A Simple Model of Endogenous Tax Reform with Irreversible Investment and Tax Breaks

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Abstract

A model of endogenous tax reform is created, characterized by irreversible investment and firms' purchases of tax breaks from the government. As the amount of tax breaks increases, the likelihood of a reform abolishing these breaks increases. The value of tax breaks and hazard rate of tax reform are calculated. The assumption of government revenue-maximization is then adopted and the government's revenue maximization problem is solved.

1 Introduction

In discussions of tax reform, tax reform is normally exogenous and a reform is devised to satisfy some optimality criteria.[1] A lone exception is Rogers [7], in which a revenue-maximizing government switches endogenously between an income tax and a consumption tax.[2] This paper considers a business income tax model in which the government sells tax breaks, subject to (stochastic) revocation. As the volume of tax breaks increases, the revenues lost to the government increase, and the probability of a tax reform increases.[3]

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[1] See, for example, the essays in Boskin [2]. Boskin himself refers to tax code instability, but does not model it (e.g. Boskin [2] p. 13).

[2] In some cases, the tax rate is endogenous as a result of decisions made by taxpayers (Jones and O'Neill [4], Mueller [6], Leuthold [5]). However, the government's tax schedule remains fixed.

[3] The probability of a tax reform is given a precise definition in the next Section.

The economy is characterized by a continuum of identical firms. Each firm makes an irreversible investment at the beginning of the period and receives a deterministic, declining revenue stream.[4] Each firm pays the government the full value of a tax cut at a random time during the period. The accumulation of tax breaks creates the pressure for reform.

A process of this sort appears to have been at work during the creation of the Tax Reform Act of 1986. This tax reform is distinguished from other tax reforms by its emphasis on simultaneously eliminating tax breaks and decreasing marginal rates. The Senate Finance Committee began its deliberations by protecting existing tax breaks and handing out new tax breaks to constituents.[5] This process came to a halt as the proposed revenue loss was too great for passage by the full Senate. The Finance Committee then responded by proposing a new tax bill which substantially eliminated tax breaks.

Section 2 develops and solves the model. Section 3 finds the government's revenue-maximizing tax rules and rates. Section 4 concludes this paper. The Appendix describes the method used to solve the model.

2 The Model

Assume that the government sells tax rate reductions, subject to a positive probability of future reform. A firm purchases a reduction in its tax rate by paying a one-time fee to the government. As firms purchase rate reductions, the deadweight loss of the reductions increases the probability of a tax reform, which eliminates all the tax preferences, restoring the pre-cut rates.[6] This is modeled as the sum of two projects, with identical gross revenues. One project is taxed at the full tax rate τ_f and has deterministic, exponentially decaying revenues. The other project has the same revenue stream, is taxed at the rate $(1 + \tau_i - \tau_f)$, where τ_i is the pre-tax cut tax rate, but is subject to stochastic termination.[7] Project value V is the weighted sum of V^i and V^f, the projects described above:

$$V = (1 - \tau_f)V^f + (1 + \tau_i - \tau_f)V^i \tag{1}$$

One way to think of this model is that it represents an industry consisting of a continuum of identical firms with unique entry times and whose entry does not affect prices. While this is a very unrealistic assumption, it has great analytic convenience and approximates many firms entering many industries at various points in

[4] Stochastic revenues result in a stochastic differential equation, which can only be solved statistically (Coleman [3]).

[5] This discussion relies on Birnbaum [1] pp. 176–233.

[6] This paper assumes only one cut-and-reform cycle for simplicity.

