B O Ğ A Z İ Ç İ   U N İ V E R S İ T Y
E L E C T R I C A L & E L E C T R O N I C   E N G I N E E R I N G   D E P A R T M E N T

E E 3 2 7 - N E T W O R K S   L A B .   R E P O R T

Number of The Experiment : ***5***
Name of The Experiment :***RLC Filters***

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Do not fill in the grading table. It is for the instructor

Grading

General		/20
Data		/30
Discussion		/20
Answers		/30
Total		/100
Delay		
SCORE		/100

1. THE EQUIPMENT USED IN THE LAB

1. HAMEG HM-203-7 Dual Trace Oscilloscope with x10 probe
2. Escort EDM 168A Digital Multimeter
3. Black Star Jupiter 2000 Signal Generator
4. Resistors, capacitors and inductors of different values (see section 2 for these values)

2. THEORY AND METHOD

Theory:

Filters are electric (or electronic) circuits used to obtain output signals within a frequency range. In this experiment, a simple type of filters implemented with energy storage elements, namely capacitors and inductors, is examined. The quality of a filter, i.e. the narrowness of the transition region between the pass-band and stop-band regions, depends on the number of the energy storage elements used in the filter design. This number equals the degree of the filter.

There are essentially 4 types of filters:

1. Low-Pass filters: The output follows the input signal up to the cut-off frequency of the filter. Signals beyond the cut-off frequency are stopped, i.e. filtered out. Ideally, there must be a discontinuity at the cut-off frequency and no transition region between the pass-band and stop-band regions. But practically, this is impossible to obtain.
2. High-Pass filters: These filters allow signals with frequencies beyond the cut-off frequency to appear at the output.
3. Band-Pass filters: Signals with frequencies within a certain range, bandwidth, appear at the output.
4. Band-Stop filters: Signals with frequencies not falling within a range appear at the output.

Cut-off frequencies, or frequencies determining the bandwidth are functions of the element values of the filter circuit. These characteristic frequencies are defined to be the –3dB frequencies where the power is half the input power. Butterworth filters and Tchebyshev filters are two type of filters with specific transfer functions. Although implemented with same types of components, the way the component values are determined results in filters with different characteristics.

In this experiment, 3rd order Butterworth low-, high-, and band-pass filters are examined.

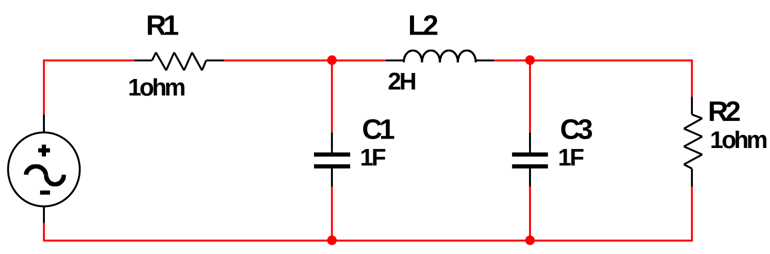
An *n*'th order normalized Butterworth filter has an amplitude function:

$$|H(j\omega)|^2 = B_n(\omega) = \frac{1}{1+\omega^{2n}} \qquad n=1,2,3,...$$

A. 3rd order Butterworth low-pass filter design:

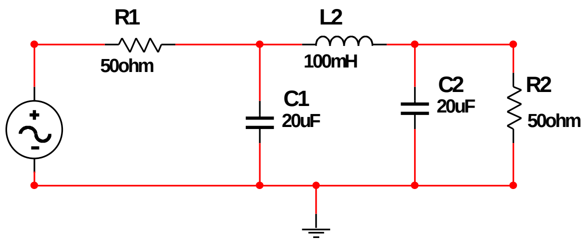
$R_S=R_L=50\Omega$, L=100mH.

For a given cut-off frequency ω, the component values would be $R_S=R_L=1\Omega$, $L=2/\omega$ and $C=1/\omega$.



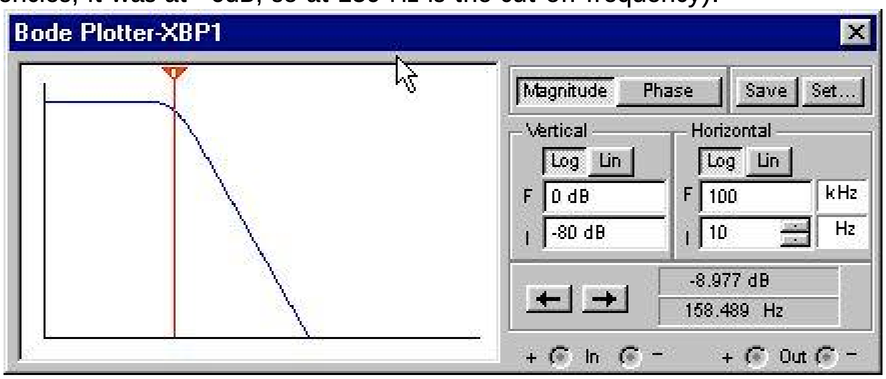
Normalized low-pass filter

After impedance scaling with scaling factor 50, these values would be $R_S=R_L=50\Omega$, $L=50*2/\omega$, and $C=1/50*\omega$. Since it is given that L=100mH, $\omega=100/L=100/100*10^{-3}=1000$ rad/sec. The corresponding cut-off frequency $f=\omega/2\pi=1000/2\pi \cong 159$ Hz. $C=1/50*\omega=20\ \mu F$

