

BOĞAZIÇI UNIVERSITY
ELECTRICAL&ELECTRONIC ENGINEERING DEPARTMENT

EE327-ELECTRICAL NETWORK LABORATORY REPORT

Number of The Experiment : 4.....
Name of The Experiment : Resonance Circuits.....

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Delay

Grading

General		/20
Data		/30
Discussion		/20
Answers		/30
Total		/100
Delay		
SCORE		/100

1. THE EQUIPMENT USED IN THE LAB

- 1.1 DC Power Supply
- 1.2 AC Voltmeter
- 1.3 Signal Generator
- 1.4 100Ω Resistor
- 1.5 47 nF Capacitor
- 1.6 22mH (2), 10 mH (1) Inductor
- 1.7 Oscilloscope
- 1.8 Breadboard

2. THEORY AND METHOD

In this experiment we examined the response of series and parallel RLC circuits to various frequencies. The property that reactive elements, i.e. the inductor and capacitor, show different impedances to different frequencies, makes the circuit to have different equivalent impedances for different frequencies. Thus the current and automatically the voltage over the elements also changes with frequency. We've already known that $Z_L = j\omega L$ and $Z_C = -j / \omega C$. Then the total impedance for a series RLC circuit is given by

$$Z_{eq} = R + j(\omega L - \frac{1}{\omega C})$$

From this equation, it is obviously seen that for $\omega = \frac{1}{\sqrt{LC}}$, Z_{eq} has the smallest value,

R . This frequency is called the resonance frequency. Another quantity called the *Quality Factor* is also defined for the circuits. Mathematically, it is given as $Q = f_o / \Delta f$ More information about the quality factor is given in Question 1.

We worked on the series and parallel RLC circuits in the following figures.

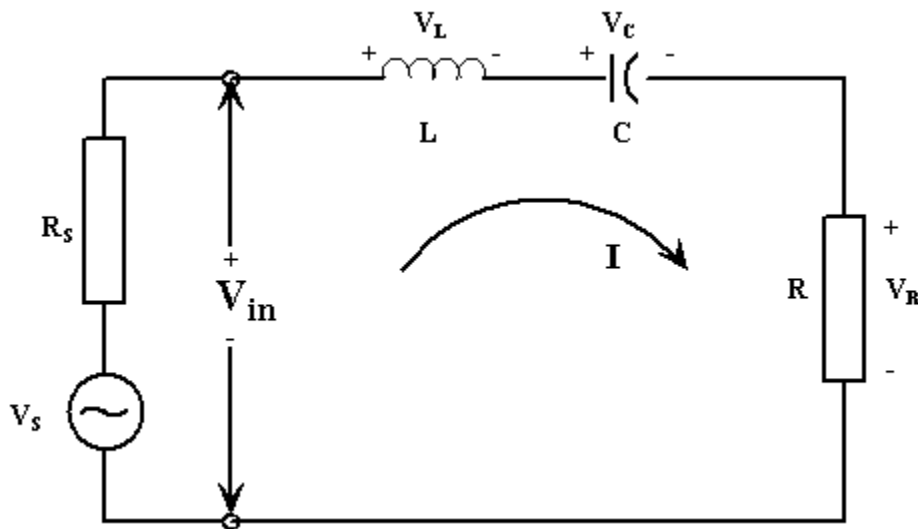


Figure: Series RLC Resonance Circuit

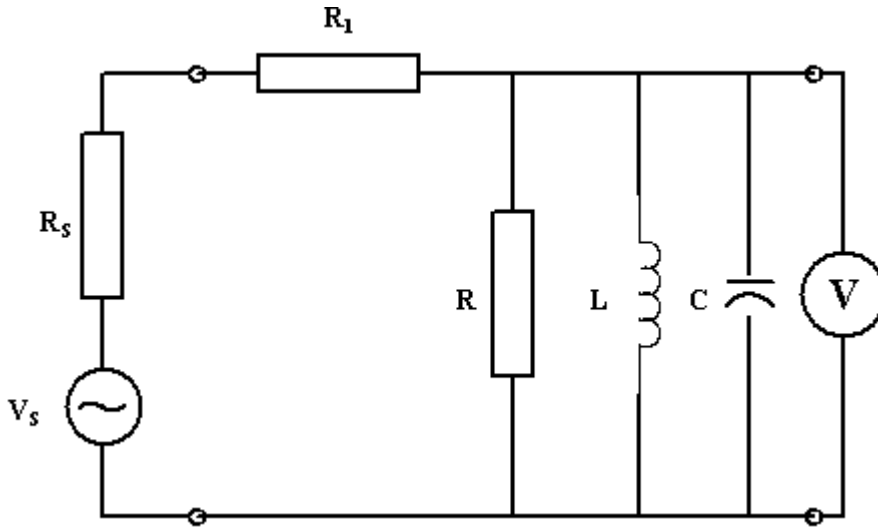


Figure: Parallel RLC Resonance Circuit

For the parallel RLC circuit, in order to obtain a current source, we used $R_1=10K\Omega$. Since $10K\Omega$ is very larger than $R=100\Omega$, I_{R_1} can be assumed to be constant.

