«Ενεργός έλεγχος του ήχου σε μικρά παραλληλεπίπεδα δωμάτια: πειράματα στο αξονικό στάσιμο κύμα (1, 0, 0)»

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Παρουσιάζονται πειραματικά δεδομένα για την τοπική ενεργή καταστολή του ήχου σε παραλληλεπίπεδα δωμάτια. Αρχικά υπολογίζεται η συχνοτική απόκριση του αντηχητικού θαλάμου διαστάσεων (3.95 x 2.38 x 3.15, όλες οι διαστάσεις σε μέτρα) καθώς επίσης επαληθεύονται και από τις μετρήσεις.

Εφαρμόζεται η κυματική ανάλυση προκειμένου να καθοριστεί η συνεισφορά του καθενός στάσιμου κύματος. Σε αυτή την μελέτη δίνεται έμφαση στο αξονικό στάσιμο κύμα (1,0,0). Χρησιμοποιείται η ακουστική δυναμική ενέργεια E_p, *σαν το μέτρο για την αναγνώριση της βέλτιστης θέσης των πηγών για το ελεγχόμενο πεδίο.*

Μελετώνται τέσσερις περιπτώσεις. Σε κάθε περίπτωση, εξετάζονται διαφορετικές θέσεις της πρωτεύουσας και δευτερεύουσας πηγής και του σημείου επίλυσης (μικροφώνου). Οι περιπτώσεις 1 και 2 αναφέρονται αντίθετες θέσεις μεταξύ των πηγών και του σημείου επίλυσης κοντά στην δευτερεύουσα και πρωτεύουσα πηγή αντίστοιχα. Οι περιπτώσεις 3 και 4 αναφέρονται σε κοντινές θέσεις μεταξύ των πηγών και του σημείου αντίστοιχα κοντά και αντίθετα από τις πηγές.

Η συχνότητα του αξονικού στάσιμου κύματος ορίσθηκε στα 44.75 Hz. Τα ελεγχόμενα και τα μη ελεγχόμενα ηχητικά πεδία μετρήθηκαν για την συγκεκριμένη συχνότητα. Επιπρόσθετα με αυτές τις μετρήσεις μία εκτός συντονισμού μέτρηση στα 49 Hz έγινε για την περίπτωση 2, για να εξεταστή το εύρος του ενεργού ελέγχου.

Τα αποτελέσματα έδειξαν ότι σε όλες τις περιπτώσεις τα ελεγχόμενα ηχητικά πεδία είναι περισσότερο ομοιόμορφα από τα μη ελεγχόμενα πεδία. Η περίπτωση 4 (η οποία αντιστοιχεί στην πρωτεύουσα και δευτερεύουσα πηγή κοντά και στο σημείο επίλυσης απέναντι), έχει επιτευχθεί μέγιστη μείωση των 14.7 dB από την μέγιστη τιμή του μη ελεγχόμενου πεδίου.

«Active control of sound in small rectangular rooms: experiments on axial mode (1,0,0).»

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ABSTRACT

Experimental results are presented for the local active control of sound in rectangular rooms.

Initially the frequency response of a reverberant chamber (3.95 x 2.38 x 3.15, all dimensions in meters) has been estimated theoretically and verified by measurement.

Modal analysis has been applied in order to determine the mode contributions. In this study, emphasis is placed on the axial mode (1,0,0). The acoustic potential energy E_p *has been used as a figure of merit for the identification of the optimum location of the sources for the controlled sound field.*

Four cases have been investigated. In each case, different positions for the primary and secondary sources pair and the solution point (microphone) are considered. Case 1 and 2 refer to opposite source locations with the solution point next to secondary and primary respectively. Case 3 and 4 refer to the placement of the sources in adjacent positions and the solution point close and opposite to the sources locations respectively.

The axial mode (1,0,0) frequency has been determined at 44.75 Hz. Both controlled and uncontrolled sound fields were measured at that specific frequency. In addition to these measurements, an off-resonance of 49 Hz measurements was performed for test case 2 to examine the bandwidth of active control.