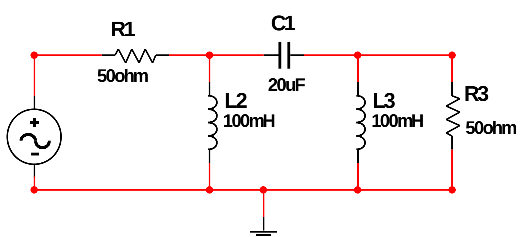


Low-pass filter with f_c=159Hz

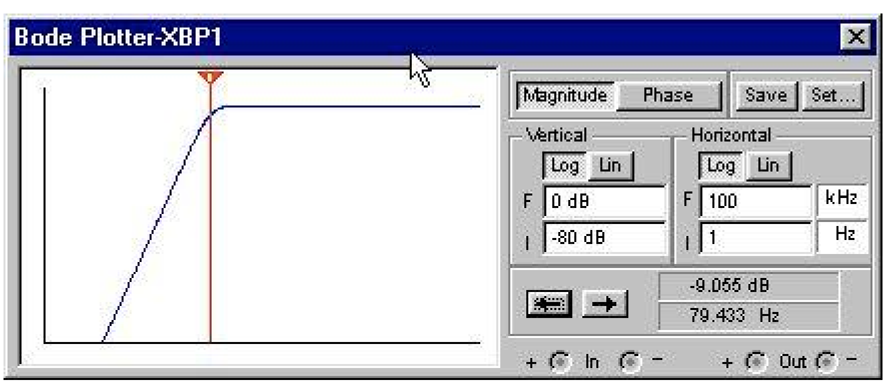
The bode plot of the low pass shows that, at the cut-off frequency, the gain is at –9 dB (at lower frequencies, it was at –6dB, so at 159 Hz is the cut-off frequency).



B. 3rd order Butterworth high-pass filter design:
After doing the transformation and impedance scaling, the component values are C=1/100ω, L=50/ω=100mH ⇒ ω=500 rad/sec, and f=ω/2π=500/2π ≅ 80Hz, C=1/100ω=20μF.
The bode plot of the output signal shows that at the cut-off frequency, the dB level 3db lower than the pass-band dB-level which is –6dB.