3. DATA

SERIES RLC CIRCUIT

X_L	X_C	$ Z = \sqrt{R^2 + X^2}$	$ I = V_{in} / Z$	V_L	V_C	V_R
340Ω	3385Ω	3047Ω	$328\mu A$	$112mV$	$1.11V$	$32.8mV$

Table: Calculated values for $V_{in} = 1 V_{RMS}$ for the series RLC circuit

Frequency (kHz)	0.2	0.5	1	2	5	10	20
Voltage(mV)	6.1	15.5	35.3	103	76	48.3	27.1
Voltage(dB)	-44.3	-36.2	-29	-19.7	-22.4	-26.3	-31.3
Current(mA)	0.062	0.159	0.357	1.080	1.044	0.487	0.275

Table: Voltage and current measurements for the series RLC circuit with $(V_{in})_{RMS} = 1V$

$$f_0 = 2.97KHz \quad V_0 = 342mV = -9.4dB \quad I_0 = 3.64mA$$

$$\text{Half Power Frequencies: } f_1 = 2.66KHz \quad f_2 = 3.44KHz$$

$$BW = 0.78KHz \quad Q = 2.97KHz / 0.78KHz = 3.81$$

The half power frequencies are measured by continuously changing the source frequency and observing the frequency for which V_R falls down to $1/\sqrt{2}$ multiple of resonant voltage V_O .

Below I have shown the theoretical graphs for the series RLC circuit.

Bode Diagrams

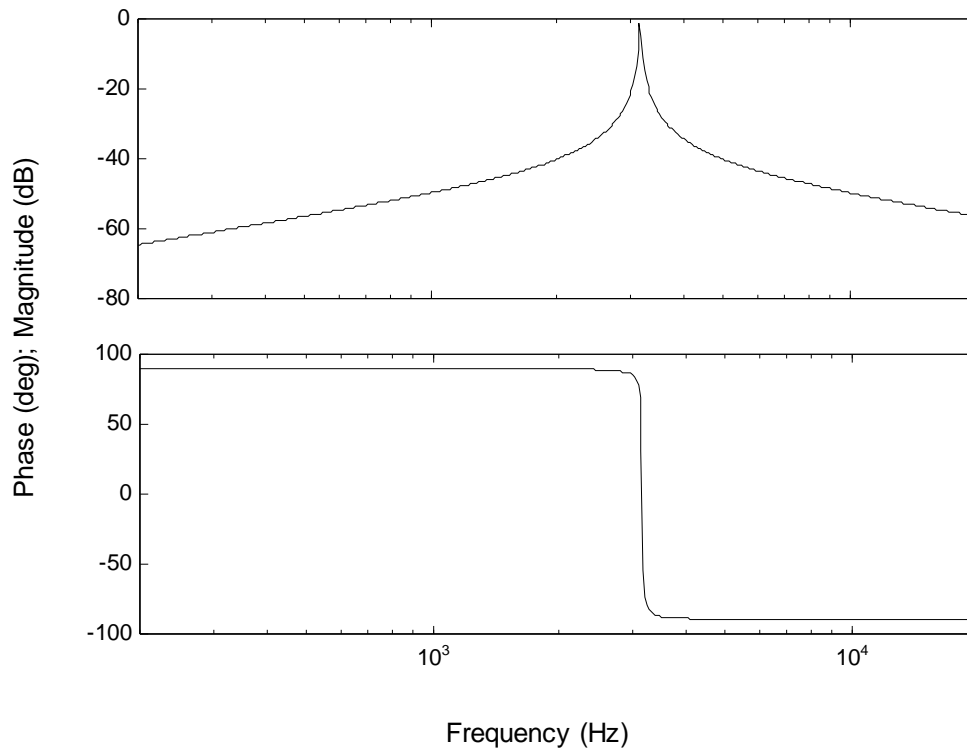


Figure: Bode plot of the transfer function of series RLC circuit over the resistor by MATLAB