The results have shown that in all cases controlled sound fields are more uniform in comparison with the uncontrolled field. Case 4 (corresponding primary and secondary sources close each other and solution point in opposite location) to has achieved 14.7 dB reduction of the maximum value of the uncontrolled field.

1. Introduction

The idea of the active cancellation of the sound field is not a new one. Lueg made the first theoretical proposals on active control in 1936. He explained that the cancellation of the plane wave in a tube by an anti-phase wave was due to the difference in the speed of propagation between the acoustic wave and the signal transmitted electronically. No other publications were written until 1953 when Olson and May introduced their electronic sound absorber. It consisted of a loudspeaker, amplifier and a microphone close to the loudspeaker, and connected so that, for an incident sound, the sound pressure at the microphone is reduced. A small level of control was achieved; quiet zones near the microphone were created.

2. Theoretical Section

2.1 The Driven Standing Wave in the rectangular room

Solving the homogeneous wave equation in an enclosure with lossy boundaries has derived the previous equations. If a sound source is placed in the room with frequency *f* and source strength q $e^{j\omega t}$, the particular solution of the wave equation should be obtained:

$$\nabla^2 p - k^2 \frac{\partial^2 p}{\partial t^2} = - \rho \frac{\partial q}{\partial t} \qquad \text{eq. 1}$$

The steady-state distribution of sound in the room can be represented as a series $\sum_{n=0}^{\infty} A_n \ \psi_n$ with the series satisfying the latter inhomogeneous equation.

Morse (1948) has shown, that the steady-state pressure equation in the room is:

$$P_{rev} = \rho c^2 \sum_{n=0}^{\infty} \frac{q\omega}{V \Lambda_n} \frac{\Psi_n(r)\Psi_n(r_o)}{2\omega_n\zeta_n + j(\omega^2 - \omega_n^2)} e^{j\omega t} \qquad \text{eq. 2}$$

where:
r: the vector of the point being considered.
r_o : the location of the primary source.
ρ: is the constant equilibrium density of the air.
c: is the phase speed of sound.
q: is the source density.
ω_n: is the resonance frequency of the mode.
ω: is the driven frequency of the source.
$\Psi_n(r)$: the normal function at point r.
$\Psi_n(r_o)$:is a constant multiplier of the mode n.
$æ_n$: the damping term in response of the n-the mode.
Λ_n: is a scaling factor equal to $1/\ \varepsilon_{nx}\varepsilon_{ny}\varepsilon_{nz}$ determined by the mode numbers.

The sound pressure at a point $r_i\,(x,\ y,\ z)$ is the sum of the waves corresponding to the different normal modes of the room, each with an amplitude proportional to the values of the standing wave at the source and at r_i *(x, y, z)* and inversely proportional to the " impedance" of the standing wave:

$$\frac{2\,\omega_n \zeta_n}{\omega} + j[\,\omega - (\frac{\omega_n^{\;2}}{\omega})] \qquad \text{eq. 3}$$

Those terms in the summation for which $\omega_n \approx \omega$ will be of significant amplitude and they called the *dominant mode.* Least significant modes, called *residuals,* but also important in summation are those which $\omega_n \neq \omega$.

2.2 The total sound pressure field in the rectangular enclosure

Equation (2) has been derived by Morse (1948), as the solution of the wave equation (1). Maa (1989) has shown that this solution of the wave equation is not the entire solution but only the part of the solution that represents the reverberant sound field of the room.

The sound source emits a spherical sound wave, propagating along radial vectors in all directions from the source. The sound pressure is inversely proportional to the distance from the source. This will be the case before the sound waves reach the boundaries. As the direct waves reach the boundaries, they cause interference with the earlier reflected waves to form a number of standing waves; the normal modes of vibration of the enclosure.