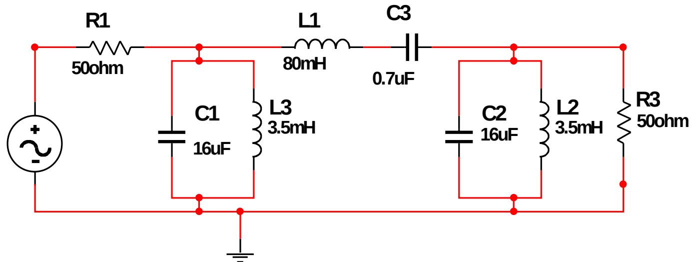


High-pass filter with f_c=80Hz

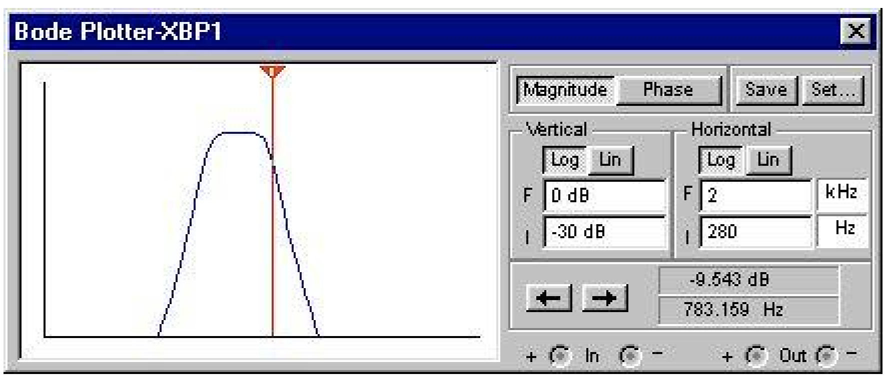


C. 3rd order Butterworth pass-band filter with B=200Hz:

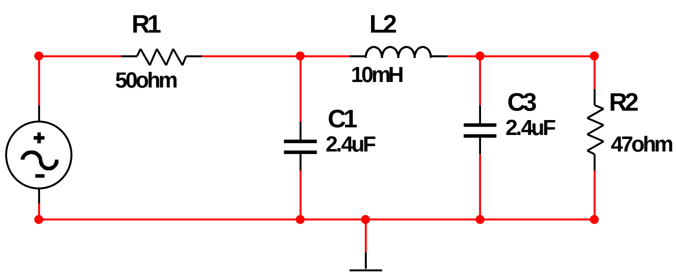
Here the bandwidth is the decisive parameter. The inductor value given for the low-pass filter cannot be applied here, since there are 3 inductors with 2 different component value. So component values are determined to obtain a band-pass filter with center frequency $\omega_c=675*2\pi=4240$ rad/sec and bandwidth B=200Hz. The bandwidth, used in the transformation calculations is $B=200*2\pi \cong 1260$ rad/sec. The capacitors in the normalized low-pass filter are substituted with a parallel capacitor-inductor configuration, $C=1/(k*B)=1/(50*1260) \cong 16\mu F$ and $L=B*k/(\omega_c^2*1) \cong 3.5mH$, where k is the impedance scaling factor k=50. The inductor in the normalized low-pass filter is substituted with a series capacitor-inductor combination, $C=B/(\omega_c^2*2*k) \cong 0.7\mu F$, $L=2*k/B \cong 80mH$.



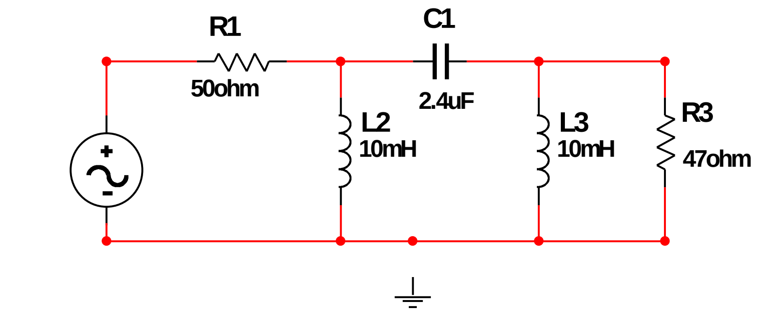
High-pass filter with f_c=675Hz and B=200Hz



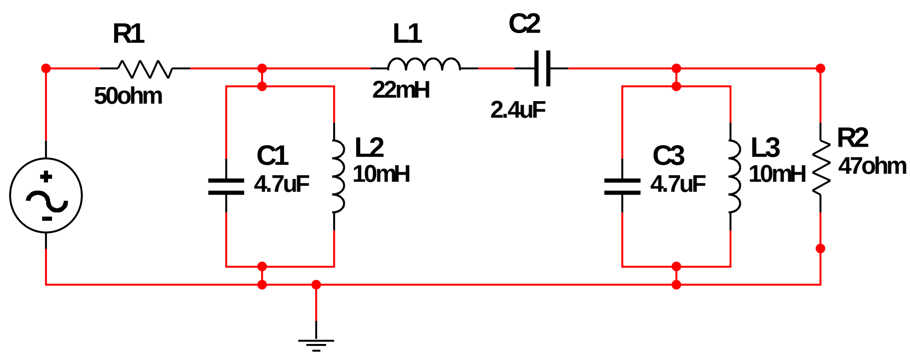
However the following circuits were implemented during the experiment:



Low-pass filter



High-pass filter



Band-pass filter

3. DATA

For all of the filters, V_{peak}=1 V was the amplitude of the input signal.

Frequency (kHz)	0.1	0.2	0.5	1	2	5	10	20
Output (V_{peak})	0.35	0.32	0.3	0.25	0.14	0.05	0.035	0.015
Output (dB)	-9.1	-9.9	-10.46	-12	-17.1	-26	-29.1	-36.5

f_c=1.1 kHz

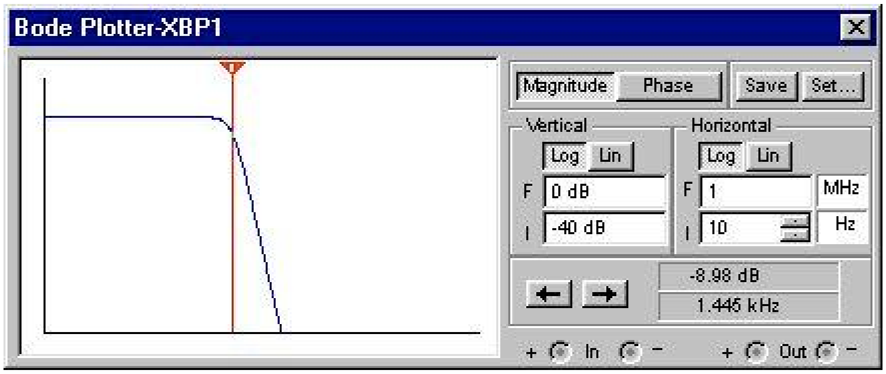
Frequency (kHz)	0.1	0.2	0.5	1	2	5	10	20
Output (V_{peak})	0.02	0.08	0.2	0.55	0.75	0.77	0.8	0.83
Output (dB)	-33.9	-21.9	-13.9	-5.19	-2.5	-2.27	-1.94	-1.62

