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## How to juggle the three ball cascade

You need three balls, ideally different colours. Pick up one of them and throw it back and forth from your right to your left hand and vice-versa. All the throws should be the same height, roughly eye level. The idea of this exercise is to get consistent throws so that your hands don't have to move a lot to catch.

Next start with a red ball in your right hand and a yellow ball in your left hand. Throw the red ball from right to left. As this ball peaks throw the yellow ball from left to right. Make both throws the same height as before. Catch the red ball with your left hand and then catch the yellow ball with your right hand. Repeat this exercise making the first throw with your left hand. There should be a steady rhythm, throw, throw, catch, catch.

At this point there are three common mistakes:

1. Passing the second ball from one hand to another.

This is a two ball shower. It's possible to juggle three balls like this, but it's much harder than the cascade. If you find it really hard to stop doing this, for the moment just concentrate on throwing the second ball and forget about catching the first one (one less thing to worry about);
2. Throwing the second ball much lower than the first.

You must throw them to the same height;
3. Throwing both balls at the same time.

Say to yourself throw, throw, catch, catch as you throw and catch the balls.
Remember to alternate starting with the right and the left hands when doing this two ball exercise. Always throw the red ball first.

Once you've mastered this exercise pick up three balls. Start with a red ball and a green ball in your right hand and a yellow ball in your left hand. As before throw the red ball from right to left and as this peaks throw the yellow ball from left to right. Now when the yellow ball peaks, throw the green ball from right to left. You make this third throw about the same time that you catch the red ball with your left hand. This is a bit tricky at first, so if you manage to throw the green ball, but don't catch the red one, that's good. Some people stubbornly hang on to the third ball, so you're making progress.

Once you've made three throws and three catches, essentially you know what to do and it's just a matter of practice, trying to make one more throw and/or catch each time.

To help you improve:

1. Try starting with two balls in your left hand and one ball in your right hand, making the first throw with your left hand.
2. If you get stuck at three throws, forget about catching the second ball and make sure you throw the fourth (remember: one less thing to worry about!).

## Ladder diagrams



Figure 1

The ladder diagram for a three ball cascade

## Basics

A ladder diagram shows what happens to each ball ${ }^{1}$ in a solo juggling pattern. Figure 1 is the ladder diagram for a three ball cascade.

In a ladder diagram:

- large unfilled circles are throws;
- small filled circles are catches;
- left hand throws and catches are drawn on the left of the diagram and right hand ones on the right;
- the first throws are at the top of the diagram;
- a line joining a throw to a catch shows a ball in the air;
- a line joining a catch to a throw shows a ball held in a hand;
- If you draw a horizontal line through a ladder diagram, the number of lines it crosses tells you the number of balls needed to juggle the pattern.

In figure 1 every ball is thrown by one hand and caught by the other. These throws are known as cascade throws. They alternate between the right and left hand, as do the catches. Indeed at the moment that the right hand throws a ball the left hand catches one and vice-versa. These regular intervals, when the hands can throw or catch a ball, are called beats.

Patterns like the three ball cascade, where the right hand throws on one beat and the left hand throws on the next beat, are known as asynchronous patterns.

In the three ball cascade each ball is thrown once every three beats and caught two beats later. Since each ball is in the air for two beats, they are all thrown to the same height.

[^0]

Figure 2
Before and after swapping two throws


## Swapping throws in ladder diagrams

You can swap two throws in a ladder diagram to create a new trick. In the first diagram in figure 2, a line joins throw a to catch $a$ and another line joins throw $b$ to catch $b$. These lines have been erased from the second diagram and replaced by a line joining throw $a$ to catch $b$ and another joining throw $b$ to catch $a$.

The throw labelled $a$ is now caught three beats later, so it's higher than the cascade throws, whereas the throw labelled $b$ is caught on the next beat so it's lower. Also both these throws are thrown and caught by the same hand, they are fountain throws.

You can miss out the low throw $b$ and hold the ball for those two beats without changing any other throws in the pattern. If you do this, the trick shown in figure 2 is one left hand throw of two balls in one hand.

Figure 3 shows another example of swapping two throws in a three ball cascade. In this case throw a becomes a high cascade throw. It is followed by a normal cascade throw and then by throw $b$. Throw $b$ is a left hand throw caught by the right hand on the same beat. This throw is a zip* . To do this you bring your hands together and pass the ball from one hand to the other. Alternatively you can make a very fast, low, almost horizontal throw.

You can only swap two throws if the first of the two catches is on the beat before the second throw or later. So you can't swap throw $c$ with throw $b$ in figure 3 .
Figure 3
Another example of swapping throws
*Also known as a feed. Some club jugglers call it a hand across.


Figure 4
Right middle left

## Continuously repeating a trick

You can create a new pattern by continuously repeating a trick. Figure 4 shows a pattern created by continuously repeating the trick in figure 2. This pattern is right middle left.

To juggle right middle left, start holding a red ball and a green ball in your right hand and a yellow ball in your left hand:

- throw the red ball straight up on the right;
- catch the green ball with your right hand (miss this step at the start);
- throw the green ball from right to left;
- catch the red ball with your right hand;
- throw the yellow ball straight up on the left;
- catch the green ball with your left hand;
- throw the green ball from left to right;
- catch the yellow ball with your left hand;
- repeat.

The pattern on the left in figure 5 is created by continuously repeating the trick in figure 3.

To juggle this pattern, start holding a red ball and a green ball in your right hand and a yellow ball in your left hand:

- throw the red ball quite high from right to left;
- throw the yellow ball from left to right;
- zip the green ball from right to left;
- throw the green ball quite high from left to right;
- throw the yellow ball from right to left;
- zip the red ball from left to right;
- repeat


Figure 5
Swapping two throws in another pattern

The catches are not included in this explanation, but one hand catches at the same time that the other hand throws.

You can swap two throws in any pattern. The diagram on the right in figure 5 shows a trick created by swapping two throws in the pattern on the left. In this trick you throw one ball straight up on the left, throw one ball straight up on the right and zip one ball from left to right. Of course you could repeat this new trick continuously to create another pattern!


Figure 7
Rewiring every third catch in a cascade to get the
four ball pattern rockets

## Rewiring ladder diagrams

You can rewire the catches in a ladder diagram to create a new pattern. The ladder diagram on the left in figure 6 is a three ball cascade with the catches labelled from a to $h$. In the diagram on the right, the throw that was connected to catch a has been connected to catch $b$, the throw that was connected to catch $b$ has been connected to catch $c$ and so on. The new pattern is a four ball pattern, it's a four ball fountain.

To juggle a four ball fountain, start holding a red ball and a green ball in your right hand and a yellow ball and a blue ball in your left hand:

- throw the red ball straight up on the right;
- throw the yellow ball straight up on the left;
- throw the green ball straight up on the right;
- throw the blue ball straight up on the left;
- repeat.

You can rewire every second or third catch to create a new pattern. By rewiring every third catch in the three ball cascade you get the four ball pattern, rockets (see figure 7).

To juggle rockets, start holding a red ball and a green ball in your right hand and a yellow ball and a blue ball in your left hand:

- throw the red ball quite high on the right;
- throw the yellow ball from left to right;
- throw the green ball from right to left;
- throw the blue ball quite high on the left;
- throw the yellow ball from right to left;
- throw the green ball from left to right;
- repeat.