Thus the sound field in the room consists of direct waves and the normal modes, so the complete solution of the wave equation should be:

$$P_{tot} = \frac{j\omega\rho q}{4\pi\,|\,r - r_o\,|} e^{j\omega(t - \frac{|r - r_o|}{c})} + \rho\, c^2 \sum \frac{q\omega}{V\,\Lambda_n} \frac{\Psi_n(r)\Psi_n(\,r_o\,)}{2\,\omega_n \zeta_n + j(\,\omega^2 - \omega_n^{\;2}\,)} e^{jomegat} \qquad \text{eq. 4}$$

The direct sound pressure has a continuous spectrum within the frequency band of the source, while the modal reverberant pressure appears as peaks on the spectrum of the reverberant sound.

The spectra of the sound at increasing distances from a primary source in an enclosure as Maa(1994) testified experimentally has the following properties:

The spectra contain both the continuous background of the direct sound and peaks of the normal modes.

The heights of the peaks remains more or less unaffected as the distance increases, and

the continuous background becomes lower in sound pressure, making the peaks more and more conspicuous.

2.3 An Active Local Control System for the perfect cancellation

The local control strategy is considered for a small rectangular enclosure. The simple situation is that, two simple sources excite the sound field in the enclosure. The primary source is located at P, has constant source strength q_p. The secondary source is located at S and has controllable source strength q_s. There is also a microphone is located at M, which should be the solution point.

The sound field at the point M due to the primary and the secondary source is given by applying the principle of linearity by the formula:

$$P_M = q_p Z_{PM} + q_s Z_{SM} \qquad \text{eq. 5}$$

where:

Z_{PM} :is the acoustic transfer impedance from the primary source to the cancellation point M.

Z_{SM} :is the acoustic transfer impedance from the secondary source to the cancellation point M.

Each of these has two components:

$$Z_{PM} = Z_{PM\,(dir)} + Z_{PM\,(rev)} \qquad \textit{eq. 6}$$

The first term of equation (6) corresponds to the direct field of the primary source. The second term refers to the reverberant field produced by reflection to the room boundaries and described with a series of plane waves $Z_{(rev)} = \Sigma\, b_n\, \psi_n$.

In order for the secondary source to drive the pressure at M to zero the optimum velocity for the secondary source is calculated as:

$$q_s = -\frac{Z_{PM}}{Z_{SM}} q_p \qquad \text{eq. 7}$$

2.4 Acoustical Potential Energy as an index of better control

Global control is achieved when the sound pressure level is reduced at all positions in the enclosure. In practical implementation, the minimization of the total acoustic potential energy E_p of the enclosure is applied. This strategy has the effect of levelling out the spatial variation of the sound pressure in the enclosure. Accordingly the sound level should be increased at some points where it was low, but falls to reduce it where it was high.

As has been shown in section 2.1 that the reverberant sound pressure field in the enclosure can be written as the sum of a series of the acoustic modes:

$$p(r) = \sum_{n=0}^{\infty} A_n \psi_n(r) \qquad \text{eq. 8}$$

where:

r: is a point in the rectangular enclosure,

A_n : are complex coefficient that quantify the extent to which a given mode is excited. They are the mode amplitudes and depend on the frequency.

Assuming that in the enclosure the sound field is excited by a single point primary source of complex source strength q_p, located at y_p and by a single point secondary source of complex source strength q_s,,located at y_s, coefficient A_n should be written:

$$A_n = \frac{\rho_o c_o^2 \omega}{\Lambda_n V [2 \omega_n k_n + j(\omega^2 \omega_n^2)]} [q_p \psi_n(y_p) + q_s \psi_n(y_s)] \qquad \text{eq. 9}$$

The extent to which a particular mode can be driven by a single source depends on the value of the shape function at the position of the source. The particular mode could be attenuated by making the coefficient A_n equal to zero. This could be obtained by choosing an appropriate value for the secondary source strength such as:

$$q_s = \frac{\psi_n(y_p)}{\psi_n(y_s)} q_p \qquad \text{eq. 10}$$

In general when an appropriate value for the secondary source strength is selected, for a particular mode to be zero everywhere, this does not assume that an increase does not happen to other modes.