[7] The subscripts i and f refer to the initial and final (i.e. with and without cut) tax rates. These are superscripted when applied to V.

time. Alternatively, instead of the firms entering at various times, their purchases of tax breaks are staggered exogenously.[8]

Let $S(t)$ be the cumulative distribution function of reform times. The *hazard rate* is defined as $S'(t)/S(t) \equiv \lambda(t)$. The hazard rate may be interpreted as the conditional probability of a reform during the infinitesimal interval $(t,\, t+dt)$, given that reform has not occurred by time t. Let X be the event of reform:

$$\Pr(X \in (t,\, t+dt)|X > t) = \frac{\Pr\{X \in (t, t+dt),\, X > t\}}{\Pr\{X > t\}} = \frac{\Pr\{X \in (t, t+dt)\}}{\Pr\{X > t\}} = \frac{S'(t)}{S(t)}dt \tag{2}$$

Conversely, the hazard rate also completely determines distribution of reform times. $S(t)$ is given by $S(t) = 1 - \exp\left\{-\int_0^t \lambda(s)\, ds\right\}$.[9]

The hazard rate for the second process must be increasing in time. One way to ensure this is to assume that it increases at a rate proportionate to the value of the tax cut.[10] That is

$$\frac{d\lambda(t)}{dt} \propto V^i(P(t)) \tag{3}$$

After integration (3) becomes

$$\lambda(t) = \Upsilon \int_0^t V^i\left(P(s)\right) ds \tag{4}$$

where Υ is a constant. P is assumed to be generated by an exponential process $P = P_0 e^{\alpha t}$, where P_0 is the initial price. Assuming that instantaneous cost is a constant c results in instantaneous profit $\pi(P, t) = P - c$.

The equation for the pre-reform project value is obtained by dynamic programming. The project value at time t, $V^i(P, t)$, is equal to its instantaneous profit $(P - c)dt$ plus the discounted expected value of the project dt later:

$$V^i(P, t) = (P - c)dt + e^{-r\,dt}\mathbf{E}_t\left[V^i(P + dP, t + dt)\right] \tag{5}$$

Since the only uncertainty comes from ending the process with hazard rate $\lambda(t)$, equation (5) becomes

$$V^i(P, t) = (P - c)dt + e^{-[r+\lambda(t)]dt}V^i(P + dP, t + dt) \tag{6}$$

[8] This could occur simply because government activities take time and government resources are limited so that there is a limit on the number of breaks that can be granted per unit of time.

[9] This exposition follows Ross [8] pp. 168–170.

[10] Linear proportionality is the simplest assumption, though not the only one.

Expanding the right hand side by Taylor's rule obtains

$$
\begin{aligned}
&(P-c)dt + e^{-[r+\lambda(t)]dt} V^i(P+dP, t+dt) \\
&= (P-c)dt + [1-(r+\lambda(t)dt]\left[V^i(P,t) + V_t^i(P,t)dt + V_P^i(P,t)\alpha P dt\right] \\
&= V^i(P,t) + \left[\alpha P V_P^i(P,t) + V_t^i(P,t) - rV^i(P,t) + P - c\right] dt
\end{aligned}
$$

Substitution into equation (5) obtains

$$\alpha P V_P^i + P - c + V_t^i - [r+\lambda(t)]\, V^i = 0 \tag{7}$$

Since $P = P_0 e^{\alpha t}$, $dP/dt = \alpha P_0 e^{\alpha t}$ and

$$V_P^i \equiv \frac{\partial V^i}{\partial P} = \frac{\partial V^i}{\partial t}\frac{dP}{dt} = \alpha P_0 e^{\alpha t} V_t^i \tag{8}$$