Figure 8
Rewiring right middle left to get a beautiful four ball pattern

You can rewire any pattern. Figure 8 shows a beautiful four ball pattern that you get if you rewire every catch of right middle left.

To juggle this pattern, start holding two balls in each hand:

- throw one ball quite high from right to left;
- throw one ball from left to right;
- throw one ball straight up on the right;
- throw one ball quite high from left to right;
- throw one ball from right to left;
- throw one ball straight up on the left;
- repeat.

In these examples the pattern is rewired by reconnecting the catches: $a$ to $b ; b$ to $c ; c$ to $d$; and so on. Each time the new pattern requires one more ball. You can rewire a pattern by reconnecting the catches: a to $c$; $b$ to $d ; c$ to $e$; and so on. Then the new pattern requires two more balls. Or reconnect $a$ to $d ; b$ to $e ; c$ to $f$; and so on. Then the new pattern requires three more balls.


Figure 9


Figure 10

Swapping two throws on the same beat in a three ball half shower

## Synchronous patterns

You can slide all the throws and catches for one hand down one beat to create a new pattern. If you do this with the ladder diagram for a three ball cascade, you get a three ball half shower (see figure 9). In this pattern both hands throw on one beat and they both catch on the next beat. Patterns like this are synchronous patterns.

In the three ball half shower all the throws are cascade throws. However the right hand throws are in the air for three beats, whereas the left hand throws are in the air for only one, so the right hand throws are higher. The left hand throws, caught by the right hand one beat later, are known as snaps*. They are very fast, low, almost horizontal throws.

To juggle a three ball half shower, start holding two balls in your right hand one in your left hand:

- throw one ball from right to left, then repeat the following:
- throw one ball from right to left and at the same time snap a ball from left to right;
- catch a ball in each hand.

In synchronous patterns you can still swap two throws to create a new trick. Sometimes the two throws and/or the two catches are on the same beat. When the two throws are on the same beat, the new right hand throw is the same height as the old left hand throw and vice-versa (see figure 10). The trick in figure 10 is one left hand throw of two in one hand in a three ball shower.

You can also rewire the ladder diagrams of synchronous patterns to create new patterns.

[^1]

Figure 11
A four ball multiplex pattern created by adding a ball to right middle left

## Multiplex patterns

In multiplex patterns, on some beats one hand throws more than one ball at the same time. These throws are multiplex throws. To make a two ball multiplex throw, catch a ball while holding a ball and then throw two balls at once.

In the three ball pattern right middle left, there are times when one hand holds a ball for three beats (see figure 4, page 5). There are many other patterns with these long holds. You can create a multiplex pattern by adding a ball to one of these patterns so that it is always caught when you are holding a ball (see figure 11).

To juggle this pattern, start holding a red ball and a green ball in your right hand and a yellow ball and a blue ball in your left hand:

- throw the red ball from right to left;
- at the same time catch the yellow ball with your left hand while holding the blue ball (miss this step at the start);
- throw a two ball multiplex with your left hand (turn your hand anticlockwise as you throw, so that the yellow ball goes from left to right and the blue ball goes straight up on the left);
- hold the green ball in your right hand;
- throw the red ball from left to right;
- at the same time catch the yellow ball with your right hand while holding the green ball;
- throw a two ball multiplex with your right hand (turn your hand clockwise as you throw, so that the yellow ball goes from right to left and the green ball goes straight up on the right);
- hold the blue ball your left hand;
- repeat.


## Squeeze patterns



Figure 12

## A four ball squeeze

 patternIn squeeze patterns, on some beats one hand catches more than one ball at the same time! These catches are squeeze catches.

You can create a squeeze pattern using the ladder diagram for a multiplex pattern. To do this, turn the ladder diagram upside down and swap the symbols for throws and catches. Figure 12 shows the squeeze pattern created by doing this to the ladder diagram in figure 11.

To juggle this pattern, start holding a red ball and a green ball in your right hand and a yellow ball and a blue ball in your left hand:

- throw the red ball straight up on the right;
- at the same time catch the yellow and blue balls with your left hand (miss this step at the start);
- throw the yellow ball from left to right while holding the blue ball;
- throw the green ball from right to left;
- throw the blue ball straight up on the left;
- at the same time catch the red and yellow balls with your right hand;
- throw the yellow ball from right to left while holding the red ball;
- throw the green ball from left to right;
- repeat.


## Siteswaps

## Vanilla siteswaps

In the section about ladder diagrams, juggling patterns are described using coloured balls. For example, using one letter for each colour, the sequence of throws for right middle left is: RYGYRG (see page 5). Siteswap replaces each letter with a number, it's the number of beats before the same ball can be thrown again. In this example the red ball, thrown on the first beat, can be thrown again four beats later, so the first $R$ is replaced by 4. The yellow ball, thrown on the second beat, can be thrown again two beats later, so the first $Y$ is replaced by 2 . The green ball, thrown on the third beat, can be thrown again three beats later, so the first G is replaced by 3 . Now this pattern can be written: 423423 (you can check that the balls thrown on the fourth, fifth and sixth beats can be thrown four, two and three beats later respectively). Usually siteswaps are written without repeats, so the siteswap for right middle left is: 423.

The siteswap for an asynchronous pattern is known as a vanilla siteswap. Table 1 shows the siteswaps for all the asynchronous patterns in the section about ladder diagrams.

| Pattern | Sequence of <br> throws | Letters replaced by <br> numbers | Siteswap |
| :--- | :--- | :--- | :--- |
| Three ball cascade | RYG | 333 | 3 |
| Right middle left | RYGYRG | 423423 | 423 |
| Pattern in figure 5 | RYGGYR | 531531 | 531 |
| Four ball fountain | RYGB | 4444 | 4 |
| Rockets | RYGBYG | 633633 | 633 |
| Pattern in figure 8 | RYGBYRGYB | 534534534 | 534 |
| Table 1 |  |  |  |
| The siteswaps for some popular asynchronous patterns |  |  |  |

In vanilla siteswaps:

- odd numbers are cascade throws;
- even numbers are fountain throws;
- the higher the number, the higher the throw;
- a 2 is usually a hold (it can be thrown as a low fountain throw);
- a 1 is a zip;
- a 0 is an empty hand, also known as a gap;
- letters are used for throws higher than $9, a$ for $10, b$ for $11, c$ for 12 and so on;
- the number of throws in a siteswap is its period. For example the three ball cascade and four ball fountain are period one siteswaps, all the other patterns in table 1 are period three siteswaps;
- if you add up the throws in a siteswap and divide by its period you get the number of balls needed to juggle the pattern. This is known as the average rule;
- in club juggling a 3 is a single spin throw, a 4 is a double, a 5 is a triple and so on.

[^2]
## Swapping throws in vanilla siteswaps

You can swap two throws in a vanilla siteswap. The throw that moves to the right decreases by one with each step, and the throw that moves to the left increases by one with each step. A throw may not move if it would decrease below zero.

This is the same as swapping two throws in the ladder diagram for an asynchronous pattern (see page 4). I like to call this the downright rule because the throw that moves to the right decreases.