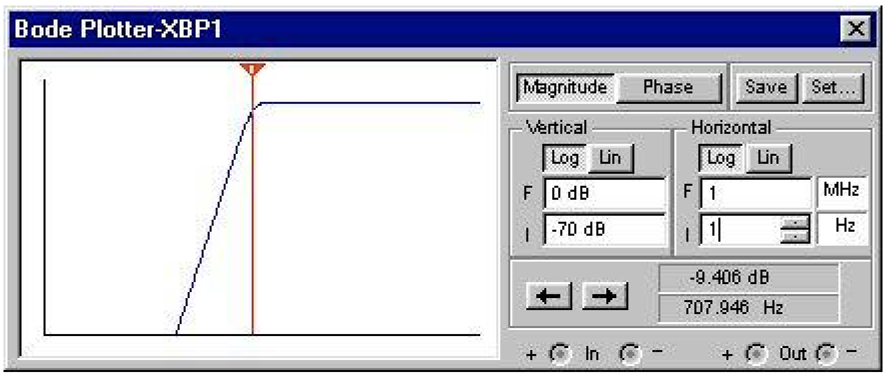
f_c=1.2 kHz

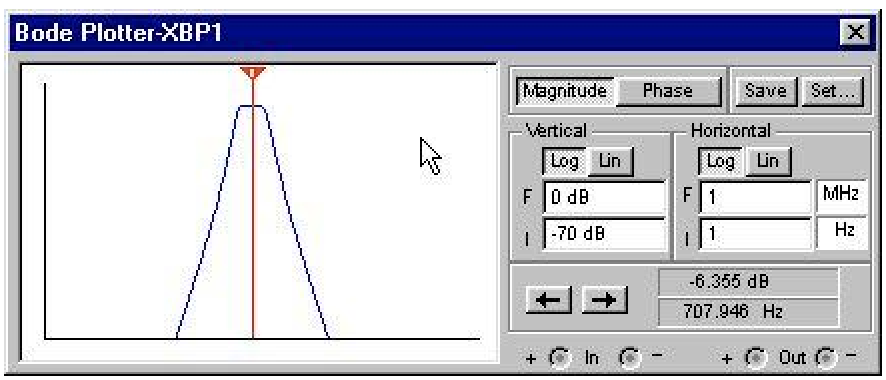
Frequency (kHz)	0.1	0.2	0.5	1	2	5
Output (V_{peak})	0.06	0.125	0.30	0.5	0.055	0.008
Output (dB)	-24.4	-18	-10.5	-6.0	-25.2	-41.9

f_c=675 Hz B=570 Hz (970-400=570)

The graphs depicting the frequency response of the ideal filters:

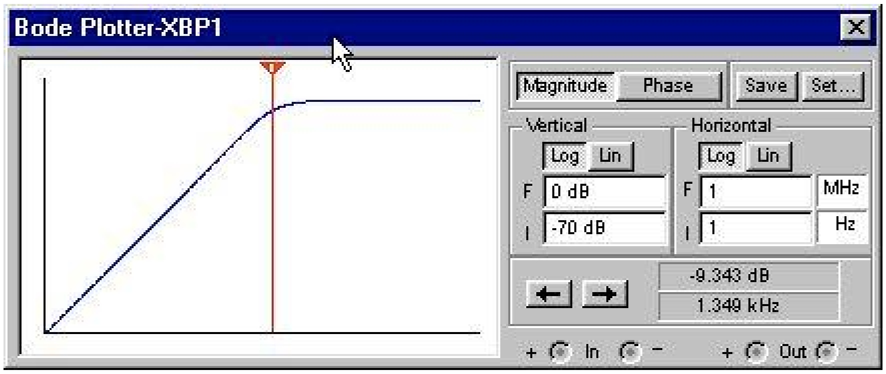




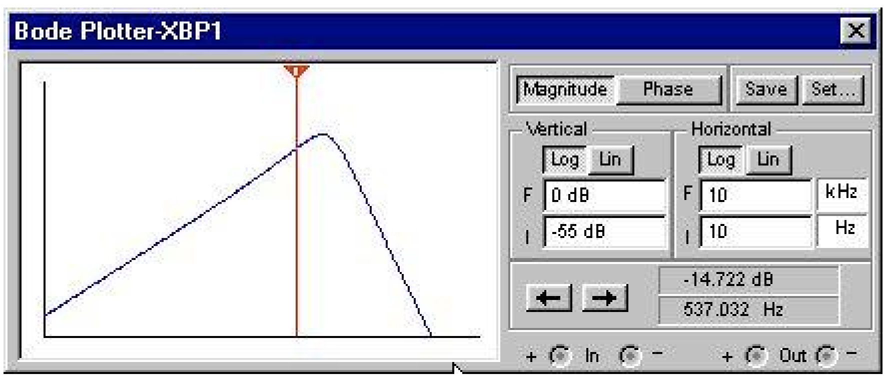


4. ANALYSIS AND DISCUSSION

The experimental values show that the internal resistance of the inductors used affects the outcome considerably. For example, although 1 V_{peak} was applied to the low-pass filter, only $0.35V_{peak}$ was obtained at the output. The reason was that the 10mH inductor had an internal resistance of 90Ω whereas the load was only 47Ω. So much of the voltage drop at low frequencies was across the inductor due to its resistance. Especially, the high-pass filter cut-off frequency deviates considerably from the ideal one. The internal resistance of the inductors, again, affected the results very much. To see this, the internal resistance of the 10mH inductors were added to the filter circuit and the bode plot of the output signal was obtained.



-9.3 dB corresponds to the –3dB level because at higher frequencies the dB level stabilizes at ca. –6.3dB. The cut-off frequency in this case is 1.35 kHz which is much closer to the measured cut-off frequency of 1.2 kHz. A similar correction in the filter model for the band-pass filter results in the following bode plot:



In this bode-plot the bandwidth is 515 Hz (1050-535=515 Hz) which is very close to the measured bandwidth of 570 Hz. Moreover, the shape of the bode plot resembles the one drawn using the experimental result in which the slopes on both sides of the pass band were different. [The bode plot of the ideal (without internal resistances of the inductors added) band-pass filter was given in part 3.] The inclusion of the internal resistances explained much of differences between theoretical and experimental values. Since the determination of the frequency values was realized through calculations made from the signal images on the oscilloscope screen, reading errors must have affected the accuracy of the results to a great extend.

5. ANSWERS TO THE QUESTIONS

1. A 3rd order Butterworth low pass filter of has the transfer function:

$$H(s) = \frac{\omega^3 / 2}{s^3 + 2\omega s^2 + 2\omega^2 s + \omega^3},$$

where ω is the cut-off frequency in rad/sec.
A 3rd order Butterworth high-pass filter has the transfer function:

$$H(s) = \frac{s^3 / 2}{s^3 + 2\omega s^2 + 2\omega^2 s + \omega^3},$$

where ω is the cut-off frequency in rad/sec.

A 3rd order Butterworth band-pass filter has the transfer function:

$$H(s) = \frac{1}{2}\frac{B^3 s^3}{s^6 + 2Bs^5 + 2B^2 s^4 + B^3 s^3 + 3\omega^2 s^4 + 4B\omega^2 s^3 + 2B^2\omega^2 s^2 + 3\omega^4 s^2 + 2B\omega^4 s + \omega^6},$$

where ω is the central frequency and B is the bandwidth in rad/sec.

A 3rd order Tchebyshev low-pass filter has the transfer function:

$$H(s) = 1.25*10^8 \frac{\omega^3}{(3349s + 1000\omega)(298061s^2 + 8.9*10^4\omega s + 2.5*10^5\omega^2)},$$

where ω is the cut-off frequency in rad/sec.

A 3rd order Tchebyshev high-pass filter has the transfer function:

$$H(s) = 1.25*10^8 \frac{s^3}{(1000s + 3349\omega)(2.5*10^5 s^2 + 8.9*10^4\omega s + 298601\omega^2)},$$

where ω is the cut-off frequency in rad/sec.

A 3rd order Tchebyshev band-pass filter has the transfer function:

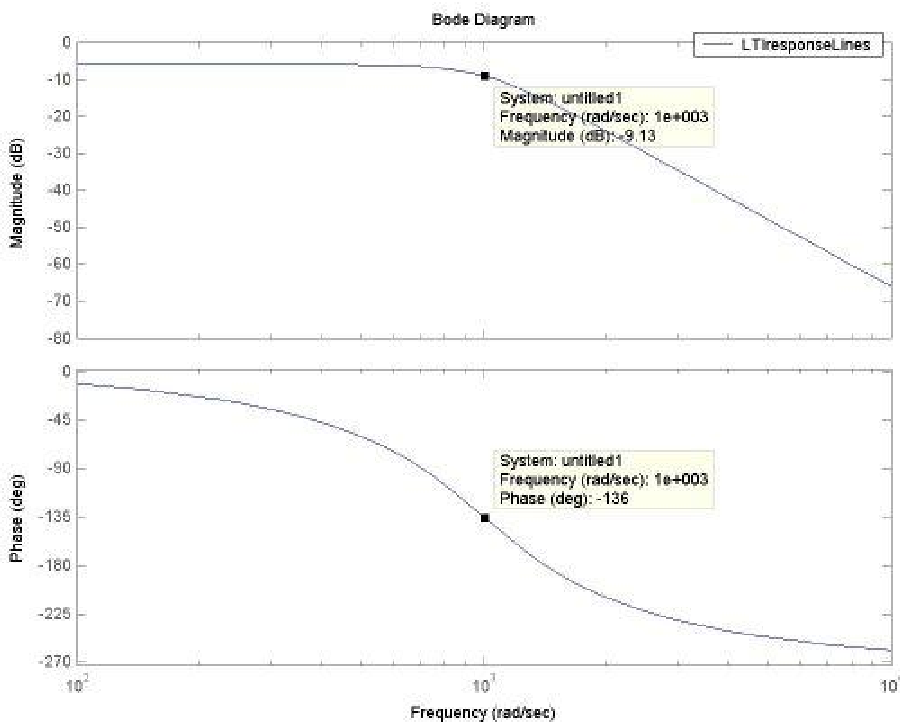
$$H(s) = \frac{1.25*10^8 B^3 s^3}{(3349s^2 + 1000Bs + 3349\omega^2)(298061s^4 + 8.9*10^4 Bs^3 + 2.5*10^5 B^2 s^2 + 596122\omega^2 s^2 + 8.9*10^4\omega^4 Bs + 298061\omega^4}$$

where ω is the central frequency and B is the bandwidth in rad/sec.

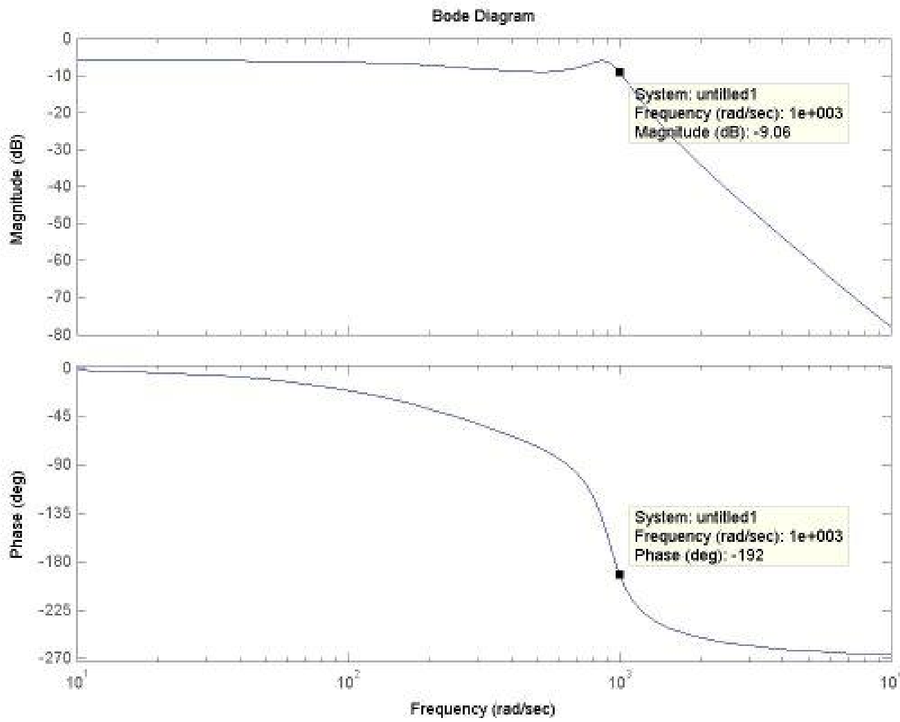
Note: These transfer functions are calculated from the normalized filter model, element values tables and transformation table from the laboratory manual. The software MATLAB is used to perform symbolic calculations including the scaling factor *k*, frequency *w*, *L* and *C* values in the normalized design, bandwidth *B* (for band-pass filters), and the complex variable *s*.

