Ch05: Risk and return

- (Total) risk is quantified by the variance or the std dev of returns, not prices. Reason: prices are distributed *log-normally* while returns are distributed *normally*.
- Return ≡ Δ in value of an asset plus any cash distribution over a given period, expressed as a % of the initial value.
- One-period return is estimated as:

  $$r_t = \left(\frac{V_t - V_{t-1} + C_t}{V_{t-1}}\right)*100\%$$ where

  V ≡ value; C ≡ cash distributions or dividends; and t ≡ time index

E.g., $$r_1 = \left(\frac{V_1 - V_0 + C_1}{V_0}\right)*100\%$$

$$r_2 = \left(\frac{V_2 - V_1 + C_2}{V_1}\right)*100\%$$

- When V_t denotes stk price and C_t denotes cash dividends per share, then $[(V_t - V_{t-1})/V_{t-1}]$ is called the *capital gain/loss yield* whereas the (C_t/V_{t-1}) is called the *dividend yield*.

<u>Example</u>: You paid $75 for one share of MSFT a year ago. You received $2/share of cash dividend today, and you sold MSFT at $100/share today. Then,
r_1 = [(100 - 75 + 2)/75] * 100%
= [(25/75) + (2/75)] * 100%
= 33.33% + 2.67% = 36%
The 33.33% is called capital gain yield while the 2.67% is called dividend yield.

- *Valuation*, aka *pricing*, is the process that links risk and return to determine the actual worth of an asset which can be a bond, a common stk, or a preferred stk.

- A *portfolio* is a combination, or collection, or group of assets.

- Generally, there're 3 attitudes toward risk: *risk averse*, *risk neutral* and *risk seeking/loving*.
- Most fin. mgrs, or individuals for that matter, are or can be assumed to be risk averse.

Risk of a single asset

- Risk of a single asset is quantified by the variance or the std dev of the returns of the asset.
- Another measure, called coefficient of variation (CV), captures the risk per unit return. Specifically,
- $CV = \frac{\sigma_k}{k}$ where σ_k = std dev of returns and k = expected or avg return of an asset.
- Risk and return can be divided into 2 types: *ex post* (historical) and *ex ante* (expected).

***Ex post*, or historical, or average risk and return**

Year, t	k_t
1	10%
2	8
3	12
4	16
5	14

Avg return, $\bar{k} = \frac{1}{T}\sum_{t=1}^{T} k_t$ = (1/5)*(10+8+12+16+14) = 12%

Variance of return, $s^2 = (\frac{1}{T-1})\sum_{t=1}^{T=5}(k_t - \bar{k})^2$ = $(1/4)*[(10-12)^2 + (8-12)^2 + \ldots.. + (14-12)^2] = 10\%^2$

s = 3.16%

- In the above e.g., we assumed that the observations came from a sample. If the observations were obtained from a population, then [1/(T-1)] is replaced by (1/T) and s is replaced by σ.

Ex ante, or expected risk and return

State, i	**Probability for state i to occur, P_i**	**Expected return under state i, k_i**
Depression	.05	-20%
Recession	.25	-5
Normality	.30	8
Mild growth	.25	12
Boom	.15	32
	$\sum P_i = 1.00$	

Expected return, $\bar{k} = \sum_{i=1}^{i=N} P_i * k_i$ = .05(-20) + .25(-5) + .3(8) + .25(12) + .15(32)

= -1 - 1.25 + 2.4 + 3 + 4.8

= 7.95%

Variance of expected return, $\sigma^2 = \sum_{i-1}^{i=N} P_i * (k_i - \bar{k})^2$

$= .05(-20-7.95)^2 + .25(-5-7.95)^2 + .3(8-7.95)^2 + .25(12-7.95)^2 + .15(32-7.95)^2$

= 39.060125 + 41.925625 + .00075 + 4.100625 + 86.760375

$= 171.8475\%^2$

Std dev of returns, σ = 13.11%

- The numerals .05, .25, .30, .25, and .15 collectively form the *probability distribution*.
- The P_i and/or k_i can be changed to reflect the best and the worst scenarios. The process of doing so is called *sensitivity analysis*. The arithmetic difference between the best-case k_i and the worst-case k_i is called the *range*.
- When sensitivity analysis is iterated numerous times to reflect a more comprehensive spectrum of possible outcomes, it is called *simulation*.
- Simulation is most efficiently performed by the computer.