To create a three ball, period three siteswap, start with three throws of a three ball cascade: 333 . You can swap the throw on beat one with the throw on beat three to get 531. The 3 on beat one moves two steps to the right, so it decreases by two and the new throw on beat three is 1 . The 3 on beat three moves two steps to the left, so it increases by two and the new throw on beat one is 5 (see figure 13). Figure 3 shows this example using ladder diagrams (see page 4).


Figure 13
Swapping two throws in the three ball cascade

Starting with this new pattern, 531, you can swap the throw on beat one with the throw on beat two to get 441. The 5 on beat one moves one step to the right, so it decreases by one and the new throw on beat two is 4 . The 3 on beat two moves one step to the left, so it increases by one and the new throw on beat one is 4 (see figure 14). Figure 5 shows shows this example using ladder diagrams (see page 5).


Figure 14
Swapping two throws in 531

## The circular rule

To create a three ball, period two siteswap, start with two throws of a three ball cascade: 33. You can swap the throw on beat one with the throw on beat two to get 42 . This is two balls in one hand, with the other hand holding a ball. Now if you start with 42 and swap the throw on beat one with the throw on beat two, you get 33. However there are other three ball, period two siteswaps. To find them you need to use the circular rule:

You can move a throw from the front to the back of the pattern.
By this rule 531,315 and 153 are all siteswaps for the same pattern. Similarly 42 and 24 are siteswaps for the same pattern. So you can start with 24 and swap the throw on beat one with the throw on beat two to get 51 . This is the siteswap for the three ball shower.

To juggle the three ball shower, start holding a red ball and a green ball in your right hand and a yellow ball in your left hand:

- throw the red ball quite high from right to left;
- hold the yellow ball in your left hand. Now repeat the following steps:
- throw the green ball quite high from right to left;
- zip the yellow ball from left to right;
- throw the yellow ball quite high from right to left;
- zip the red ball from left to right;
- throw the red ball quite high from right to left;
- zip the green ball from left to right.

You can create any vanilla siteswap by combining the downright and circular rules. For example to create a four ball, period six siteswap, start with six throws of a four ball fountain: 444444. Then apply these two rules as many times as you wish!

## Rewiring vanilla siteswaps

You can add 1 to every throw of a vanilla siteswap to create a pattern with an extra ball. This is the same as rewiring every throw of a ladder diagram (see page 6). Applying this rule to the three ball pattern 423, you get the four ball pattern 534 (see figure 8, page 7).

You can also take 1 from every throw of a vanilla siteswap to create a pattern with one less ball, providing no throw decreases below zero. Applying this rule to the four ball pattern 633, you get the three ball pattern 522. This is a slow cascade. Each throw is a high cascade throw and you throw once every three beats.

You can add the period of a siteswap to one throw to create a pattern with an extra ball. Starting with the period two, three ball pattern 51, if you add 2 to the 5 you get the four ball pattern 71 . This is a four ball shower. Every right hand throw is a very high cascade throw and every left hand throw is a zip. Applying this rule to a period two siteswap is the same as rewiring every second throw in a ladder diagram.

You can think of three throws of a three ball cascade as a period three siteswap. So starting with 333 , if you add the period to one of the throws, you get the four ball pattern 633. Applying this rule to a period three siteswap is the same as rewiring every third throw in a ladder diagram (see figure 7, page 6).


Figure 15

The three ball pattern 504

## Checking vanilla siteswaps

To check a vanilla siteswap, for each throw write that number under the next throw that can be made with the same ball. A vanilla siteswap will have one number under each throw in the pattern.

Let's check if 51234 is a vanilla siteswap. The throw five beats after the 5 is a 5 , so write 5 under itself. The throw one beat after the 1 is a 2 , so write 1 under the 2 . The throw two beats after the 2 is a 4 , so write 2 under the 4 . The throw three beats after the 3 is a 1 , so write 3 under the 1 . Finally the throw four beats after the 4 is a 3 , so write 4 under the 3 . There is one number under every throw, so 51234 is a vanilla siteswap (see figure 16). As the average of the throws is 3,51234 is a three ball siteswap.

| 51234 | 51234 | 51234 | 51234 | 51234 |
| :--- | :--- | :--- | :--- | :--- |
| 5 | 51 | 512 | 5312 | 53142 |

Figure 16
Checking a vanilla siteswap
Note that if you read the numbers in the checking line from right to left, it's always a vanilla siteswap. In this case it's 24135 (or 52413), but sometimes it's the same as the pattern you are checking.

Let's check if 55235 is a vanilla siteswap. The throw five beats after each 5 is a 5 , as the period is five, so write 5 under each 5 . The throw two beats after the 2 is the last 5 . There is already a 5 under this throw, so 55235 is not a vanilla siteswap (see figure 17).

| 55235 | 55235 | 55235 |  |
| :--- | :--- | ---: | ---: |
| 55 | 5 | 55 | 5 |
|  |  | 55 | 5 |
|  |  | 2 | 2 |

Figure 17
A pattern that fails this test
Note that the average of these numbers is 4 but 55235 in not a vanilla siteswap. When the average of a set of numbers is a whole number there's always at least one arrangement of those numbers that's a siteswap. In this case it's: 55253.

## Linking vanilla siteswaps together

You can use the method for checking patterns to work out how to link patterns together. Suppose you are juggling a three ball cascade and want to switch to 441 . First assume that you can switch straight from one pattern into the other. Write down several throws of a three ball cascade followed by several throws of 441, then use the method for checking patterns to see if any throws clash (see figure 18).

```
pattern .....333441441....
check .........333144144....
```

Figure 18

## Linking the three ball cascade and 441

In this example, no throws clash, so you can switch straight from the three ball cascade to 441. Also reading the numbers in the checking line from right to left shows that you can switch straight from 441 back to the cascade.

Suppose you are juggling a three ball cascade and want to switch to 504 . Once again assume that you can switch straight from one pattern into the other. In this example there's a clash (see figure 19).


Figure 19
The second 3 clashes with the first 0 and there's no number under the second 5 . You can change one of the throws that clash in order to link the patterns. The second 5 is five beats after the second 3 , so you can change the 3 into a 5 (see figure 20).

$$
\begin{aligned}
& \text { pattern ....353504504.... } \\
& \text { check ........ } 3035054054 \ldots
\end{aligned}
$$

Figure 20

## Linking the three ball cascade and 504

Reading the checking line from right to left shows that you can switch from 504 back to the cascade by juggling: ...50450530333...

You can find another link between these patterns by changing the first 0 . The second 5 is two beats after the first 0 , so you can change the 0 into a 2 to link the patterns (see figure 21).

> pattern .... $333524504 \ldots$
> check ....... $3332054054 \ldots$

Figure 21

## Another way to link the three ball cascade and 504

Reading the checking line from right to left shows that you can switch from 504 back to the cascade by juggling: ...504502333...

## Multiplex siteswaps

In siteswap a multiplex throw is shown by enclosing the numbers in square brackets. For example the pattern in figure 11 is [43]23 (see page 9). The [43] on the first beat is a multiplex throw. One hand throws a fountain throw and a cascade throw at the same time.