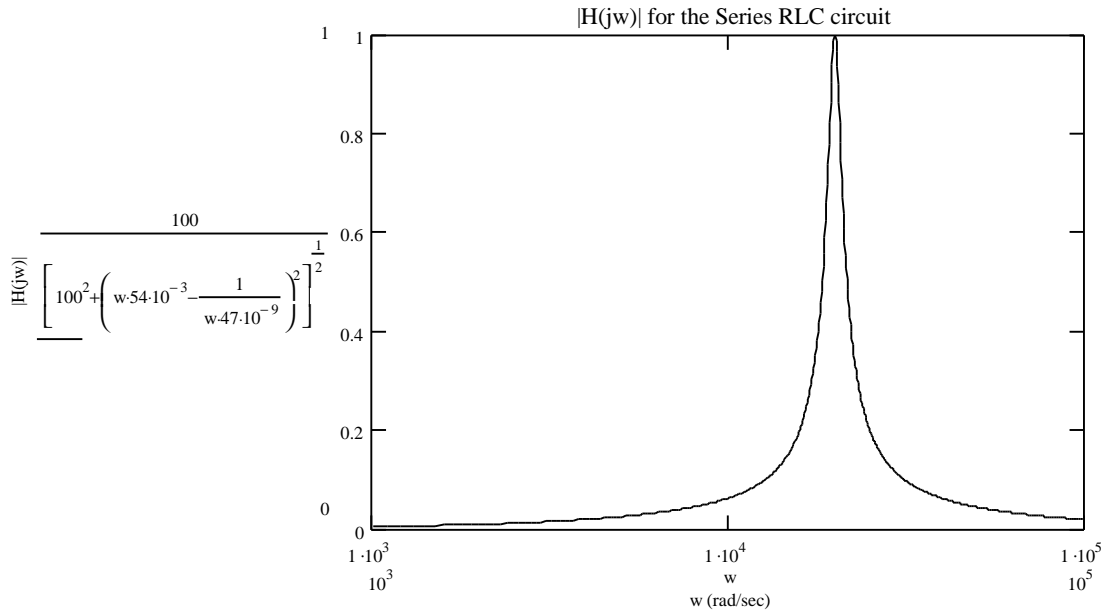


Figure: Plot of $|H(jw)|$ of the series RLC circuit over the resistor by Mathcad

These graphs are obtained with the transfer function;

$$H(jw) = \frac{R}{R + jwL + \frac{1}{jwC}}$$

We had the similar graphs in the experiment. From the graphs, I found 3159Hz=19848rad/sec as the resonant frequency. This frequency corresponds to the peak values of the graphs.

PARALLEL RLC CIRCUIT

V_L	V_C	V_R	$I = V_R/R$	$X_C = V_C/I$	$X_L = V_L/I$	$Z = V_{in}/I$
0.123V	0.120V	0.031V	310 μ A	3613 Ω	393 Ω	3225 Ω

Table: Calculated values for $V_{in} = 1 V_{RMS}$ for the parallel RLC circuit

Frequency (kHz)	0.1	0.2	0.5	1	2	5	10	20
Output(mV)	107	122	174	315	908	503	107	75
Output(dB)	-19.4	-18.3	-15.1	-10.0	-0.8	-6.0	-19.4	-22.5

Table: Voltage and current measurements for the parallel RLC circuit with $(V_{in})_{RMS} = 6.78V$

$$f_0 = 2.82 \text{ KHz} \quad V_0 = 20 \log \left(\frac{2.65}{6.78} \right) = -8.15 \text{ dB}$$

$$\text{Half Power Frequencies: } f_1 = 2.53 \text{ KHz} \quad f_2 = 3.33 \text{ KHz}$$

$$BW = 0.8 \text{ KHz} \quad Q = 2.82 \text{ KHz} / 0.8 \text{ KHz} = 3.53$$

Below I have shown the theoretical graphs for the parallel RLC circuit.