Substituting this into (7) obtains

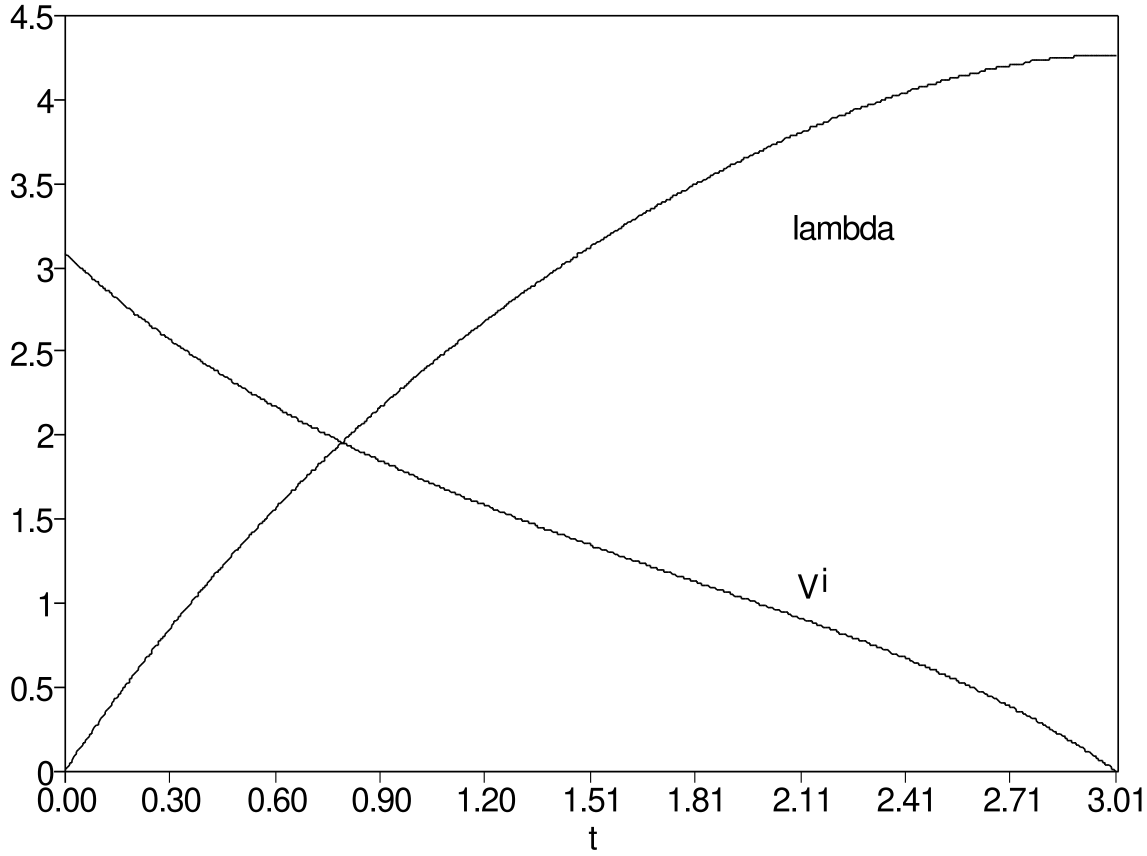
$$P_0 e^{\alpha t} - c + (\alpha^2 P_0^2 e^{2\alpha t} + 1)V_t^i - [r+\lambda(t)]\, V^i = 0 \tag{9}$$

Together with (4) this forms a first-order ordinary differential equation system. Its solution is obtained by substituting an expression for V^i in terms of $\lambda(t)$. If $d\lambda(t)/dt = \Upsilon V^i(t)$ then (9) becomes

$$P_0 e^{\alpha t} - c + (\alpha^2 P_0^2 e^{2\alpha t} + 1)\lambda_{tt} - \Upsilon^{-1}\,[r+\lambda]\,\lambda_t = 0 \tag{10}$$

This is a nonlinear ordinary differential equation which lacks an apparent analytic solution. Instead, it must be solved numerically and requires an assumption for $\lambda(0)$. Assuming that the hazard rate for reform is zero in the absence of tax breaks obtains $\lambda(0) = 0$. If $\alpha < 0$, $P = c$ when $t = T = (\log c - \log P_0)/\alpha$ and the project shuts down. Thus, we can introduce the boundary condition $V^i(T) = 0$. That is, the tax break becomes valueless when the project shuts down.[11] Figure 1 illustrates a solution to (10). This solution was obtained by the shooting method. An initial value for $V^i(0)$ was assumed, the equations for λ and V^i integrated, and the resulting $V^i(T)$ obtained. This was also done for $V^i(0) + \epsilon$. The two values of $V^i(T)$ were then used to iterate $V^i(0)$ by Newton's algorithm until the value of $V^i(0)$ was obtained such that $V^i(T) = 0 \pm 0.001$. The full details are given in the Appendix.