You can create a multiplex pattern by combining two siteswaps which have the same period. On any beat that both patterns have a non-zero throw, there will be a multiplex throw in the new pattern. Also the throw two beats before each multiplex throw must be a 2 . Table 2 shows some examples.

| pattern 1 | pattern 2 | multiplex <br> pattern |
| :---: | :---: | :---: |
| 40303 | 30020 | $[43] 0323$ |
| 6020 | 4000 | $[64] 020$ |
| 500304 | 400020 | $[54] 00324$ |
| 423 | 300 | $[43] 23$ |
| 504 | 420 | $[54] 24$ |
| Table 2 |  |  |
| Some popular multiplex patterns |  |  |

You can use the average rule to work out how many balls are needed to juggle a multiplex pattern. For example in the case of [43]0323, the sum of the throws is 15 and the pattern is a period five siteswap, so it's a three ball pattern.

When you check a multiplex pattern, one of the numbers under the multiplex throw must be 2 (see figure 22).
[43]23
2343
Figure 22

## Squeeze siteswaps

You can create a squeeze pattern by reading the checking line for a multiplex pattern from right to left. This is the same as turning the ladder diagram for a pattern upside down and swapping the symbols for throws and catches (see page 10). If you do this with the example in figure 22 you get the four ball pattern 34[23] (see figure 12, page 10).

You can create a pattern with squeeze catches and multiplex throws from this new pattern by adding the period to the 2. In this case you get the five ball pattern 34[53].


Figure 23
Crossing throws in a four ball fountain

## Synchronous siteswaps

If the numbers in a vanilla siteswap are all even, the throws in the pattern are all fountain throws. This means each hand has it's own pattern, so both hands can throw at the same time. To show this in siteswap, write the throws in brackets. The first throw in each bracket is a right hand throw and the second throw is a left hand throw. For example the synchronous four ball fountain is written: $(4,4)$.

In these patterns, the hands also catch at the same time, so some of the throws can cross. In siteswap write an x after each crossing throw. Figure 23 shows crossing throws in a synchronous four ball fountain. This trick is: $(4,4)(4 x, 4 x)$.

## Swapping throws in synchronous siteswaps

If you swap a right hand throw with a left hand throw, a crossing throw changes to a fountain throw and vice-versa.

This is the handedness rule. For example, start with the three ball pattern: $(2,4)$. You can swap the right hand throw with the left hand throw to get $(4 x, 2 x)$. The 2 in $(2,4)$ is a right hand, fountain throw, so the left hand throw in the new pattern, $(4 x, 2 x)$, is a crossing 2 . This throw, $2 x$, is a snap. The 4 in $(2,4)$ is a left hand, fountain throw, so the right hand throw in the new pattern is a crossing 4 . This pattern, $(4 x, 2 x)$, is the three ball half shower.

In this case the throws are on the same beat, so their heights are unchanged (see page 8). To swap throws on different beats you need the following rule:

You can swap two throws in a synchronous siteswap. The throw that moves to the right decreases by two with each bracket, and the throw that moves to the left increases by two with each bracket. A throw may not move if it would decrease below zero.

This is the synchronous downright rule. It works because throws in one bracket are two beats before throws in the next bracket. For example, start with the four ball pattern: $(4,4)(4 x, 4 x)$. You can swap the right hand throws to get: $(6 x, 4)(2,4 x)$. The right hand throw in the first bracket, 4 , moves right one bracket, so it decreases by two and the new right hand throw in the second bracket is 2 . The right hand throw in the second bracket, $4 x$, moves left one bracket, so it increases by two and the new right hand throw in the first bracket is $6 x$.


Figure 24

## Swapping two throws in a synchronous pattern

## The circular rule

You can move a bracket from the front to the back of the pattern.
By this rule $(6 x, 4)(2,4 x)$ and $(2,4 x)(6 x, 4)$ are siteswaps for the same pattern.
You can create any synchronous siteswap by combining the handedness, downright and circular rules. For example, to create a three ball period 4 pattern start with $(2,4)(2,4)$. If you swap the throws on the first beat, by the handedness rule, you get: $(4 x, 2 x)(2,4)$. This is the trick shown in the ladder diagram in figure 10 (see page 8). Now if you swap the right hand throws, by the downright rule, you get: $(4,2 x)(2 x, 4)$. This is the three ball box.

To juggle the box, start holding a red ball and a green ball in your right hand and a yellow ball in your left hand:

- snap the red ball from right to left and at the same time throw the yellow ball straight up on the left;
- catch the green ball in your right hand (miss this step out at the start) and at the same time catch the red ball in your left hand;
- snap the red ball from left to right and at the same time throw the green ball straight up on the right;
- catch the yellow ball in your left hand and at the same time catch the red ball in your right hand;
- repeat.

Applying these rules can give rise to a $0 x$ throw. For this throw a ball is caught by one hand but thrown by the other hand on the next beat. It feels like a zip.

## Rewiring synchronous siteswaps

You can add two to every throw of a synchronous siteswap to get a pattern with two extra balls.
Applying this rule to the three ball box: $(4,2 x)(2 x, 4)$, you get the five ball pattern: $(6,4 x)(4 x, 6)$.
You can also take two from every throw of a synchronous siteswap to get a pattern with two less balls, providing no throw decreases below zero.

You can add the period of a synchronous siteswap to any throw to get a pattern with an extra ball.
Applying this rule to the three ball box: $(4,2 x)(2 x, 4)$, if you add the period to the first left hand throw, you get the four ball pattern: $(4,6 x)(2 x, 4)$.

You can also take the period of a synchronous siteswap from any throw to get a pattern with one less ball, providing that throw doesn't decrease below zero.

Applying this rule to the five ball pattern: $(6,4 x)(4 x, 6)$, if you take the period from the second left hand throw, you get the four ball pattern: $(6,4 x)(4 x, 2)$.

## Checking synchronous siteswaps

To check a synchronous siteswap, for each throw write that number under the next throw that can be made with the same ball. A synchronous siteswap will have one number under each throw in the pattern.

Let's check if $(6,4 x)(6 x, 2)(4,2)$ is a synchronous siteswap. The ball thrown by the right hand as a 6 can be thrown again by the right hand six beats later. So write 6 under itself, as it's the right hand throw three brackets later. The ball thrown by the left hand as a crossing 4 can be thrown again by the right hand four beats later. So write $4 x$ under the 4 , the right hand throw two brackets later. The ball thrown by the right hand as a crossing 6 can be thrown again by the left hand six beats later. So write $6 x$ under the 2 in the same bracket, as it's the left hand throw three brackets later. The ball thrown by the left hand as a 2 in the second bracket, can be thrown again by the left hand two beats later. So write 2 under the 2 in the next bracket. The ball thrown by the right hand as a 4 can be thrown again by the right hand four beats later. So write 4 under the $6 x$, as it's the right hand throw two brackets later. The ball thrown by the left hand as a 2 in the third bracket, can be thrown again by the left hand two beats later. So write 2 under the $4 x$, as it's the left hand throw in the next bracket. there is one number under every throw, so $(6,4 x)(6 x, 2)(4,2)$ is a synchronous siteswap (see figure 25).