At low frequencies near to the natural frequencies where the modal density is low and the sound field is dominated by the response of a single mode, the global control has successfully achieved.

The acoustic potential energy E_p, is proportional to the volume integrated mean square acoustic pressure in an enclosure, being defined as:

$$E_p = \frac{1}{4 \rho_o c_o^2} \int |p|^2 dV \qquad \text{eq. 11}$$

The above equation (11), using equation (8) and (9) is arranged for the primary and secondary source as:

$$E_p = \frac{V}{4 \rho_o c_o^2} \sum_{0}^{\infty} |A_n|^2 \qquad \text{eq. 12}$$

The measurement of this energy and the minimization of a suitable quadratic cost function is used when the global control in the enclosure applied in practice.

Elliott *et al* (1987) showed that the acoustic potential energy can be approximated by a quantity J_p proportional to the sum of the mean square pressures at a finite number of discrete points, defined as:

$$J_p = \frac{V}{4\rho_o c_o^2 L} \sum_{l=1}^{L} |p_l|^2 \qquad \text{eq. 13}$$

where: pl is the pressure at the l-th position in the enclosure and L is the total number of the measurement points.

At low modal densities in a lightly damped enclosure the sum of the squares of the pressures at a relative small number of carefully selected positions is also a good approximation to the acoustic potential energy Ep. These locations are the maxima of the primary sound field and for the rectangular enclosure is each corner.

3.1 Simulations

3.1 Spectral analysis of the best attenuation in a local control system, using the acoustic potential energy Ep

The reduction of the total acoustic potential energy has been suggested as a criterion by which the effectiveness of the active control systems in enclosures can be evaluated.

In the following, simulations have been done for four different cases, by locating the primary, secondary source and solution point in different places. The acoustic potential energy E_p is calculated with equation (12) using the theory developed in section 2.3.

The local active control of the system was applied, by designing the secondary source strength q_s, such that the pressure at the solution point was perfectly cancelled (driven to zero). The frequency range was 0 Hz to 100 Hz, with 1 Hz step and the secondary source strength was calculated for each step. The sound pressure field was assumed to consist of the direct and reverberant part, described in equation (4) in section 2,2. A thousand modes were summed for the reverberant field calculations.

The theory presented above is applied in various cases in this section.

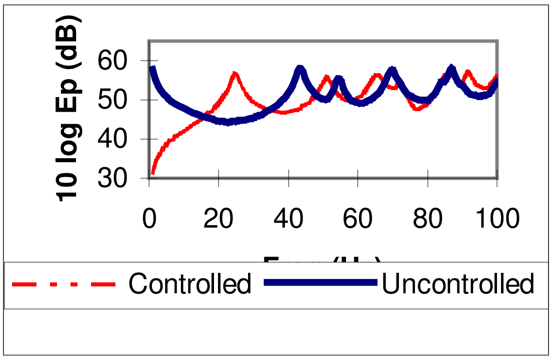
Four different cases for various primary and secondary source and microphone locations are considered:

Case 1: Microphone close to secondary source, sources opposite side.
P(3.65, 1.00, 1.00) S(0.25, 1.00, 0.85) M(1.00, 1.50, 1.00)
Case 2: Microphone close to primary source, sources opposite side.
P(3.65, 1.00, 1.00) S(0.25, 1.00, 0.85) M(3.09, 0.60, 1.00)
Case 3: Microphone close to sources, sources same side.
P(0.30, 0.50, 1.00) S(0.25, 1.00, 0.85) M(1.00, 1.50, 1.00)
Case 4: Microphone away from sources, sources same side.