2. All graphs are obtained using MATLAB.

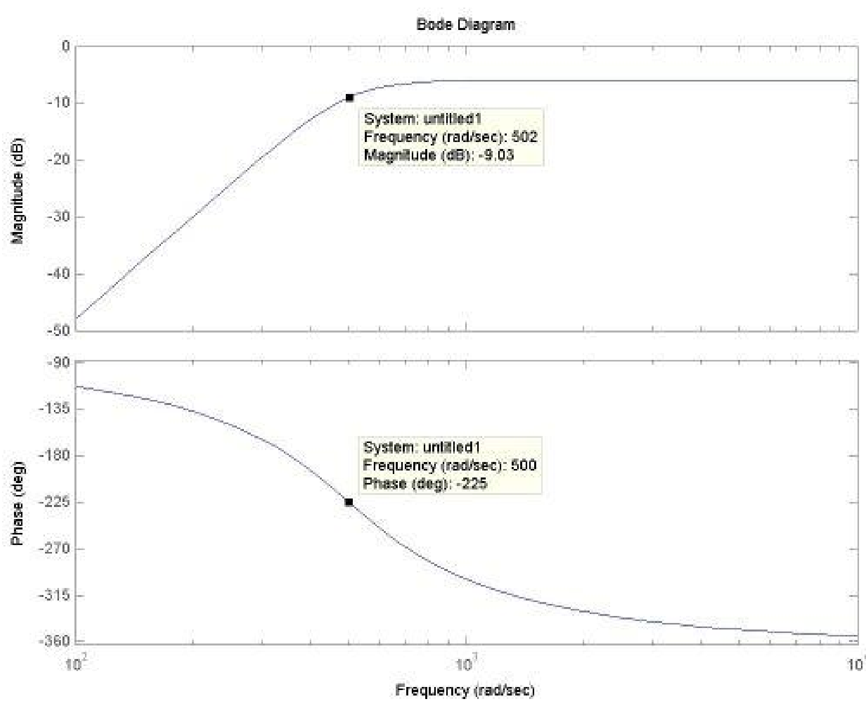
Butterworth LPF (f_c=159Hz ω_c=1000rad/sec)



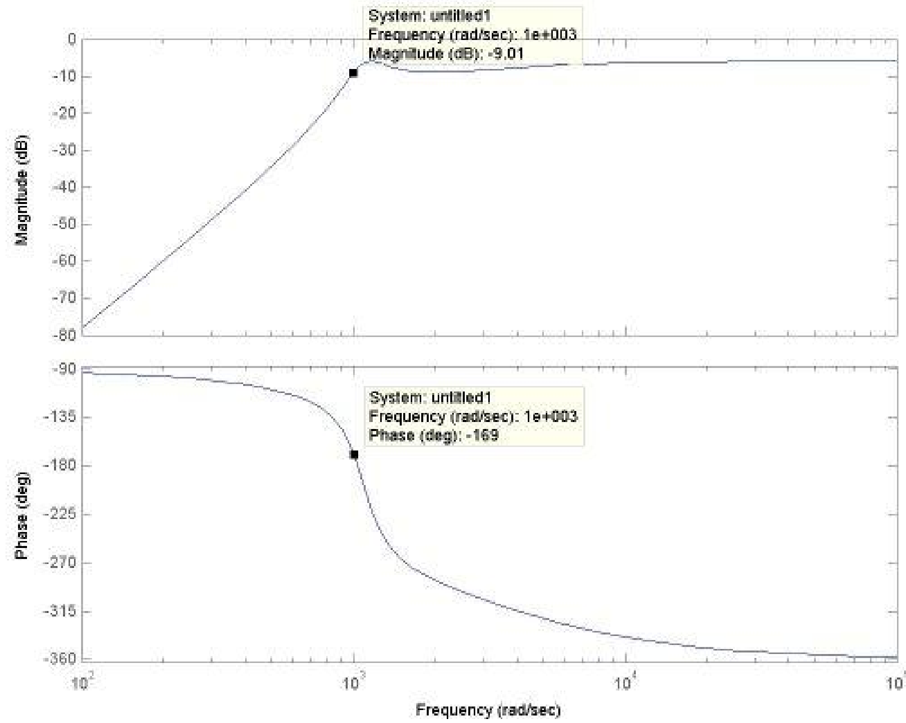
Tchebyshev LPF (f_c=159Hz ω_c=1000rad/sec)



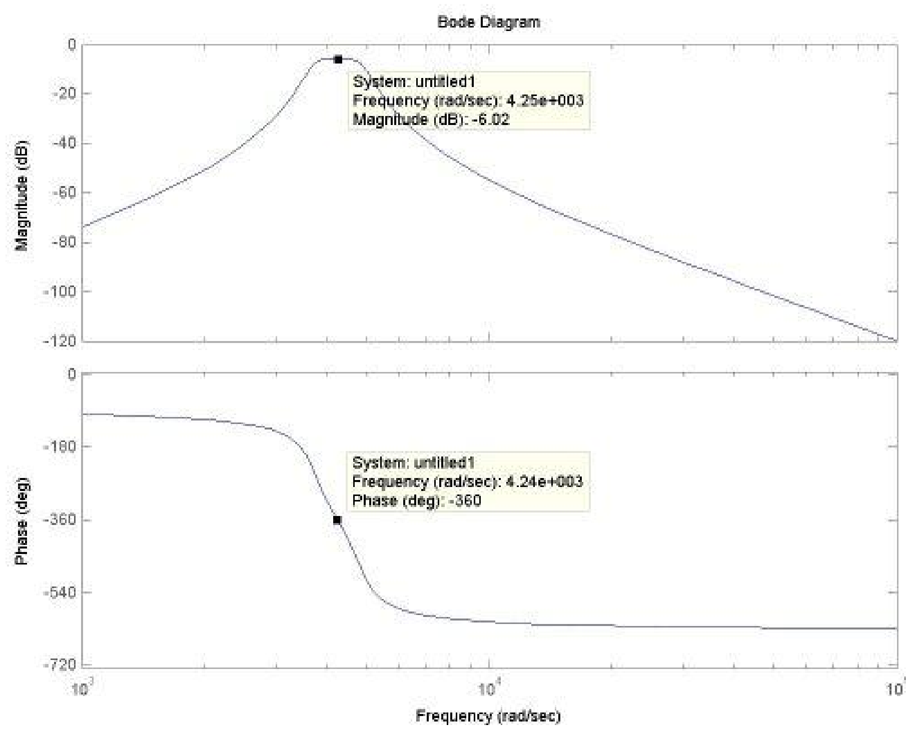
Butterworth HPF (f_c=159Hz ω_c=1000rad/sec)