Figure 25

## Checking a synchronous siteswap

## Linking synchronous siteswaps together

You can use the method for checking patterns to work out how to link patterns together. Suppose you are juggling a four ball fountain and want to switch to $(6 x, 4)(4,2 x)(4,6 x)(2 x, 4)$. First assume that you can switch straight from one pattern into the other. Write down several throws of a four ball fountain followed by several throws of $(6 x, 4)(4,2 x)(4,6 x)(2 x, 4)$, then use the method for checking patterns to see if any throws clash (see figure 26).
pattern $\ldots(4,4)(4,4)(6 x, 4)(4,2 x)(4,6 x)(2 x, 4) \ldots \ldots .$.
check $\ldots \ldots \ldots \ldots \ldots .44442 x 446 x \ldots \ldots$.

Figure 26
Linking the four ball fountain and ( $6 x, 4$ )(4,2x)(4,6x)(2x,4)
In this example no throws clash, so you can switch straight from the four ball fountain into ( $6 x, 4$ ) $(4,2 x)(4,6 x)(2 x, 4)$. Also reading the checking line from right to left shows that you can switch straight from $(6 x, 4)(4,2 x)(4,6 x)(2 x, 4)$ into the four ball fountain.

Suppose you are juggling a four ball fountain and want to switch to $(6 x, 2 x)(2 x, 6 x)$. Once again assume that you can switch straight from one pattern into the other. In this example there's a clash (see figure 27).


Figure 27
The first 2 x clashes with the third 4 and there's no number under the third 6 x . You can change one of the throws that clash in order to link the patterns. The third $6 x$ is four beats after the first $2 x$, so you can change the $2 x$ into a $4 x$ (see figure 28 ).

$$
\begin{aligned}
& \text { pattern } \ldots(4,4)(4,4)(6 x, 4 x)(2 x, 6 x)(6 x, 2 x)(2 x, 6 x) \ldots \\
& \text { check } \ldots \ldots \ldots \ldots \ldots .444444 x 2 x 2 x 6 x \ldots
\end{aligned}
$$

Figure 28
Linking the four ball fountain and $(6 x, 2 x)(2 x, 6 x)$
Reading the checking line from right to left shows that you can switch from ( $6 x, 2 x)(2 x, 6 x)$ back to the fountain by juggling: ... $(6 x, 2 x)(2 x, 4 x)(4,4)(4,4) \ldots$

You can find another link between these patterns by changing the third 4 . The third $6 x$ is six beats after the third 4 , so you can change the 4 into a 6 to link the patterns (see figure 29).

```
pattern .... \((4,4)(6,4)(6 x, 2 x)(2 x, 6 x)(6 x, 2 x)(2 x, 6 x) \ldots\).
check .................. \(442 \mathrm{2x} 4\) 6 2x \(2 \mathrm{x} 6 \mathrm{x} \ldots\)
```

Figure 29
Another way to link the four ball fountain and ( $6 x, 2 x)(2 x, 6 x)$
Reading the checking line from right to left shows that you can switch from $(6 x, 2 x)(2 x, 6 x)$ back to the fountain by juggling: ... $(6 x, 2 x)(2 x, 6)(4,2 x)(4,4) \ldots$

## Linking asynchronous and synchronous siteswaps

Siteswap shows the throws in a juggling pattern, but ignores the catches. This makes it easier to read siteswap, but difficult to work out how to link asynchronous and synchronous patterns.

In asynchronous patterns, such as 534, on each beat one hand throws and the other hand catches. Using a hyphen, '-', to show the catches the pattern can be written: $(5,-)(-, 3)(4,-)(-, 5)(3,-)(-, 4)$.

In synchronous patterns, such as $(6 x, 4)(4,2 x)(4,6 x)(2 x, 4)$, on one beat both hands throw and on the next beat both hands catch. So this pattern can be written: $(6 x, 4)(-,-)(4,2 x)(-,-)(4,6 x)(-,-)(2 x, 4)(-,-)$.

Remember, to check a siteswap, for each throw write that number under the next throw that can be made with the same ball. There should be one number under each throw in the pattern. Figure 30 shows the check for 534.

```
pattern: \((5,-)(-, 3)(4,-)(-, 5)(3,-)(-, 4)\)
check: 4354335
```

Figure 30

Checking 534
Figure 31 shows the check for the synchronous pattern ( $6 x, 4$ )(4,2x)(4,6x)(2x,4).

$$
\begin{aligned}
& \text { pattern: }(6 x, 4)(-,-)(4,2 x)(-,-)(4,6 x)(-,-)(2 x, 4)(-,-) \\
& \text { check: } 42 x \quad 6 x 4 \quad 2 x 4 \quad 46 x
\end{aligned}
$$

Figure 31
Checking ( $6 x, 4$ )(4,2x)(4,6x)(2x,4)
To switch from an asynchronous pattern to a synchronous pattern, you hold a ball for an extra beat so you can throw two balls at once. This hold is a 1 fountain throw, written $1 \mathrm{x}^{*}$. It is 'thrown' and 'caught' by the same hand on the same beat. It's a shorter hold than a 2, which is 'thrown' on one beat and 'caught' on the next beat, but it increases the normal 'dwell time' between a catch and a throw.

Suppose you are juggling 534 and want to switch to $(6 x, 4)(4,2 x)(4,6 x)(2 x, 4)$. First assume you can switch straight from 534 to $(6 x, 4)(4,2 x)(4,6 x)(2 x, 4)$ (see figure 32$)$.

Figure 32
There is a clash. The check for the left hand 5 is under a hyphen and there is no number under the 4, the right hand throw in the next bracket. This throw is six beats after the left hand throw, so you can change the left hand 5 to a $6 x$ (see figure 33). Note the $6 x$ throw in an asynchronous pattern.

$$
\begin{gathered}
\ldots(5,-)(-, 3)(4,-)(-, 6 x)(3,-)(-, 4)(1 x,-)(6 x, 4)(-,-)(4,2 \mathrm{x})(-,-)(4,6 \mathrm{x})(-,-) \ldots \\
3 \\
5 \quad 4 \quad 1 \times 3 \quad 6 \times 4 \quad 2 \times 4
\end{gathered}
$$

Figure 33
Linking 534 and $(6 x, 4)(4,2 x)(4,6 x)(2 x, 4)$

[^3]Suppose you want to switch from juggling an asynchronous to a synchronous four ball fountain. First assume that you can switch straight from 4 into $(4,4)$ (see figure 34 ).

$$
\begin{array}{r}
\ldots(4,-)(-, 4)(4,-)(-, 4)(1 \times,-)(4,4)(-,-)(4,4)(-,-) \ldots \\
4 \quad 1 \times 44
\end{array}
$$

Figure 34
In this case the check for the second right hand 4 is under a hyphen and there is no number under the 4 , the right hand throw in the next bracket. This throw is five beats after the right hand throw, so you can change the right hand 4 to a $5 x$. It's written $5 x$ to show that it's a fountain throw (see figure 35).

$$
\begin{array}{r}
\ldots(4,-)(-, 4)(5 x,-)(-, 4)(1 x,-)(4,4)(-,-)(4,4)(-,-) . . \\
4 \quad 1 \times 4 \quad 5 \times 4
\end{array}
$$

Figure 35

## Linking the asynchronous and synchronous fountain

To switch from a synchronous to an asynchronous pattern, on one beat you throw only one ball and hold the other. Once again you 'throw' a 1x.