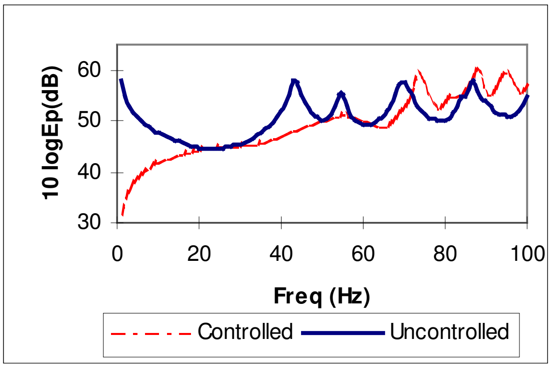
P(0.30, 0.50, 1.00) S(0.25, 1.00, 0.85) M(3.09, 0.60, 1.00)

The sensors for calculations of the acoustic potential energy were placed in the eight corners of the rectangular enclosure: M_1 (0.1, 0.1,0.1), M_2 (3.8, 0.1,0.1), M_3 (3.8, 2.18,0.1), M_4 (0.1, 2.18, 0.1), M_5 (0.1, 0.1,3), M_6 (3.8, 0.1,3), M_7 (3.8, 2.18,3), M_8 (0.1, 2.18, 3).

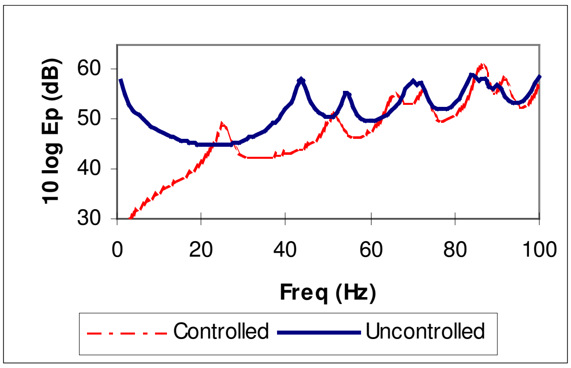
Figure 2 shows the acoustic potential energy E_p due to the primary source alone and with the primary and secondary source adjusted to cancel the pressure at a point in the enclosure.



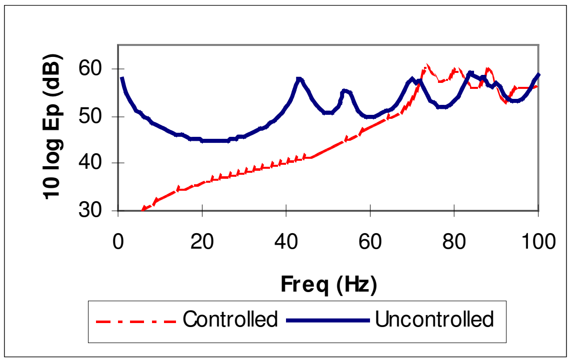
Case 1: The microphone is placed close to the secondary source and the sources are placed to the opposite side.



Case 2: The microphone is placed close to the primary source and the sources are placed to the opposite side.



Case 3: The microphone is placed close to the sources and the sources are placed to the same side.



Case 4: The microphone is placed away form the sources and the sources are placed to the same side.

Figure 2: The total acoustic potential energy E_p, of the uncontrolled and the controlled sound field.

When the microphone is close to the secondary source and the sources are placed far away, there is poor control of the sound field. It is observed 10 dB reduction at 43.42 Hz and almost no reduction is achieved at the other resonant frequencies. The attenuation becomes better with the microphone close to the primary source and the sources placed far away. Excepting of the 10 dB reduction at 43.42 Hz, there is also about 3 dB reduction of the resonant mode at 54.44 Hz. The control of the resonant frequencies is improved as the sources are placed close each other and the microphone close to them. The reductions are 13 dB at 43.42 Hz, 9 dB at 54.44 Hz and 3 dB around 70 Hz. Finally, the best control is achieved when the sources are close each other and the microphone is placed far away, to the other side of the enclosure. The reductions are observed about 18 dB at 43.42 Hz, 10 dB at 54.44 Hz and 6 dB at 70 Hz. There is also a significant reduction at off- resonant frequencies.

Comparing the above four control cases the best attenuation over the biggest range of frequencies, is obtained when the sources are close each other and the microphone far away from them. This happens because the sources are coupled to most of the modes in the same way and the solution point far away detects the reverberant field rather than the direct fields of the sources.