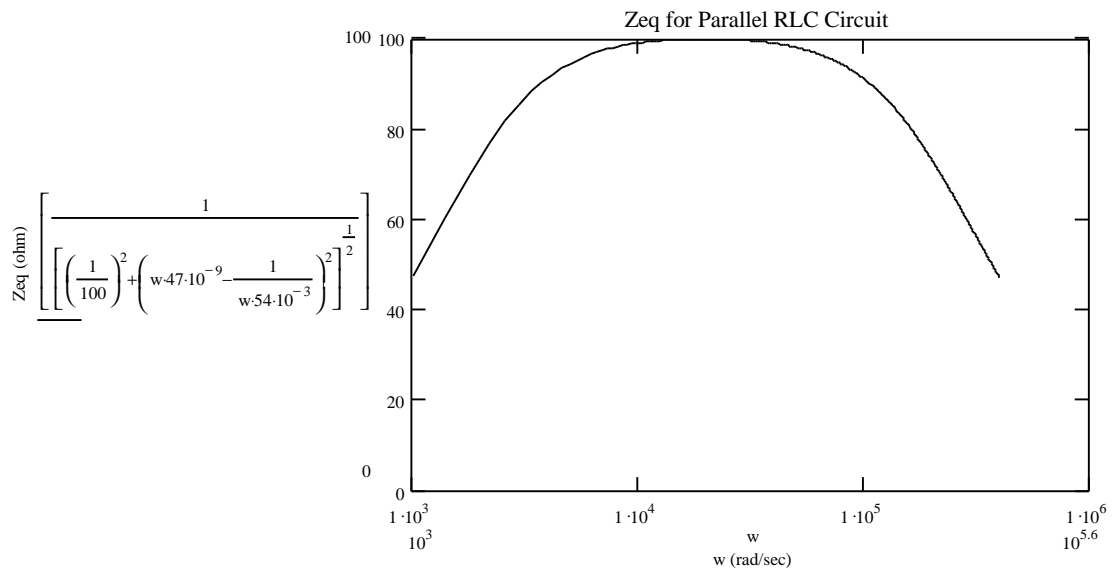


Figure: Plot of the equivalent impedance of the parallel RLC circuit by Mathcad

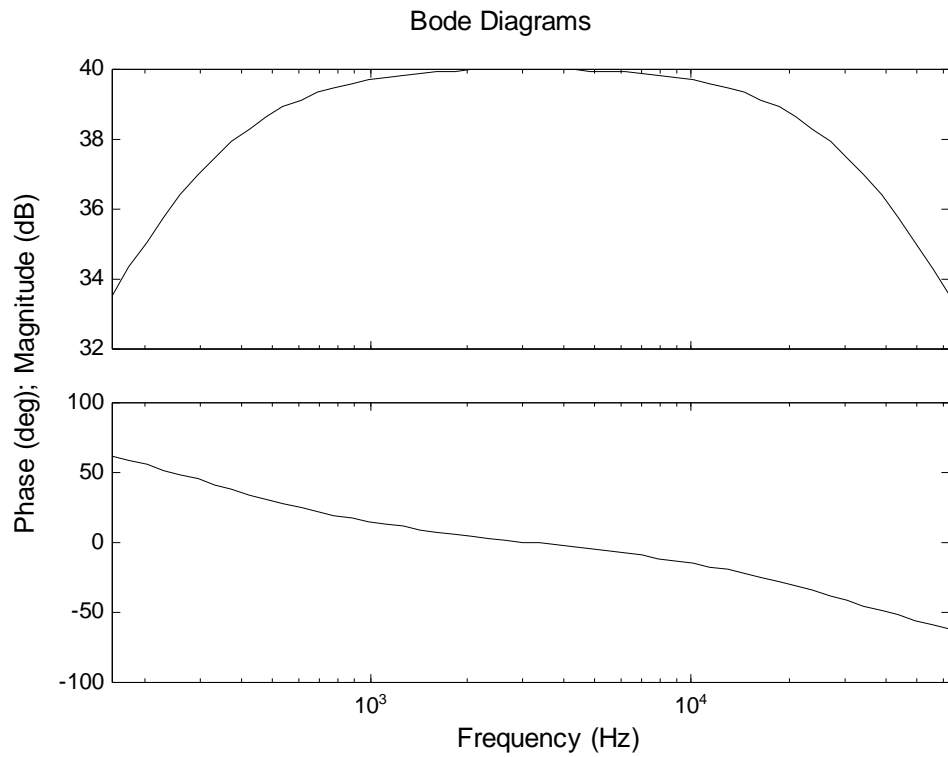


Figure: Bode plot of the transfer function (V_O/I_S) of the parallel RLC circuit by MATLAB

These graphs are obtained with the transfer function;

$$H(j\omega) = \frac{V_o}{I_s} = \frac{1}{\frac{1}{R} + j\omega C + \frac{1}{j\omega L}}$$

From the graphs, I found 3159Hz=19848rad/sec as the resonant frequency, which is same as the series RLC circuit. This frequency corresponds to the peak values of the graphs.