[11] It is interesting to note that $\lambda_{tt}(T) = 0$, as well, although Figure 1 does not show this due to numerical inaccuracy.



Parameters: $\alpha = -.1$, $c = 10$, $P_0 = 20$, $\Upsilon = 1$, $T \approx 3.01$, $\Delta t = T/1000$, $\epsilon = .0001$, $tol = .001$.

Figure 1: λ and V^i

3 A Tax Break-Selling Government

Assume that a government sells tax breaks. At the margin, firms should be willing to pay the government the exact amount of the present value of the tax cut. For simplicity, we assume that the bribe is paid at the moment of initial investment. Assume that the government charges a price of θ, $0 < \theta < 1$ for each dollar of the present value of the tax cut. A firm invests at price P if

$$V(P) - I - \theta(1 + \tau_i - \tau_f)V^i(P) \geq 0 \tag{11}$$

where I is the cost of investment, assumed identical across the economy. This equation simply states that investment occurs if the net present value is nonnegative.[12] Substituting (1) into the above obtains

$$(1 - \tau_f)V^f(P) + (1 - \theta)(1 + \tau_i - \tau_f)V^i(P) - I \geq 0 \tag{12}$$

The government's tax receipts are the sum of bribes and tax revenues. In the case of one firm, the government receives $\theta(1 + \tau_i - \tau_f)V^i(P)$ at the moment of investment. Thereafter, it receives the stream $\tau_i(P - c)$ until the tax reform. Then, it receives the greater stream $\tau_f(P - c)$. If the government's discount rate is large enough, it may be optimal for the government to seek bribes for tax cuts, and then eliminate those cuts at a later, stochastic time.

Formally, we wish to compare the two income streams with and without the bribe and tax cut. We continue to use the deterministic model. We assume that investment is immediately optimal given the parameters. In the absence of the bribe and tax cut, the present value of the government's tax stream is

$$\int_0^T \tau_f e^{-dt} \left(P(t) - c\right) dt \tag{13}$$

where d is the government's discount rate. τ_f is used as it is the government's pre-tax cut tax rate. Substituting this and $P(t) = P_0 e^{\alpha t}$ obtains

$$\begin{aligned} \int_0^T \tau_f e^{-dt} \left(P(t) - c\right) dt &= \tau_f \int_0^{T_e} e^{-dt} \left(P_o e^{\alpha t} - c\right) dt = \tau_f \left(\frac{P_0}{\alpha - d} e^{(\alpha - d)t} + \frac{c}{d} e^{-dt} \right) \Bigg|_0^T \\ &= \frac{P_0 \tau_f}{\alpha - d} \left[\left(\frac{P_0}{c} \right)^{\frac{\alpha}{\alpha - d}} - 1 \right] + \frac{c\tau_f}{d} \left[\left(\frac{P_0}{c} \right)^{\frac{\alpha}{d}} - 1 \right] \end{aligned} \tag{14}$$

[12] The net present value rule is appropriate because of the assumption of a deterministic, declining P. Coleman [3] pp. 29–31 has a general discussion of investment rules when $P(t) = P_0 e^{\alpha t}$ and there are no taxes.

