8}$$

2d) $f(x) = x.e^{\frac{x}{2}}$ $x = 0$ e $x = 1$

$$[f(x)]^2 = x^2.e^x \qquad V = \pi \int_0^1 x^2 e^x dx$$

integração por partes:

$$\int udv = uv - \int vdu$$ tomando u = x^2 du = $2x$ v = e^x $dv = e^x$

sendo assim: $\int_0^1 x^2.e^x dx = x^2.e^x \Big]_0^1 - \int_0^1 e^x.2x$

$$= x^2.e^x \Big]_0^1 - 2.\left[\int_0^1 xe^x dx \right] = x^2.e^x \Big]_0^1 - 2.\left[xe^x \Big]_0^1 - \int_0^1 e^x dx \right] =$$

$$= x^2.e^x\Big]_0^1 - 2.\left[xe^x\Big]_0^1 - \int_0^1 e^x dx\right] =$$

$$V = \pi\left[x^2e^x\Big]_0^1 - 2\left(xe^x\Big]_0^1 - e^x\Big]_0^1\right)\right]$$

$$V = \pi\{e - 2[e - (e-1)]\}$$

$$V = \pi(e-2)$$

2e) $f(x) = x^{\frac{1}{2}}.e^{\frac{x}{2}}$ **$x = 1$ e $x = 2$**

$$[f(x)]^2 = xe^x$$

$V = \pi \int_1^2 xe^x$ tomando u = x du = 1dx v = e^x dv = e^xdx

temos:

$$V = \pi \int_1^2 xe^x dx = \pi\left[xe^x\Big]_1^2 + \int_1^2 e^x dx\right]$$

$$V = \pi\left[xe^x\Big]_1^2 - e^x\Big]_1^2\right]$$

$V = \pi [(2e^2 - e) - (e^2 - e)]$

$V = \pi [2e^2 - e - e^2 - e]$ $V = \pi e^2$