4. Experimental Section

4.1 Description of apparatus

The measurements were carried out in the rectangular reverberant chamber with volume 29.6 m^3 and dimensions (3.95 x 2.38 x 3.15 m). The dimensions of the two loudspeakers are: primary source (0.6 x 0.34 x 0.30 m) with cone diameter r_p=0.105 m and secondary source (0.525 x 0.305 x 0.23 m) cone diameter r_s=0.0815 m.

The instruments used are as follows: Dual Phase Oscillator, Oscilloscope, Power Amplifier, Frequency Meter, Phase Meter, ONOSOKKI FFT Analyser, B&K 4165 Microphone. Additionally a purpose built rectifier that connects an array of equally spaced electrets microphones via an A/D card to a computer is used. The computer is used to collect the data measured for the uncontrolled and controlled field.

Ten electrets (Maplin) Microphones of diameter 6mm are placed ever 25cm on a wooden beam of 2.28 m length. This dimension represents the y-axis of the enclosure. Thus, if the wooden beam is slightly moved perpendicular to the x-axis of the enclosure the whole enclosure is easily scanned from collection of contour plots.

The primary and secondary sources are connected to a dual phase oscillator that allows the amplitude and the phase of the secondary source to be adjusted in order to cancel the acoustic pressure at a point in the field. The sources are driven with a sinusoidal wave. A frequency meter is connected to the oscillator to measure the frequency of excitation accurately. The oscilloscope is used to measure the relevant amplitude and phase of the signal fed to the primary and secondary source. A B&K phase meter is also used for more accurate measurement between the signal phases, because is important for the active control of the sound. The B&K microphone is located at the solution point and is connected to the Onosokki FFT Analyser. The 10 equal spaced microphone array is connected though a rectifier to an A/D PC card.

4.2 Frequency response of the reverberation chamber

First the low frequency response (spectrum) of the primary source in the rectangular enclosure is measured. A white noise of frequency range of 0 Hz -200 Hz is applied in the chamber and the spectrum is measured at the point M (1.0, 1.5, 1.0).

There are obvious resonances at 44.75 Hz, 55 Hz, 70 Hz, and so on.

In Figure 1 is shown the frequency response measured in the reverberant chamber with the FFT Analyser.

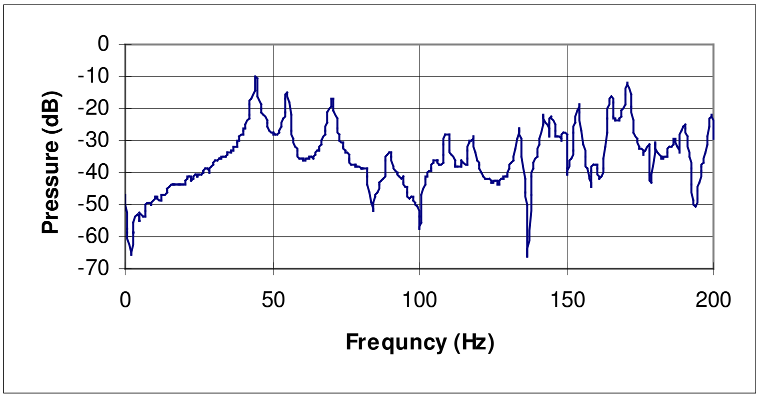


Figure 1: Frequency response of the reverberant chamber measured with the FFT Analyser.

The y-axis decibel scale in figure 1 is arbitrary.