The present value of the government's revenues with bribes and a stochastic reform is

$$\theta(1+\tau_i-\tau_f)V^i(P_0)+\int_0^T \tau_f e^{-dt}\,(P(t)-c)\,dt+\mathbf{E}_s\int_0^s e^{-dt}(\tau_i-\tau_f)\,(P(t)-c)\,dt \tag{15}$$

The government's net profit from selling tax cuts is the difference between (13) and (14):

$$\theta(1+\tau_i-\tau_f)V^i(P_0)+\mathbf{E}_s\int_0^s e^{-dt}(\tau_i-\tau_f)\,(P(t)-c)\,dt \tag{16}$$

We evaluate (16) by noting that the cumulative distribution function of tax reform at time t is $\exp(-\int_0^t \lambda(s)\,ds$:

$$\mathbf{E}_s\int_0^s e^{-dt}(\tau_i-\tau_f)\,(P(t)-c)\,dt=(\tau_i-\tau_f)\int_0^T e^{-\int_0^s\lambda(t)\,dt}ds\int_0^s\left(P_0e^{(\alpha-d)t}-ce^{-dt}\right)dt$$

$$=(\tau_i-\tau_f)\int_0^T e^{-\int_0^s\lambda(t)\,dt}\left(\frac{P_0}{\alpha-d}(e^{(\alpha-d)s}-1)+\frac{c}{d}(e^{-ds}-1)\right)ds \tag{17}$$

This integral has to be evaluated numerically.

Assume that the demand for tax breaks is a continuous function $D(\theta,\,\tau_i)$. We impose the following conditions on D:

$$\begin{aligned}
D(\theta,\,\tau_f) &= 0 && (18)\\
D(1,\cdot) &= 0 && (19)\\
\frac{\partial D}{\partial\theta} &< 0 && (20)\\
\frac{\partial D}{\partial\tau_i} &< 0 && (21)
\end{aligned}$$

Condition (18) states that there is zero demand for a null tax break. Likewise, when a firm pays the full value of the tax break, it has zero demand for it as shown by condition (19). Condition (20) states that demand is declining in the price of tax breaks. Condition (21) states that the demand for tax breaks decreases in the initial tax rate. The government's choice problem becomes

$$\max_{\theta,\,\tau_i}W=D(\theta,\,\tau_i)\left[\theta(1+\tau_i-\tau_f)V^i(P_0)+(\tau_i-\tau_f)\int_0^T e^{-\int_0^s\lambda(t)\,dt}\left(\frac{P_0}{\alpha-d}(e^{(\alpha-d)s}-1)+\frac{c}{d}(e^{-ds}-1)\right)ds\right] \tag{22}$$

Given that W satisfies appropriate conditions, the revenue maximizing $(\theta^*,\,\tau_i^*)$ can be found. It is quite possible that $\tau_i^*<0$. That is, the government subsidizes the firm. The first-order conditions for this problem are

$$\frac{\partial W}{\partial \theta} = D(\theta, \tau_i)\,(1+\tau_i-\tau_f)\,V^i(P_0) + D_\theta(\theta,\,\tau_i)\left[\theta(1+\tau_i-\tau_f)V^i(P_0)+(\tau_i-\tau_f)K\right] = 0 \quad (23)$$

$$\frac{\partial W}{\partial \tau_i} = D(\theta,\,\tau_i)\left(\theta V^i(P_0)+K\right) + D_{\tau_i}(\theta,\,\tau_i)\left[\theta(1+\tau_i-\tau_f)V^i(P_0)+(\tau_i-\tau_f)K\right] = 0 \quad (24)$$

where K is the outer integral. One choice for $D(\theta,\,\tau_i)$ is $(1-\theta)(\tau_i-\tau_f)$. In this case, the first-order conditions become

$$\frac{\partial W}{\partial \theta} = (1-\theta)(\tau_i-\tau_f)\,(1+\tau_i-\tau_f)\,V^i(P_0) - (\tau_i-\tau_f)\left[\theta(1+\tau_i-\tau_f)V^i(P_0)+(\tau_i-\tau_f)K\right] = 0 \quad (25)$$

$$\frac{\partial W}{\partial \tau_i} = (1-\theta)(\tau_i-\tau_f)\left(\theta V^i(P_0)+K\right) + (1-\theta)\left[\theta(1+\tau_i-\tau_f)V^i(P_0)+(\tau_i-\tau_f)K\right] = 0 \quad (26)$$

