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Conservation of Solar Cell Optoelectronic Parameters

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Abstract

Application of the standard one-diode model on real solar cell current-voltage characteristics often leads to physically meaningless results and remains a subject of active research. Although approximating shunt resistance to infinity and photocurrent by short circuit current is highly acceptable, implications of such approximations on the model and the errors introduced in the model have not received much attention. This paper reports (a) a violation of energy conservation between the terminals of the diode by approximating shunt resistance to infinity (b) the need to conserve solar cell parameters, like the electric charge and energy on whose foundation the standard model is formulated (c) that the simultaneous use of infinite shunt resistance and replacement of short circuit current by photocurrent leads to a 63.21% violation of the standard one-diode real solar cell equation. (d) curve fitting is necessary but not sufficient for the correct analysis of solar cell parameters.

Keywords: Solar cell parameters, combinatorics, characterization of solar cells, shunt resistance, photocurrent, short circuit current.

1. **Introduction**

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Application of the standard one-diode model on real solar cell current-voltage characteristics often leads to physically meaningless results [1-2]. In this paper, combinatorics is used to analyze some of the requirements of the standard one diode real solar cell equation based on foundations of the model, without emphasis on the nature of solutions. Combinatorics refers to the branch of mathematics that studies finite collections of objects that satisfy specified criteria, and is in particular concerned with "counting" the objects in those collections and with deciding whether certain "optimal" objects exist and falls under the general umbrella of *Discrete/Finite Mathematics*, which also includes graph theory, discrete algorithms, etc [3]. The “collection of objects” to be studied are the (discrete) optoelectronic solar cell parameters $\{I_{ph}, I_o, R_s, G_{sh}, n\}$, in the standard solar cell equation. Here, I_{ph} is the photocurrent, I_o is the reverse saturation current, R_s is the series resistance, G_{sh} is the shunt conductance, and n is the diode quality factor.

The standard one-diode model of solar cells is given by fig 1 [4-8]. Applying Kirchhof’s Current Law (KCL) in fig 1 leads to

$$I_{ph} - I_D - I_{sh} - I = 0 \qquad (1)$$

$$\Rightarrow \quad I = I_{ph} - I_D - I_{sh} \qquad (2)$$

Applying Kirchhof’s Voltage Law (KVL) in fig 1 (to the loop forming the diode, the series resistor and load resistance), leads to