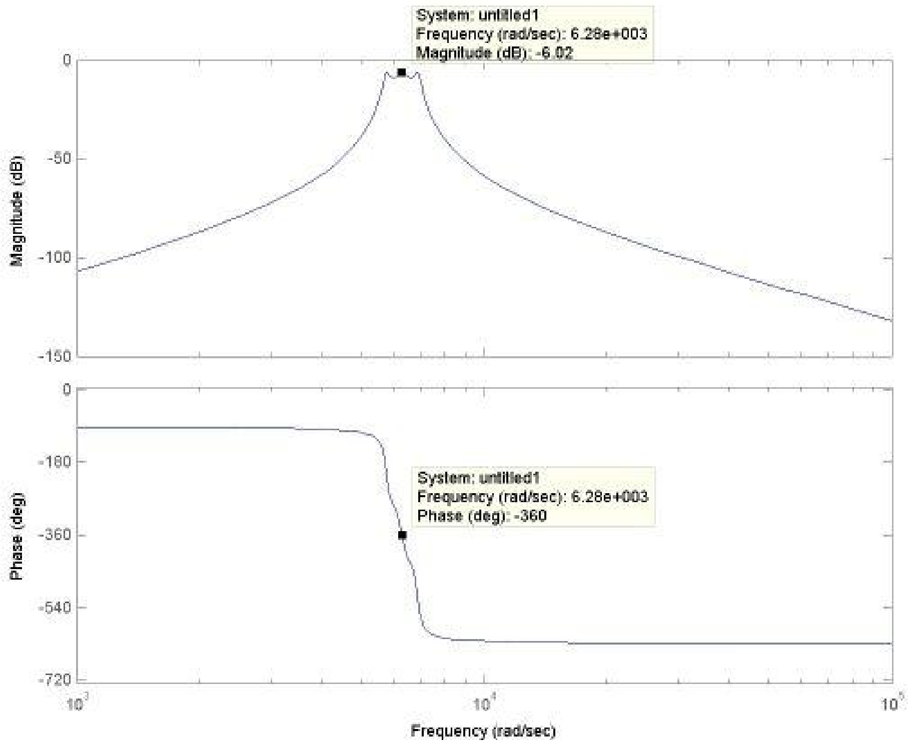
Tchebyshev HPF (f_c=159Hz ω_c=1000rad/sec)



Butterworth BPF (f_c=675Hz ω_c=4240rad/sec, BW=200Hz B=1260rad/sec)



Tchebyshev BPF (f_c=1kHz ω_c=6280rad/sec, BW=200Hz B=1260rad/sec)



Butterworth LPF introduces a -135° phase difference, whereas the Butterworth HPF intreduces a hase shift of 135° Tchebyshev LPF and HPF's introduce asymmetrical phase differences, 190° and 170° respectively. BPF's of both types don't cause any phase difference.

3. All the filters examined in this experiment introduce phase shifts to the output, i.e. their frequency responses are not real but complex functions with nonzero phase angle. This fact is clearly observed in the above graphs, in each of which the lower graph shows the phase angle of the transfer functions versus frequency. The maximum phase difference occurs at the cut-off (or center) frequency. The delay can be given as a function of the frequency as *q(w)*. Then a measure for the linearity of the phase shift would be the group delay which is defined as the rate of change of the phase:

$$\tau(\omega) = -\frac{d\theta(\omega)}{d\omega}\bigg|_{\omega=\omega_C}$$

Appendix:

The 3rd order Tchebyshev Low-Pass Filter with 3dB ripple

For L=100mH, the cut-off frequency and the component values are calculated:

$L=(0.712*k)/\omega \Rightarrow \omega=(0.712*k)/L=(0.712*50)/100*10^{-3}$ =356rad/sec $\Rightarrow f_c=356/2\pi$=57Hz

C= 3.349/(k∗ω) =3.349/(50∗356) = 188μF