Suppose you want to switch from juggling $(6 x, 4)(4,2 x)(4,6 x)(2 x, 4)$ to 534 . First assume that you can (see figure 36).

$$
\begin{gathered}
\ldots(6 x, 4)(-,-)(4,2 x)(-,-)(4,6 x)(-,-)(2 x, 1 x)(-, 5)(3,-)(-, 4) \ldots \\
2 x 4 \quad 46 x \quad 1 \times 42 x \\
\text { Figure } 36
\end{gathered}
$$

In this case the check for the right hand 2 x is under a hyphen and there is no number under the 4 , the left hand throw in the next bracket. This throw is three beats after the right hand throw, so you can change the right hand $2 x$ to a 3 (see figure 37). Note the 3 in a synchronous pattern.

$$
\begin{array}{r}
\ldots(6 x, 4)(-,-)(4,2 x)(-,-)(4,6 x)(-,-)(3,1 x)(-, 5)(3,-)(-, 4) \ldots \\
2 \times 4 \quad 46 x \quad 1 \times 4 \quad 3
\end{array}
$$

Figure 37
Linking (6x,4)(4,2x)(4,6x)(2x,4) and 534
Note that this is the same as reading the checking line for the switch from 534 to $(6 x, 4)(4,2 x)(4,6 x)$ $(2 x, 4)$ from right to left.

Suppose you want to switch from juggling a synchronous to an asynchronous four ball fountain. First assume that you can switch straight from $(4,4)$ into 4 :

$$
\begin{array}{r}
\ldots(4,4)(-,-)(4,4)(-,-)(1 x, 4)(4,-)(-, 4)(4,-)(-, 4) \ldots \\
441 x \quad 44
\end{array}
$$

Figure 38
In this case the check for the second right hand 4 is under a hyphen and there is no number under the 4 , the right hand throw in the next bracket. This throw is five beats after the right hand throw, so you can change the right hand 4 to a $5 x$ (see figure 39 ).

$$
\begin{array}{r}
\ldots(4,4)(-,-)(5 x, 4)(-,-)(1 \mathrm{x}, 4)(4,-)(-, 4)(4,-)(-, 4) \ldots \\
441 \mathrm{x} \quad 45 \mathrm{x} \quad 4
\end{array}
$$

## Figure 39

## Linking the synchronous and asynchronous fountain

Note that this is the same as reading the checking line for the switch from the asynchronous to the synchronous fountain from right to left.

## Passing patterns

## Basic patterns

In a basic two person, six club* passing pattern:

- you stand facing your partner about 10-15 feet apart;
- you both start with two clubs in your right hand and one in the left;
- you start together by raising the clubs above your head, swinging them down to your side and then throwing the first pass. This sequence: 'up, down, pass', is the juggling equivalent of 'ready, steady, go';
- passes from your right hand are thrown to your partner's left hand and vice-versa. These are tramline passes;
- passes spin one and a half times but are known as singles;
- normal cascade throws are called selfs;
- you pass to an agreed rhythm, for example four count, where every fourth throw is a pass.

The rhythms can be right handed or ambidextrous. Table 3 shows the most popular basic rhythms.

| rhythm | throwing sequence | handedness | other names |
| :---: | :---: | :---: | :---: |
| four count two count three count one count pass, pass, self | pass, self, self, self <br> pass, self <br> pass, self, self <br> pass <br> pass, pass, self | right handed right handed ambidextrous ambidextrous ambidextrous | every others <br> everys, shower <br> waltzing <br> ultimates, thunder shower <br> ogilvys, $2 / 3$ count |
| Table 3 <br> Popular passing patterns |  |  |  |

## Passing etiquette

When passing:

- always keep count. For example chant 'pass, self, self' to yourself when waltzing;
- don't pass if you only have two clubs. This is known as passing the gap or holding through the gap;
- don't look away. Your partner might pass clubs to you, even if you think the pattern is finished!

It's useful to give your partner some feedback about their passes. For example, tell them if you'd like shorter, higher, or more spinny passes. Maybe hold your hand up and say: 'l'd like your passes here'. Also encourage them to comment on your passes by asking: 'are my passes alright?'

[^4]
## Causal diagrams

## Basics

A causal diagram shows how each throw in a passing pattern causes another throw. For example, a right hand, tramline pass to you causes you to throw the club in your left hand so you can catch the pass.

In a causal diagram:

- small circles in the same row show one juggler's throws. They are labelled $R$ or $L$ for right or left hand throws;
- small circles in the same column show throws on the same beat;
- the first throws are on the left of the diagram;
- an arrow pointing from one circle to a circle that's one, or one and a half beats to the right is a single spin throw;
- an arrow pointing to a circle two, or two and a half beats to the right is a double spin throw;
- an arrow pointing to a circle three, or three and a half beats to the right is a triple spin throw and so on;
- an arrow pointing to a circle in the same row is a self;
- an arrow pointing to a circle in another row is a pass;
- an arrow pointing from one circle back to itself is a hold;
- an arrow pointing to a circle one beat to the left is a zip;
- an arrow pointing to a circle two beats to the left is a gap;
- if you draw a vertical line through a causal diagram, the number of forward pointing arrows minus the number of backward pointing arrows that the line crosses, is the number of clubs in the air. Add this to the number of hands to get the number of clubs needed to juggle the pattern.

Figure 40 shows some examples of causal diagrams for two person, right handed patterns.


Figure 40
Causal diagrams for the popular right handed, six club passing patterns
In these examples Anne's first throw is a right handed, tramline, single pass to Ben. It causes Ben to throw a left hand self on the second beat. Similarly Ben's first throw is a right hand pass, which causes Anne to throw a left hand self. In the four count pattern these left hand selfs cause right hand selfs on the third beat, whereas in the two count pattern they cause right hand passes. In both these patterns the jugglers always pass at the same time.
Anne:

Three count



Figure 41
Causal diagrams for the popular ambidextrous, six club patterns
Figure 41 shows the causal diagrams for some ambidextrous patterns. These patterns include left hand passes, for example the fourth beat of all three patterns.

You can simplify causal diagrams by drawing lines instead of arrows. You only need arrows to show zips and gaps. Also if each juggler's throws always alternate right, left, right, left, etc., as above, you don't need to label all the throws R and L .


Figure 42

## Passing an early double

## Swapping throws in causal diagrams

You can swap two throws in a causal diagram to create a new trick. In the first diagram in figure 42, an arrow joins a to $b$ and another arrow joins $b$ to $c$. These arrows have been erased from the second diagram and replaced by an arrow joining $a$ to $c$ and another joining $b$ to itself.

The first diagram shows three beats of a pattern. Anne and Ben throw selfs on the first beat and single spin, tramline passes on the second beat. It could be part of any of the patterns in table 3 except one count. The labels R and $L$ are not shown because in ambidextrous patterns the first beat could be a right or left hand throw. In the second diagram Anne's throws are a double spin, crossing pass on the first beat and a hold on the second beat. The pass crosses because the first and third beats are thrown by the same hand. The only change to Ben's pattern is that he has to catch Anne's double pass. Figure 43 shows this trick in four count.