4. ANALYSIS AND DISCUSSION

To my opinion, this experiment was a well-prepared and teaching experiment. We were able to see our theoretical knowledge about the RLC circuits and resonance works in practical situations. What I learned newly is the physical meaning of the Quality Factor. Previously, I only knew that $Q = f_o/\Delta f$. Another thing that I noticed is that in the experiment we obtained similar graphs for both series and parallel RLC circuits. But only after examining the graphs I had with MATLAB and Mathcad, I understood that the graphs have different forms. One is concave and the other is convex. What this implies is that Parallel RLC circuit should have larger bandwidth compared to Series RLC circuit. This is the case in our experimental results but $0.8 > 0.78$ is not a that much difference. In my opinion this difference should be much more than we'd found.

The error we made in the measurements is, I think, in a reasonable range. For example, For series RLC circuit, error of the resonant frequency is,

$$\text{error}\% = \frac{\Delta f}{f_o} = \frac{|2.97\text{KHz} - 3.16\text{KHz}|}{3.16\text{KHz}} \times 100 = 6\%$$

This is due to instrument errors, incorrectness in the values of given components and our reading errors.

5. ANSWERS TO THE QUESTIONS

1.

The Quality Factor of a circuit is a measure of ratio of its energy-storing capability to energy dissipation as heat.

$$Q = \frac{\text{energy stored in the reactive component}}{\text{energy dissipated by resistor in one cycle}}$$

$$= \frac{X_L \cdot i^2}{R \cdot i^2} = \frac{X_L}{R} = \frac{\omega_o L}{R} = \frac{1}{\sqrt{LC}} \cdot \frac{L}{R}$$

$$Q = \frac{1}{R} \sqrt{\frac{L}{C}} \quad (\text{For series RLC circuit})$$

$$Q = \frac{\text{energy stored in the reactive component}}{\text{energy dissipated by resistor in one cycle}}$$

$$= \frac{V^2/X_C}{V^2/R} = \frac{R}{X_C} = \frac{R}{1/\omega_o C}$$

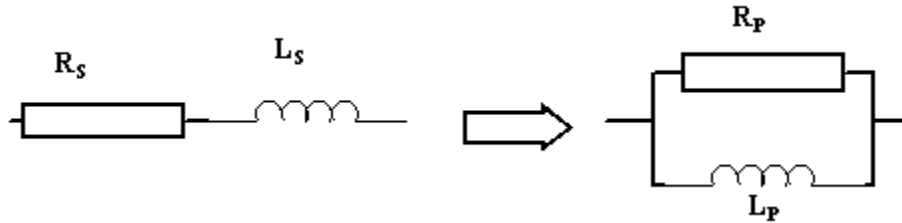
$$Q = \omega_0 CR \quad (\text{For parallel RLC circuit})$$

2.

$$\begin{aligned} \text{(For series)} \quad I_{\max} &= \frac{V_S}{R} \Rightarrow (V_L)_{\max} = \frac{V_S}{R} \cdot j\omega L \\ &\Rightarrow (V_C)_{\max} = \frac{V_S}{R} \cdot \frac{1}{j\omega C} \end{aligned}$$

$$\begin{aligned} \text{(For parallel)} \quad V_{\max} &= I_S R \Rightarrow (I_L)_{\max} = \frac{V_{\max}}{j\omega L} = \frac{I_S R}{j\omega L} \\ &\Rightarrow (I_C)_{\max} = \frac{V_{\max}}{1/j\omega C} = I_S R \cdot j\omega C \end{aligned}$$

3.



$$\begin{aligned} R_S + j\omega L_S &= R_P // L_P \\ &= \frac{R_P \cdot j\omega L_P}{R_P + j\omega L_P} \\ &= \frac{j\omega L_P R_P (R_P - j\omega L_P)}{R_P^2 - \omega^2 L_P^2} \end{aligned}$$

(Real parts of both sides should be equal)

$$\begin{aligned} R_S &= \frac{\omega^2 L_P^2 R_P}{R_P^2 - \omega^2 L_P^2} \\ \omega &= \frac{1}{\sqrt{L_P C}} \end{aligned}$$

Substituting ω in the equation, and solving R_P for $C=670\text{nF}$, $R_S=10\Omega$, $L_P=10\text{mH}$, we find $R_P= 4.68\Omega$ -hopefully no calculation errors.