$$V_D - V - IR_s = 0 \qquad (3)$$

$$\Rightarrow \quad V_D = V + IR_s \qquad (4)$$

From the loop containing the shunt resistance and diode, we have

$$I_{sh} R_{sh} = V_D \qquad (5)$$

so that by virtue of equation (4), equation (5) becomes

$$I_{sh} R_{sh} = V + IR_s \qquad (6)$$

or

$$I_{sh} = \frac{V + IR_s}{R_{sh}} \tag{7}$$

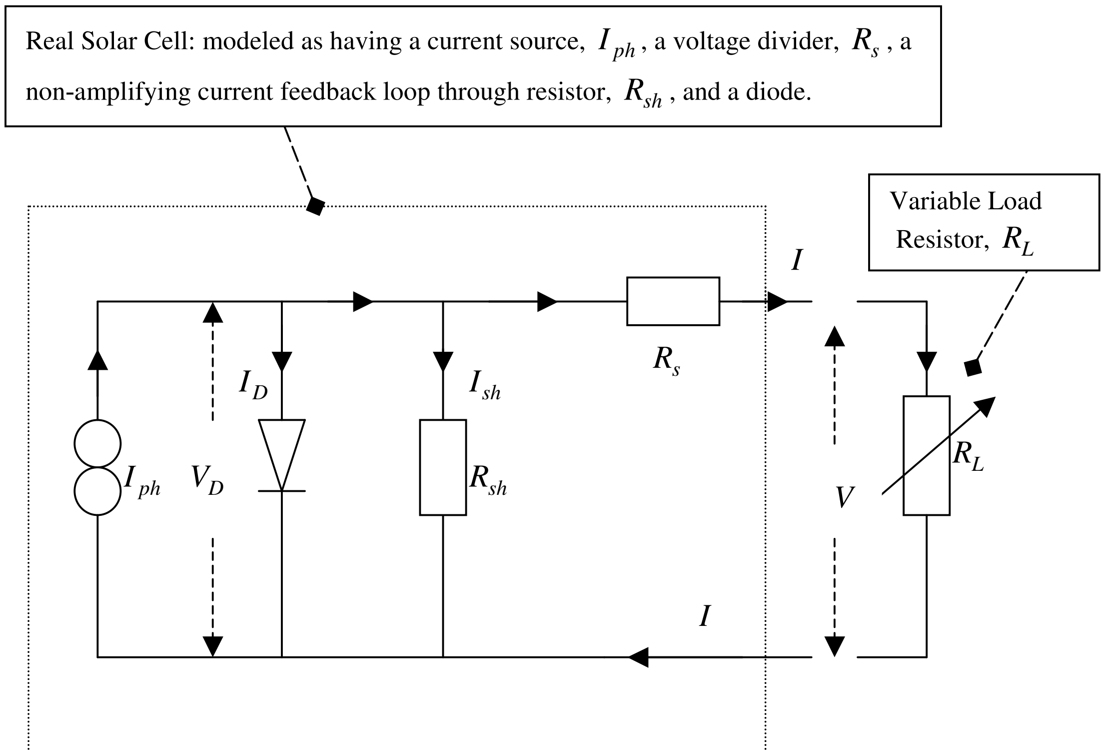

Fig 1. The Standard “One-Diode Model” of a Real Solar Cell. The only variables obtained from experiment are the output current, I and output voltage, V .

The diode current is then given by the Shockley equation [4]

$$I_D = I_o\left[\exp\left(\frac{qV_D}{nkT}\right) - 1\right], \tag{8}$$

where q, k and T are the electron charge, Boltzmann constant and solar cell operating temperature respectively. Substituting equation (4) into (8), leads to

$$I_D = I_o\left[\exp\left(\frac{q(V + IR_s)}{nkT}\right) - 1\right] \tag{9}$$

$$I = I_{ph} - I_o\left[\exp\left(\frac{q(V + IR_s)}{nkT}\right) - 1\right] - G_{sh}(V + IR_s) \tag{10}$$

where

$$G_{sh} = \frac{1}{R_{sh}} \qquad (11)$$

Equation (10) is what is considered as the standard one-diode real solar cell equation. Before proceeding with the mathematical analysis and/or utility of the standard one-diode real solar cell equation, there is need to understand the physical information contained therein, from the foundations on which it is derived. This is the main objective in this paper.

2. The General Connectedness of Solar Cell Parameters

Since current is the rate of change of electric charge, equation (2) may be expressed as

$$\frac{dQ}{dt} = \frac{dQ_{ph}}{dt} - \frac{dQ_D}{dt} - \frac{dQ_{sh}}{dt} \qquad (12)$$

where Q is the electric charge extracted at the terminals of the solar cell, Q_{ph} is the photo generated electric charge , Q_D is the electric charge passing through the diode and Q_{sh} is the electric charge through the shunt resistance. Integrating equation (12) from an initial time, t_o, to an arbitrary time, t, leads to

$$Q(t) - Q(t_o) = \left[Q_{ph}(t) - Q_{ph}(t_o)\right] - \left\{\left[Q_D(t) - Q_D(t_o)\right] + \left[Q_{sh}(t) - Q_{sh}(t_o)\right]\right\} \qquad (13)$$

which is the conservation law of electric charge from the in accordance to the requirements of Kirchhof's current law. Since equation (10) is partly based on equation (2) (and impliedly on equation (13)), it follows that equation (10) is partly represents the conservation of electric charge, a general law of physics that must not be violated.

Additionally, multiplying equations (4) and (6) by q leads to

$$qV_D = \begin{cases} qV + qIR_s \\ \\ qI_{sh}R_{sh} \end{cases} \tag{14}$$

which can be expressed in the form

$$E_D = \begin{cases} E_o + E_{R_s} \\ \\ E_{sh} \end{cases} = \text{Constant} \tag{15}$$

where E_D is the energy between the terminals of the diode, E_o is the energy collected at the terminals of the solar cell, E_{R_s}, is the energy lost from the solar cell due to R_s, while E_{sh} is the energy between the terminals of the shunt resistance, R_{sh}. Equation (15) represents the energy conservation law, in accordance to Kirchhof's laws, between the terminals of the diode and/or shunt resistance, through the series resistance, to the output terminals (see figure 1). Equation (15) must equal to a constant, because it gives the energy balance between the terminals of a diode, the energy lost through the series resistance and the energy delivered at the terminals of the solar cell. By the same token, the energy between the terminals of the shunt resistance must be equal to a constant, which implies that the product $I_{sh}R_{sh}$ must be constant! This is a fundamental point, in accordance with the conservation law of energy and is one of the most important contributions in this paper. It follow that we cannot use the much acceptable approximation [4-8] $R_{sh} = \infty$ in the standard one diode-real solar cell because it leads to undefined values of diode voltage and/or energy, according to equation (14) and/or (15) respectively. Hence, under all conditions, we must have I_{sc} and R_{sh} restricted within the open intervals $0 < I_{sh} < \infty$ and $0 < R_{sh} < \infty$ respectively, so that in all situations, $0 < I_{sh}R_{sh} < \infty$. This condition must be satisfied in order for the diode voltage to be well defined. Therefore, it is incorrect to drop any term from the standard one-diode real solar cell equation.