Figure 43
An early double in four count

becomes


Figure 44
A late double


Figure 45
A late double in one count


Figure 46
An early triple


OR

becomes


Figure 47
Self triples

Figure 44 shows another trick created by swapping two throws in a causal diagram. This time Anne passes a crossing double at the same time that Ben passes a tramline single. This changes Ben's pattern, he has only two clubs for one beat, so he holds through the gap. You can pass a late double in all the patterns in table 3. Figure 45 shows a late double in one count.

You can pass triples or quads too. Figure 46 shows an early triple. It also shows how to create this combination starting with the causal diagram for a late double and swapping two throws. You can throw an early triple in three count or four count.

When you create a trick with a pass, the new pass must land on the same beat as the original pass or later. Your partner will not be able to catch a pass that arrives earlier than expected.

You can swap throws in a causal diagram to create tricks with self passes. Figure 47 shows two possible self triples.

The first example can be thrown in four count. Anne throws a triple self on the first beat, a single self on the second beat and a zip on the third beat. This is the same as the solo siteswap 531. Anne could throw other period three siteswaps on these three beats, for example 441 or 522.

The second example can be thrown in four count, three count or two count. Anne throws a triple self on the first beat, a single pass on the second beat and a zip on the third beat.


Figure 48
A combination in two count


Figure 49
A combination in three count


Figure 50

A late single pass

## Combining tricks

You can combine tricks in interesting ways. For example, in two count Anne can throw an early double followed by a late double then an early triple (figure 48). Note the gap in this trick, shown by an arrow pointing two beats to the left. You can work out similar combinations in any count by repeatedly swapping throws in the causal diagram.

A nice combination in three count is pass, double self, double pass, zip (figure 49). The beauty of tricks in ambidextrous patterns is that you can start them with either hand. In figure 49 Anne throws the self double with her right hand, but could throw the same trick three beats later with her left hand.

An unusual combination, that works in any of the patterns in table 3, is a late single. On a beat when she should pass, Anne throws a self double followed by a single pass on the next beat. This causes Ben to hold through the gap. Figure 50 shows how to create this combination starting with the causal diagram for a late double and swapping two throws.

## Rewiring causal diagrams

You can rewire one juggler's passes in a causal diagram to create a pattern with an extra club. The first causal diagram in figure 51 is a six club four count with some beats labelled from a to $d$. In the second diagram the pass that was connected to a has been connected to $b$, the pass that was connected to $b$ has been connected to $c$ and so on. The new pattern is a seven club four count.

Anne:
Ben:


Figure 51

## Rewiring a causal diagram for four count

In this new pattern, all Anne's passes are quins, whereas Ben's are singles. You can slide all Ben's throws two beats to the left to get a more balanced pattern, with Anne and Ben both passing triples. This pattern is called triple self (see figure 52). In triple self Anne starts with four clubs and her first throw is a triple pass. Ben starts at the same time, but throws two selfs and then a triple pass on the third beat.


Figure 52
The seven club pattern triple self
We know that it's Anne who starts with four clubs, because in figure 52 Anne's first left hand throw is not caused by another throw, so she starts the pattern with two clubs in her left hand and four clubs altogether.

You can rewire causal diagrams to create seven club patterns for all the passing rhythms in table 3. In the case of three count you have to slide Ben's throws one and a half beats to the left to get a balanced pattern (see figure 53). In this pattern Ben starts with four clubs and his first throw is a tramline double pass. Anne waits one and a half beats before throwing a crossing double pass. All of Ben's passes are tramline passes whereas Anne's are crossing.


Figure 53

## Seven club three count

Note that there is one throw every half a beat and the four hands take it in turns to throw: Anne's right hand, Ben's right hand, Anne's left hand, Ben's left hand, and so on. Patterns with this sequence of throws are asynchronous passing patterns.

## Four handed siteswaps

## Vanilla siteswaps

You can think of vanilla siteswaps as passing patterns for two people, or in other words four hands. Vanilla siteswaps are siteswaps for asynchronous patterns and in asynchronous passing patterns there's one throw every half a beat, so each number in a four handed siteswap is the number of half beats before the same club can be thrown again.

Seven club three count is an asynchronous passing pattern so let's work out it's siteswap. Looking at the causal diagram for seven club three count, each pass causes a self two and a half beats later, so the pass is caught three and a half beats later. This means the same club can be thrown again four and a half beats later, or nine half beats later, so each pass is a 9 . Each self causes a throw on the next beat, so it's caught two beats later. This means the same club can be thrown three beats later, or six half beats later, so each self is a 6 . Now the sequence of throws in seven club three count is: Anne's right hand crossing double pass, Ben's right hand self, Anne's left hand self, Ben's left hand tramline double pass, Anne's right hand self, Ben's right hand self, Anne's left hand crossing double pass, Ben's left hand self, Anne's right hand self, Ben's right hand tramline double pass, Anne's left hand self, Ben's left hand self, and so on. Replacing each throw with a number, you get: 966966966966, so the siteswap for seven club three count is 966 .

In vanilla four handed siteswaps:

- the four hands take it in turns to throw, for example: Anne's right hand, Ben's right hand, Anne's left hand, Ben's left hand;
- there is one throw every half a beat;
- even numbers are selfs: 6 is a single self, 8 is a double self, $a$ is a triple self and so on;
- 4 is a hold;
- 2 is a zip;
- 0 is a gap;
- odd numbers are passes: 7 is a single pass, 9 is a double pass, $b$ is a triple pass and so on;
- 5,3 and 1 are impossible passes in practice;
- one juggler throws tramline single passes, crossing double passes, tramline triple passes and so on, whereas the other throws crossing single passes, tramline double passes, crossing triple passes and so on.


## Swapping throws in vanilla four handed siteswaps

You can use the circular and downright rules to create four handed siteswaps. For example, to create a period seven, six club pattern, start with: 6666666. If you swap the first two 6 s you get 7566666. This has a 5 so it's not a suitable pattern. However if you swap the 5 in this pattern with the next 6 you get 7746666 . This is a three count pattern known as Jim's three count.


Figure 54

## Jim's three count

Anne's throws in this pattern are 7466766 , or in words: pass, hold, self, self, pass, self, self. Ben's throws are 7667466, or in words: pass, self, self, pass, hold, self, self. Also Anne's passes are tramline passes whereas Ben's are crossing passes. Ben's first pass is half a beat after Anne's first pass. Note that 774,77466 and 774666666 are also valid six club siteswaps. They are Jim's one count, Jim's two count and Jim's four count.

For a period five six club pattern you could start with 77466. If you swap the 4 with the last 6 you get 77862. This is known as why not?


Figure 55
Why not?
Anne's throws in this pattern are 78276, or in words: pass, double self, zip, pass, self. Ben's throws are 76782 , or in words: pass, self, pass, double self, zip.

For a period three, seven club pattern, start with 777. If you swap the second and third 7s you get 786. This is known as French three count.


Figure 56
French three count
In this pattern Anne starts with four clubs. Her throws are 768, or in words: pass, self, double self. Ben's throws are 876, or in words: double self, pass, self.

You can also swap throws in a four handed siteswap to create tricks within the pattern. This works if all the throws that are changed are one juggler's throws and the other person's pattern is unchanged. For example in seven club three count if you start with 966966, you can swap the first 6 with the second 9 to get 9b6466. This is an early triple pass (see figure 57).