4.3Perfect Cancellation of the sound pressure field at a point

4.3.1 The resonant frequency 44.75 Hz of the Axial Mode (1,0,0)

Description		**Case 1**	**Case 2**	**Case 3**	**Case 4**
Pressure at the solution M:	***Uncontrolled:***	-17.25 dB	-13.11 dB	-17.67 dB	-18.54 dB
	Controlled:	-56.36 dB	-48.33 dB	-62.22 dB	-57.97 dB
	Attenuation:	*39.11 dB*	*35.22 dB*	*44.55 dB*	*39.43 dB*
$Vp_{(p-p)}$		0.86 V	0.8 V	0.72 V	0.6 V
$Vs_{(p-p)}$		2 V	1.8 V	1.64 V	1.68 V
Z_{pm}		$0.322\angle 11.7^{0}$	$0.395 \angle -174.6^{0}$	$0.354 \angle -163.1^{0}$	$0.378 \angle 19.2^{0}$
Z_{sm}		$0.145 \angle -146^{0}$	$0.165 \angle 27.4^{0}$	$0.155 \angle -36.7^{0}$	$0.165 \angle 142.9^{0}$
Z_{ps}		$0.431\angle 22.9^{0}$	$0.427 \angle 20.6^{0}$	$0.440 \angle 20.1^{0}$	$0.456 \angle 17.7^{0}$

Table 1: Setting of the sources for the active control of sound of the axial mode (1,0,0)

4.3.2 The off-resonant frequency 49 Hz

Description		**Case 2**
Pressure at the solution M:	***Uncontrolled:***	-28.10 dB
	Controlled:	-58.12 dB
	Attenuation:	*30.02 dB*
$Vp_{(p-p)}$		2.2 V

***Vs**$_{(p-p)}$*	2.2 V
$\boldsymbol{Z_{pm}}$	0.291e-1 $\angle$ 98.2^0
$\boldsymbol{Z_{sm}}$	0.253e-1 $\angle$ 54.3^0
$\boldsymbol{Z_{ps}}$	0.872 $\angle$21.8^0

Table 2: Setting of the sources for the active control of sound of off resonance frequency 49 Hz

4.3 Results of the controlled field

In order to classify the amount of the controlled field relevant to the uncontrolled the space averaged mean square pressure of the primary source to the secondary source is calculated

$$e = 20\log(\frac{\langle p^2{}_p \rangle}{\langle p^2{}_s \rangle} \qquad \text{eq. 14}$$

Cases	Description of sources and microphone locations	e (dB) space averaged mean square pressure
1 **44.75 Hz**	The microphone is placed close to the secondary source and the sources are placed to the opposite side.	**23.0**
2 **44.75 Hz**	The microphone is placed close to the primary source and the sources are placed to the opposite side.	**17.7**
3 **44.75 Hz**	The microphone is placed close to the sources and the sources are placed to the same side.	**23.1**
4 **44.75 Hz**	The microphone is placed away form the sources and the sources are placed to the same side.	**29.9**
2 **49.00 Hz**	The microphone is placed close to the primary source and the sources are placed to the opposite side.	**3.6**

Table 3: Attenuation achieved in controlled sound field

Conclusions

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References

[1] Morse, P.M. (1948). *Vibrations and Sound*, 2nd edn. McGraw-Hill, New York (reprinted in 1981 by the Acoustical Society of America).

[2] Maa, D. Y. (1989) *Formulae of sound Field in enclosures*, Chinese J. Acoustics 8 [4], pp 373-375.

[3] Maa, D. Y. (1994) *Sound Field in a Room and Its Active Noise Control,* Applied Acoustics 41 pp 113-126.

[4] Elliott, S.J, Cutris, R.D, Bullmore, A.J., and Nelson, P.A. (1987). The Active Minimization of the Harmonic Enclosed Sound Fields, Part III: Experimental Verification. Journal of Sound and Vibration 117, pp 35-58.

[5] Nelson, P.A. and Elliott, S.J. (1992). *Active Control of Sound*. Academic Press Limited, Southampton

[6] Avis, M. R. and Darlington, P.(1995). *Modifying low frequency room acoustics 1: active de-reverberation* . Proc IOA 17 (7), pp. 77- 86.

[7] Curtis, A.R.D., Nelson, P.A., Ellioitt, S.J and Bullmore, A.J. (1987). *The active suppresion of acoustic resonances.* Journal of the Acoustical Society of America 81, pp 624-631.