This system has solution

$$\theta^* = \frac{\sqrt{(V^i(P_0)+K)(V^i(P_0)+9K)}+V^i(P_0)-3K}{4V^i(P_0)} \quad (27)$$

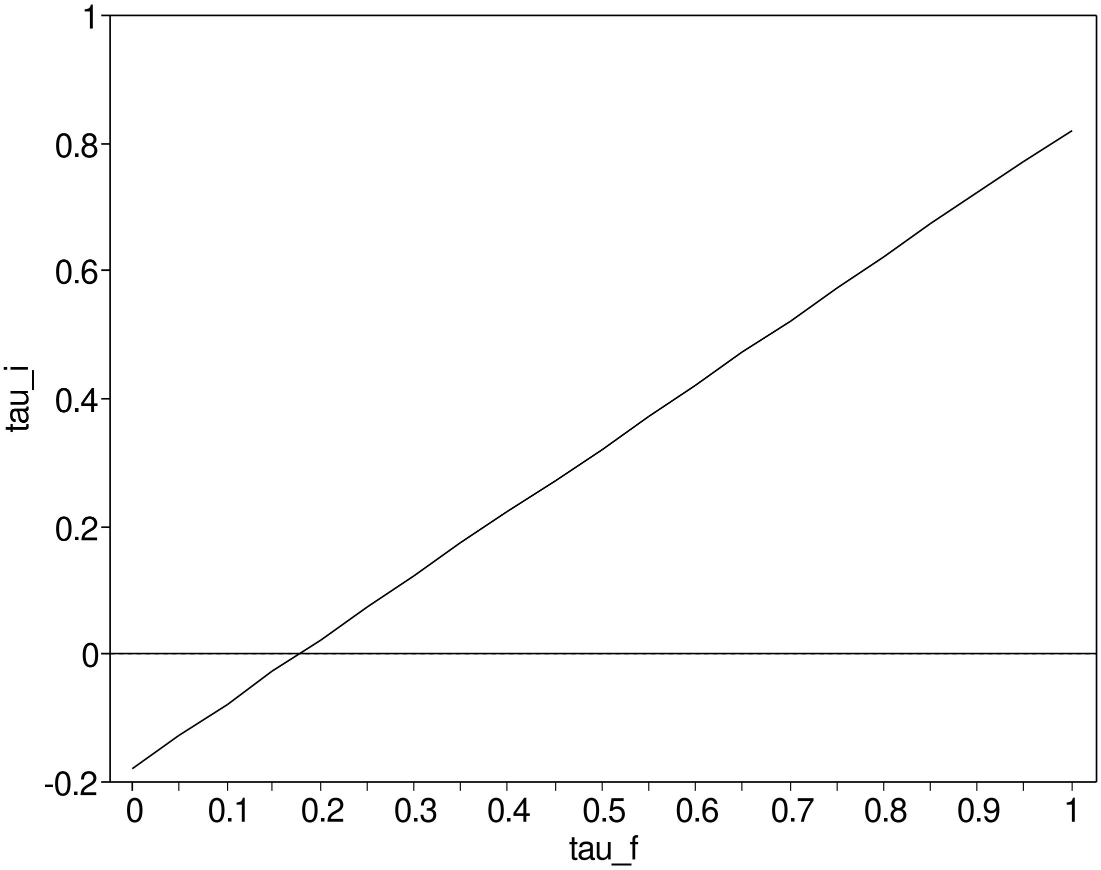
$$\tau_i^* = \tau_f + \frac{1}{4}\sqrt{\frac{V^i(P_0)+9K}{V^i(P_0)+K}} - \frac{3}{4} \quad (28)$$

The forms that these solutions take are artifacts of the linearity of D in θ, τ_i. Maintaining the assumptions used in Figure 1 obtains $V^i(P_0) = 3.089$ and $K = 3.453$, where K is obtained by trapezoidal rule integration over the grid used to solve (10). $\theta^* = .622$ and $\tau_i^*(0) = -.179$. Figure 2 plots τ_i^* as a function of τ_f, using the same assumptions as Figure 1, plus the assumption that $d = .2$.