Figure 57
An early triple in seven club three count
For Anne this triple pass is a tramline pass. If Ben throws a triple pass it's a crossing pass.
You can create tricks that change both jugglers' patterns. If the first throws of the trick are Anne's, then any throws changed in Ben's patterns must become 4s, i.e. holds. Also the first change to Ben's pattern must be at least one and a half beats after the first change to Anne's pattern. For example, starting with 7777, seven club one count, you can swap the first and the last 7 to get a774. This is a self triple. You can create another trick by swapping the a with the second 7 to get 9784 . This is a double pass followed by a double self (see figure 58). Both of these tricks cause the other juggler (Ben in this example) to hold through the gap.


Figure 58
A trick in seven club one count

## Synchronous siteswaps

If the numbers in a vanilla siteswap are all even, the throws in the pattern are all selfs. This means each juggler has their own pattern, so they can throw the same time. To show this in siteswap, write the throws in brackets. The first throw in each bracket is Anne's throw and the second throw is Ben's throw. In these patterns, the hands also catch at the same time, so some of the throws can be passes. In siteswap write an $x$ after each pass.

All the patterns in table 3 are synchronous patterns. In these patterns both jugglers throw right handed throws on one beat and left handed throws on the next beat. So the throws in the first bracket are right hand throws and the throws in the next bracket are left hand throws, and so on. As an example, the siteswap for three count is: $(6 x, 6 x)(6,6)(6,6)$, and the siteswap for four count is: $(6 x, 6 x)(6,6)(6,6)(6,6)$. In patterns like these, $6 x$ and ax are tramline passes, whereas $8 x$ and $c x$ are crossing passes. $4 x, 2 x$ and $0 x$ are impossible passes in practice.

## Swapping throws in synchronous four handed siteswaps

You can use the circular, downright and handedness rules to create synchronous four handed siteswaps. For example, to create a period four, seven club pattern, start with: $(8,6)(8,6)(8,6)(8,6)$. Swap the first two 8 s to get $(a, 6)(6,6)(8,6)(8,6)$. Now swap the other two 8 s to get $(a, 6)(6,6)(a, 6)(6,6)$. It's not a passing pattern yet as there are no passes. If you swap the a and the 6 in the third bracket you get $(a, 6)(6,6)(6 x, a x)(6,6)$. This is a passing pattern, but an unusual one. However, if you swap the $a$ and the $6 x$ you get $(a x, 6)(6,6)(6, a x)(6,6)$. This is the siteswap for seven club four count (see figure 52, page 29). On the first beat Anne throws a right handed, tramline, triple pass, ax and Ben throws a right handed, single self, 6 . On the second beat Anne and Ben both throw left handed, single selfs, $(6,6)$. On the third beat Anne throws a right handed, single self, 6 and Ben throws a right handed, tramline, triple pass, ax. On the fourth beat Anne and Ben both throw left handed, single selfs, $(6,6)$. And so on.

To create a period three, seven club pattern, start with: $(8,6)(8,6)(8,6)$. Swap the throws in the first bracket to get $(6 x, 8 x)(8,6)(8,6)$. Now swap the throws in the third bracket to get $(6 x, 8 x)(8,6)(6 x, 8 x)$. This is a version of seven club pass, pass, self. For a more well known version, swap the 8 in the second bracket with the $6 x$ in the third bracket to get $(6 x, 8 x)(8 x, 6)(6,8 x)$.


Figure 59

## Seven club pass, pass, self

In this pattern Anne starts with four clubs. On the first beat Anne throws a right handed, single, tramline pass, $6 x$ and Ben throws a right handed, double, crossing pass, $8 x$. On the second beat Anne throws a left handed, double, crossing pass, $8 x$ and Ben throws a left handed, single self, 6. On the third beat Anne throws a right handed, single self, 6 and Ben throws a right handed, double, crossing pass, $8 x$. On the fourth beat Anne throws a left handed, single, tramline pass, $6 x$ and Ben throws a left handed, double, crossing pass, 8x. And so on.

You can also swap throws in a four handed siteswap to create tricks within the pattern. This works if all the throws that are changed are one juggler's throws and the other person's pattern is unchanged. For example, starting with six club four count: $(6 x, 6 x)(6,6)(6,6)(6,6)$, by the circular rule, this is the same as $(6,6)(6,6)(6 x, 6 x)(6,6)$. Now swap the third 6 with the first $6 x$ to get $(6,6)(8 x, 6)(4,6 x)(6,6)$. This is an early double in four count (see figure 43, page 26).

You can create tricks that change both jugglers' patterns. If the first throws of the trick are Anne's, then any throws changed in Ben's patterns must become 4s, i.e. holds. Also the first change to Ben's pattern must be at least one beat after the first change to Anne's pattern. For example, starting with six club one count: $(6 x, 6 x)(6 x, 6 x)$, if you swap the first $6 x$ with the third $6 x$, you get $(8 x, 6 x)(4 x, 6 x)$. This has a $4 x$, so it's not a suitable trick. However if you swap the throws in the second bracket, you get $(8 x, 6 x)(6,4)$. This is a late double in one count (see figure 45, page 27).

## Relabelling causal diagrams of synchronous patterns

You can create a variation of a synchronous passing pattern by sliding all the labels (the Rs and Ls) for one juggler's throws one beat to the left. Figure 60 shows a variation of seven club pass, pass, self with all of Anne's throws relabelled in this way.


Figure 60
Seven club pass, pass, self variation
In this pattern on one beat Anne throws with her right hand and Ben throws with his left hand and on the next beat Anne throws with her left hand and Ben throws with his right hand. Also the single passes are crossing passes, whereas the doubles are tramline passes. The siteswap for this pattern is still the same: $(6 x, 8 x)(8 x, 6)(6,8 x)$. In this pattern the throws in the first bracket are Anne's right hand throw and Ben's left hand throw and the throws in the second bracket are Anne's left hand throw and Ben's right hand throw.

You can juggle any synchronous siteswap like this. In these patterns $6 x$ and $a x$ are crossing passes whereas $8 x$ and cx are tramline passes. One popular synchronous pattern that's juggled like this is seven club two count $(8 x, 6)(6,8 x)$ (see figure 61).


Figure 61
Seven club two count
In this pattern all the passes are right handed, tramline, double passes. Anne starts with four clubs. Her first throw is a pass and Ben starts one beat later with a pass.

## Further Reading

- Siteswap Ben's guide to juggling patterns. Ben Beever. Published by the author. Ben Beever's ideas about generalising siteswap notation.
- Charlie Dancey's encyclopedia of ball juggling. Charlie Dancey. Published by Butterfingers. A lovely juggling book which includes introductions to siteswap notation, ladder diagrams and causal diagrams.
- The mathematics of juggling. Burkard Polster. Published by Springer. A very mathematical book.
- Four ball juggling. Martin Probert. Published by Veronika Probert.

Contains lots of four ball siteswaps.


[^0]:    ${ }^{1 *}$ or club or ring.

[^1]:    *Some club jugglers call them vamps.

[^2]:    * Many people juggle five clubs on double spins. They throw them with less spin, but to the same height as triples.

[^3]:    'You may prefer to write 1 f for a 1 fountain throw.

[^4]:    * or ball or ring.