Since the equation (10) is based on equations (4) and (6) (and impliedly on (15)), it follows that equation (10) partly represents the conservation law of energy, and must also never be violated.

Additionally, it is important to note that although there could be charge leakage through the shunt resistance, according to equation (13), there can never be energy loss through shunt resistance, in accordance to the conservation law of energy and according to equation (15), and hence the solar cell equation.

Further, equation (10) is partly obtained from equation (8), which accounts for the conservation of electric charge and energy as elaborated between pages 84 and 89 in [4]. Therefore, the use of equation (8) in the formulation of equation (10) equally amounts to the conservation of electric charge and energy.

From the above arguments, we observe that the foundations of the standard one-diode real solar cell equation are general laws of physics. This implies that equation (10) is basically a conservative law, one that accounts for both energy and electric charge. Consequently, equation (10) must, be expressible in terms of a constant; the constant to which all combination of parameters should add up. This constant should give "*the general connectedness of solar cell parameters*".

To obtain the desired *general connectedness* of solar cell parameters, we express equation (10) into the non-dimensional forms

$$\frac{I_{ph} + I_o - I - G_{sh}(V + IR_s)}{I_o} = \exp\left(\frac{q(V + IR_s)}{nkT}\right) \qquad (16)$$

which leads to

$$\left(\frac{q(V + IR_s)}{nkT}\right) = \ln\left|1 + \frac{I_{ph} - I - G_{sh}(V + IR_s)}{I_o}\right| \qquad (17)$$

or

$$1=\left(\frac{nkT}{q(V+IR_s)}\right)\ln\left|1+\frac{I_{ph}-I-G_{sh}(V+IR_s)}{I_o}\right| \qquad (18)$$

or

$$1=\ln\left|1+\frac{I_{ph}-I-G_{sh}(V+IR_s)}{I_o}\right|^{\left(\frac{nkT}{q(V+IR_s)}\right)} \qquad (19)$$

or

$$\left(1+\frac{I_{ph}-I-G_{sh}(V+IR_s)}{I_o}\right)^{\left(\frac{nkT}{q(V+IR_s)}\right)}=e\approx 2.7183=\text{Constant};\, I_o\neq 0 \qquad (20)$$

Equation (20) is a necessary, though not sufficient, condition that must be satisfied by solar cell parameters under all conditions. Fundamentally, it is exactly the same as equation (10), although it also gives a necessary "*connectedness*" that must be satisfied by solar cell parameters under all conditions. Any violation of the "*connectedness"* of solar cell parameters, as given by equation (20), amounts to an inherent violation of the conservation laws of electric charge and energy. This observation suggests that solar cell parameters, like electric charge and energy, must be conserved.

3. **Effect of violating the general connectedness of solar cell parameters**

Suppose $R_{sh}=\infty$, so that from equation (11), we have $G_{sh}=0$ and effectively reducing equation (20) into the form

$$\left(1+\frac{I_{ph}-I}{I_o}\right)^{\left(\frac{nkT}{q(V+IR_s)}\right)}=e \qquad (21)$$

We observe that the *parametric constant*, from equation (20), is satisfied. If we then equate the photocurrent, I_{ph}, by short circuit current, I_{sc} – a highly acceptable approximation [4-8] – equation (21) becomes

$$\left(1+\frac{I_{sc}-I}{I_o}\right)^{\left(\frac{nkT}{q(V+IR_s)}\right)} = e \tag{22}$$

Therefore, under short circuit conditions, $(V, I) = (0, I_{sc})$), equation (22) becomes

$$\left(1+\frac{I_{sc}-I_{sc}}{I_o}\right)^{\left(\frac{nkT}{qI_{sc}R_s}\right)} = e \tag{23}$$

or

$$1 = e \tag{24}$$

representing a **violation of the connectedness of solar cell parameters,** as required by equation (20), and thus a complete breakdown of the standard one diode real solar cell equation. Since equation (20) is derived from the conservation laws of energy and electric charge, it follows that the simultaneous use of $G_{sh} = 0$ and $I_{ph} = I_{sc}$ in the standard one diode real solar cell equation leads to a **violation of the conservation laws of energy and electric charge**! As explained earlier, the very use of $R_{sh} = \infty$ in the standard one-diode real solar cell equation amounts to a violation of the conservation law of energy, as required by equation (15). On the other hand, the use of $R_{sh} = \infty$ in equation (20) does not lead to a violation of the exponential *parametric connectedness* of equation (20), as can be seen in equation (20). It follows, therefore, that although equation (20) is derived from the conservation laws of energy and electric charge, not all solutions given by the equation refers to reality. This is also clear by virtue of the fact that even negative values of parameters can lead to results with good curve fitting, yet they do not refer to reality. We can calculate the error associated by the use of $G_{sh} = 0$ and $I_{ph} = I_{sc}$ at the point $(V, I) = (0, I_{sc})$, and leading to equation (24), as