Risk and time

Risk as measured by σ^2 or σ increases with time into the future.
The longer into the future, the higher is the σ^2 or the σ of the expected return, k. This is be'cos variability of returns resulting from increased forecasting errors increases with yrs into the future.

Risk of a portfolio

- The purpose of forming a portfolio is to diversify away some risk of individual assets.
- The (total) risk of an asset, as measured by σ^2, can be decomposed dichotomously as:

Total risk =	*nondiversifiable risk*, aka *mkt risk*, aka *systematic risk*	*+ diversifiable risk*, aka *unique risk*, aka *unsystematic risk*, aka *idiosyncratic risk*

- When an *efficient* portfolio is formed, diversifiable risk $\rightarrow$ 0.
- An efficient portfolio is one that either:
    - maximizes return for a given level of risk, or

- minimizes risk for a given level of return

- The statistical measure called *correlation* forms the crux of diversification with efficient portfolio formation.
- Specifically, correlation is measured by *correlation coefficient*, denoted as ρ_{ij} or r_{ij}, and ρ_{ij} or r_{ij} lies between -1, thru 0, to +1.
- For efficient portfolio formation, ρ_{ij} that is more negative is preferred to ρ_{ij} that is less negative, or even positive, i.e., ρ_{ij} = -.8 is preferred to ρ_{ij} =.-05 which in turn is preferred to ρ_{ij} = .20.
- Some technical jargons:
- When ρ_{ij} = -1, we say that assets i and j returns are negatively perfectly correlated;
- When ρ_{ij} = +1, we say that assets i and j returns are positively perfectly correlated, and
- When ρ_{ij} = 0, we say that assets i and j returns are not correlated.

Insert Fig. 5.3 here

Return of a portfolio, R_p.

$$R_p = \sum_{i=}^{i=N} w_i R_i$$

You invest $4k in MSFT, $3k in GE, and $3k in WMT. Their returns are 15%, 12%, and 10% respectively. What is the return of your portfolio?

R_p = .4(15%) + .3(12%) + .3(10%) = 6 + 3.6 + 3 = 12.6%

- The risk of a 2-asset portfolio is estimated as:

$$\sigma_p^2 = w_i^2\sigma_i^2 + w_j^2\sigma_j^2 + 2w_i w_j \rho_{ij}\sigma_i\sigma_j$$

where w_i and w_j are the proportion of one's wealth that is invested in assets i and j respectively.

- For example, if one has $10k, and one invests $4k in asset i and the remaining $6k in asset j in forming a 2-asset portfolio, then $w_i = .40$ and $w_j = .60$.
- Remember that: $\sum_{i=1}^{i=N} w_i = 1$ is called the *budget constraint*.

Some brief recaps on correlation coefficient:

yr	x	y	z	x - xbar	y - ybar	z - zbar	(x-xbar)(y-ybar)	(x-xbar)(z-zbar)
1	8	16	8	-4	4	-4	-16	16
2	10	14	10	-2	2	-2	-4	4
3	12	12	12	0	0	0	0	0
4	14	10	14	2	-2	2	-4	4
5	16	8	16	4	-4	4	-16	16
avg	12	12	12				∑=-40	∑=+40
stdev	3.16	3.16	3.16					

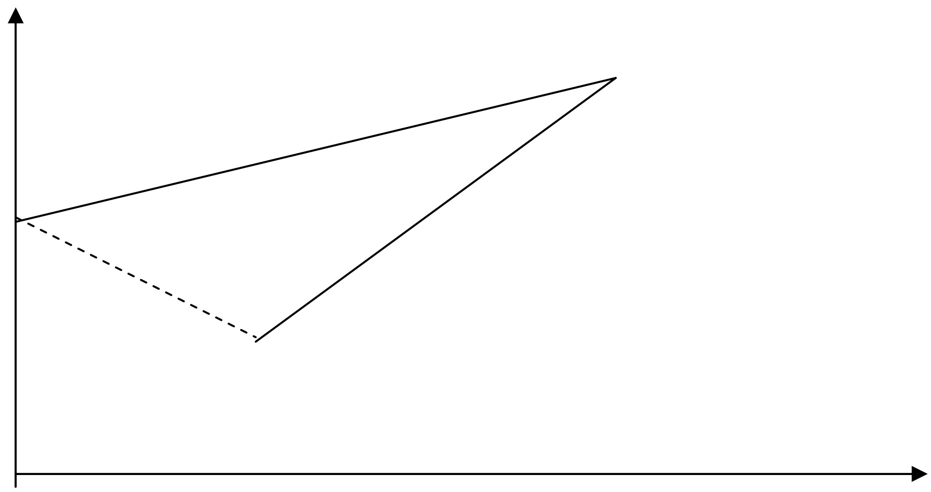
$$\rho_{xy} = \frac{\frac{1}{N-1}\sum_{1}^{N}(x-\bar{x})(y-\bar{y})}{s_x * s_y} = \frac{(\frac{1}{5-1})(-40)}{\sqrt{10}*\sqrt{10}} = -1$$