Two things about Figure 2 are noteworthy. For values of τ_f below .179, $\tau_i^* < 0$. That is, when τ_f is low enough, it is optimal for the government to sell subsidies. When $\tau_f = 1$, $\tau_i^* = .821$. In this case, the government may thought of as selling licenses to operate, with stochastic expirations. Instead of the cumulative revenue loss leading to tax reform, this scenario can be driven by a limited supply of licenses. The government every now and then expires old licenses in order to have new ones to sell. This is sensible when the government's discount rate is high enough.

4 Conclusion

This paper has constructed a simple process by which tax preferences may be generated and stochastically expired. This process is consistent with the process that led to the Tax Reform Act of 1986. If anything, it appears that history has repeated itself: the tax bills of 1990 and 1993 undid much of the 1986 reforms.



Note: Assumptions are the same as in Figure 1.

Figure 2: τ_i as a function of τ_f

It is likely that tax breaks will continue to increase over time until, at some unknowable date, the political system will be unable to accommodate additional preferences. Tax reform will ensue and the process will repeat.

Appendix

This is a type of boundary problem which has boundary conditions at both endpoints. The shooting method introduces an additional parameter over which the problem can be iterated until the optimal value of that parameter is found. This parameter itself constitutes an additional boundary condition.[13]

By substituting $\lambda_t = \Upsilon V^i$ in equation (10), we re-express it as the system

$$\frac{d}{dt}\begin{bmatrix} \lambda \\ V^i \end{bmatrix} = \begin{bmatrix} \Upsilon V^i \\ \frac{(r+\lambda)V^i - P_0 e^{\alpha t} + c}{\alpha^2 P_0^2 e^{2\alpha t} + 1} \end{bmatrix}$$

We use Euler's forward method to integrate this system by the approximation[14]

$$\begin{bmatrix} \lambda(t+\Delta t) \\ V^i(t+\Delta t) \end{bmatrix} \approx \begin{bmatrix} \lambda(t) \\ V^i(t) \end{bmatrix} + \Delta t \begin{bmatrix} \Upsilon V^i(t) \\ \frac{[r+\lambda(t)]V^i(t) - P_0 e^{\alpha t} + c}{\alpha^2 P_0^2 e^{2\alpha t} + 1} \end{bmatrix} \tag{29}$$

where $\Delta t = T/n$ for some integer n. We set $\lambda(0) = 0$ and choose a positive value for $V^i(0)$. Integration is then accomplished by iterating (29) until $t = T$.

The solution for $V^i(0)$ is found by Newton's algorithm. It proceeds by choosing an initial value for $V^i(0)$, and calculating $V^i(T)$ by the above integration. If $\mid V^i(T) \mid > tol$, where tol is the chosen tolerance, a new value of $V^i(0)$ is obtained from the formula

$$V_{j+1}^i(0) = V_j^i(0) - \frac{V_j^i(T; V_j^i(0))}{\left[V_j^i\left(T; V_j^i(0)\right) - V_j^i(T; V_j^i(0) + \epsilon)\right]/\epsilon} \tag{30}$$

where the subscripts are the iteration numbers and $\epsilon > 0$ is a small constant. The second argument of $V_j^i(T; \cdot)$ is the initial value of $V^i(0)$ used in the iteration. In effect, two integrations are done: one with $V_j^i(0)$ and the other with $V_j^i(0) + \epsilon$.

The numerical work was done in Quattro Pro for DOS v. 5.0 on an 80386.

[13] See Zwillinger [9] pp. 631–634 for a fuller explanation.
[14] See Zwillinger [9] pp. 653–658.

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