$$\left(\frac{e-1}{e}\right)\times 100\% \approx 63.21\% \tag{25}$$

which is an **extremely large error**. Another way to analyze the effect of approximation is as follows:

(i) *Uncovering a mathematical fallacy.*

Assuming we could use the equality

$$R_{sh} = \infty \tag{26}$$

in the standard one-diode real solar cell equation. Then for all diode voltage values in the open interval $0 < (V + IR_s) < \infty$, equation (7) becomes

$$I_{sh} = \frac{V + IR_s}{\infty} = 0 \tag{27}$$

which implies imposes a condition on equation (6) such that

$$V_D = I_{sh} R_{sh} = 0.\infty \tag{28}$$

Multiplication of zero by infinity is undefined [9].If we allow diode voltage values in the closed interval $-\infty \le (V + IR_s) \le \infty$, then in addition to the result in equation (27), equation (28) leads to $\frac{\infty}{\infty}$ or $-\frac{\infty}{\infty}$ when $(V + IR_s) = \pm\infty$ respectively, which are equally meaningless. Therefore, there is no value of diode voltage, and hence diode energy, on the real plane for which the use of $R_{sh} = \infty$ is logically consistent with the model.

(ii) *Physical ambiguities*

From a physical perspective, in terms of the model circuit of the real solar cells, the conservation law of energy according to equation (15), becomes

$$E_D = \begin{cases} E_o + E_{R_s} \\ \\ 0.\infty \text{ or } \pm\infty.\infty \end{cases} \tag{29}$$

which implies that the energy between the terminals of the diode, is not analytic. In fact, if I_{sh} is restricted in the open interval $-\infty < I_{sh} < \infty$, as long as equation (26) is valid, then we have, $I_{sh}R_{sh} = \mp\infty$, which is also meaningless. However, we observe that restricting values of I_{sh} and R_{sh} in the open intervals $0 < I_{sh} < \infty$ and $0 < R_{sh} < \infty$, respectively, then it follows that $0 < I_{sh}R_{sh} < \infty$, which is well defined as required by the conservation law of energy and electric charge, as well as mathematical analyticity.

4. **Conservation of solar cell parameters**

As discussed above, the standard one-diode real solar cell equation is basically a sandwich of the conservation laws of energy and electric charge. Therefore, when expressed as in (10) – which accounts for charge conservation – there is need to state the boundary conditions within which parameters should be obtained, according to the energy conservation given by equation (15), as well as not forgetting the parametric connectedness given by equation (20). By virtue of equations (14) and (15), and since n is constant on a given current-voltage curve measured at a temperature , T, then we can express the solar cell equation in conservative form as

$$\left(1+\frac{I_{ph}-I-G_{sh}\left(V+IR_s\right)}{I_o}\right)^{\left(\frac{nkT}{q\left(V+IR_s\right)}\right)} = e = \text{Constant} \begin{cases} 0 < I_{ph} < \infty \\ 0 < I_o < \infty \\ 0 < R_{sh} < \infty \\ 0 < R_s < \infty \\ 0 < n < \infty \\ \left(V+IR_s\right) = I_{sh}R_{sh} = \text{Constant} \\ \left(\frac{q\left(V+IR_s\right)}{nkT}\right) = \left(\frac{qI_{sh}R_{sh}}{nkT}\right) = \text{Constant} \end{cases}$$

……………………………………………………………………………………… (26).

As can be seen in equation (26), the expanded expression for the solar cell equation, takes account of energy conservation between the diode voltage and/or the shunt resistance.

5. **Conclusion**

Not all solutions obtained from the standard one-diode real solar cell equation are physically acceptable, and the simultaneous use of $R_{sh} = \infty$ and $I_{ph} = I_{sc}$ in the analysis of solar cell parameters leads to a breakdown of the standard solar cell equation as well as physically meaningless results. There is need for a detailed analysis of the solutions obtained from the standard one-diode real solar cell equation. Solar cell parameters, like electric charge and energy on whose foundation the standard solar cell equation is based, must be conserved. Curve fitting is necessary, but not sufficient for the correct analysis of the standard one-diode real solar cell equation.

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