$$\rho_{xz} = \frac{\frac{1}{N-1}\sum_{1}^{N}(x-\bar{x})(z-\bar{z})}{s_x * s_z} = \frac{(\frac{1}{5-1})(+40)}{\sqrt{10}*\sqrt{10}} = +1$$

Of course, when N is used instead of (n-1), then σ will replace s.

More on Portfolio Diversification and Correlation Coef.

Consider two assets, A and B, whose expected return and risk are shown below:



- When ρ_{AB} = +1, then efficient portfolios lie on the straight line joining points A and B.
- When ρ_{AB} = -1, then efficient portfolios lie on the straight line joining B to Z. The portfolios along Z to A are said to be ***inefficient***.
- When -1 < ρ_{AB} <+1, then efficient portfolios lie on the solid-line parabolic arc.
- A portfolio can be formed, from any two component assets, by adjusting the weights of one's wealth invested in each of the component assets. The adjustment results in movement along the parabolic arc.
- Theoretically, an ∞ # of portfolios can be formed using any 2 assets.
- The only way to form a risk-free (σ_p = 0) 2-asset portfolio is for ρ_{AB} = -1.
- When ρ_{AB} = +1, the least risky 2-asset portfolio is formed by simply investing 100% in the less risky asset alone.
- In the real world, ρ_{AB} lies within -1 and +1 bounds.

Risk and return: the CAPM (capital asset pricing model)

Universal axiom in finance:
Higher return can only be realized by bearing higher risk.

- When the axiom is violated, we say that the fin. mkt is inefficient, and *arbitrage* (i.e., money machine) opportunity exists.
- When the fin. mkt is inefficient, arbitrageurs will create trading strategies (i.e., buying and selling transactions) quickly to wipe out any arbitrage opportunity.
- Given the opportunity to form efficient portfolios to diversify away the diversifiable risk, the relevant measure for the non-diversifiable risk is now called *the beta coefficient*, β_i.
- Specifically, $\beta_i = \frac{\rho_{im}\sigma_i}{\sigma_m}$

  where ρ_{im} = the correlation coefficient between the returns of the asset i and returns of the *mkt portfolio*.
- The mkt portfolio is a hypothetical portfolio which is supposed to consist of *all assets* in the economy, including human capital.
- Practically, the returns and variance of the mkt portfolio is proxied by *S&P 500 Stk Composite Index*, or other similar stk indices.
- The Center for Research in Security Prices (***CRSP***) in the University of Chicago provides the most authoritative source for data on returns of the mkt portfolio.
- The CAPM is an economic equilibrium model that explicitly spells out the relationship between return and risk.
- In equation form, the CAPM is: $k_i = r_f + \beta_i\left(k_m - r_f\right)$

  where:
  k_i = required return for asset i
  β_i = beta coefficient for asset i;
  k_m = return on mkt portfolio, and;
  r_f = risk-free rate of return, proxied by 3-mth U.S. T-bill interest rate.

- By definition, the mkt portfolio has a beta value of 1.

- Other than capturing the non-diversifiable risk, β_i also shows the sensitivity of a security to the mkt's movement.
- When β_i =1.5, it implies asset i's return is 1.5 times as responsive as that of the mkt.
- When β_i =0.8, it implies asset i's return is 0.8 time as responsive as that of the mkt.
- The (k_m-r_f) part of the CAPM formula is called *the mkt risk premium*.
- The $\beta_i(k_m-r_f)$ part of the CAPM formula is called the *risk premium*.
- The graphical depiction of the CAPM is called the *security mkt line* (SML).

Insert Fig. 5.7 here

Example: Given r_f = 7%; k_m = 11% and β_{gm} = 1.10, find k_{gm}. k_{gm} = 7% + 1.10(11-7)

$= 7\% + 1.10(4) = 7 + 4.4 = 11.4\%$