Combinatorics of Solar Cell Optoelectronic Device Parameters

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Abstract:

We apply the recently defined Lambert W function to the standard solar cell equation and obtain new exact explicit analytic solutions of the output voltage for solar cells under both illumination and dark conditions. A comparative analysis of the solutions obtained herein and those in standard literature is also presented. Graphs that contain all solutions for any possible combination of solar cell parameters are plotted, effectively adding an extra dimension to the standard theory of solar cells under both illuminated and dark conditions.

Keywords: Photovoltaics, Non-ideal solar cells, Diode quality factor, Current-voltage characteristics, Lambert W function, Wein's displacement law.

1. INTRODUCTION

1.1 Characterization of solar cells

Measuring current – voltage (I-V) characteristics of photovoltaic solar cells, under either illuminated or dark conditions or both, and extracting a set of solar cell optoelectronic parameters from the data is the conventional method of assessing these optoelectronic devices. These parameters are the photocurrent, I_{ph}, reverse saturation current, I_o, parasitic series resistance, R_s, the shunt resistance, R_{sh}, and the diode quality factor, A [1-10]. When using the one-diode real solar cell model with a single shunt and parasitic series resistance, these parameters are related by the equation

$$I = \begin{cases} I_{ph} - I_o\left(\exp\left[\frac{q(V+IR_s)}{AkT}\right]-1\right) - \frac{V+IR_s}{R_{sh}} & \text{under illuminated conditions} \\ I_o\left(\exp\left[\frac{q(V-IR_s)}{AkT}\right]-1\right) + \frac{V-IR_s}{R_{sh}} & \text{under dark conditions} \end{cases}. \qquad (1)$$

where $k = 1.38\times10^{-23}$ J.K^{-1} is the Boltzmann constant, I is the output current (or current density) while V is the voltage at the terminals of the solar cell. However, application of Eq. (1) on real experimentally determined solar cell current – voltage curves often leads to physically meaningless values of these parameters such as negative values of series resistance R_s [8-10]. Hence, before embarking on utilizing the standard solar cell equation, it is imperative for us to study the detailed general behavior of this equation with respect to variations in solar cell parameters and/or combinations of these parameters– a process which we herein refer to as combinatorics of solar cell optoelectronic device parameters.

Combinatorics refers to the mathematical study of finite collections of objects that satisfy specified criteria, and in particular concerned with "counting" the objects in those collections as well as with deciding whether certain "optimal" objects exists. In this paper, our "collection of objects" is the set of (discrete/finite) solar cell parameters $\{I_{ph}, I_o, A, R_s, R_{sh}\}$ and the criterion subject to satisfaction is the standard solar cell equation under both illuminated and dark conditions with values of solar cell parameters restricted to physically meaningful ranges.

2. METHODOLOGY

2.1 The non-dimensional characteristic solar cell equations

Since solar cells operate at temperatures absolute zero (i.e. $T > 0$) we have the freedom to express Eq. (1) in the form

$$z = \exp[x] + xy \qquad \begin{cases} x = x_i \text{ and } z = z_i \text{ under illuminated conditions} \\ x = x_d \text{ and } z = z_d \text{ under dark conditions} \end{cases} \qquad (2)$$

where

$$y = \frac{AV_{th}}{I_o R_{sh}} \qquad (3)$$

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$$x = \begin{cases} x_i = \dfrac{V + IR_s}{AV_{th}} & \text{under illuminated conditions} \\ x_d = \dfrac{V - IR_s}{AV_{th}} & \text{under dark conditions} \end{cases} \tag{4}$$

$$z = \begin{cases} z_i = 1 + \eta_i & \text{under illuminated conditions} \\ z_d = 1 + \eta_d & \text{under dark conditions} \end{cases} \tag{5}$$

$$\eta = \begin{cases} \eta_i = \dfrac{I_{ph} - I}{I_o} & \text{under illuminated conditions} \\ \eta_d = {}^{I}\!/_{I_o} & \text{under dark conditions} \end{cases} \tag{6}$$

and

$$V_{th} = {}^{kT}\!/_{q} \quad \text{(i.e. the thermal voltage).} \tag{7}$$

Since Eq. (2) is dimensionless and valid for the solar cell equation under both illuminated and dark conditions and we herein refer to it as a characteristic non-dimensional solar cell equation. As shown in Eq. (2), the solar cell equation under illuminated conditions has the same mathematical structure with that under dark conditions. Under standing the mathematical structure of Eq. (2) is thus important to us.

2.2 Solvability of the characteristic non-dimensional solar cell equation

If $y \neq 0$, we can express Eq. (2) as

$$\frac{z}{y} = \frac{\exp[x]}{y} + x, \quad y \neq 0 \qquad \begin{cases} x = x_i \text{ and } z = z_i \text{ under illuminated conditions} \\ x = x_d \text{ and } z = z_d \text{ under dark conditions} \end{cases} \tag{8}$$

which upon exponentiation and dividing both sides of by y results in

$$\frac{\exp\left[{}^{z}\!/_{y}\right]}{y} = \frac{\exp[x]}{y}\exp\left[\frac{\exp[x]}{y}\right]. \qquad \begin{cases} x = x_i \text{ and } z = z_i \text{ under illuminated conditions} \\ x = x_d \text{ and } z = z_d \text{ under dark conditions} \end{cases} \tag{9}$$

Now Eq. (9) can be solved using the Lambert W function [11-13], denoted as $W_m[x]$, and verifies the equation

$$x = W_m[x]\exp[W_m[x]]. \tag{10}$$

with $m = 0, \pm 1, \pm 2, ...$ referring to the branches of $W[x]$. The physical branch, is called the principal branch, is denoted as $W_0[x]$ [11]. Application of the Lambert W function to Eq. (9) – restricted to the principal branch – gives

$$\frac{\exp[x]}{y} = W_0\left[\frac{\exp\left[{}^{z}\!/_{y}\right]}{y}\right] \qquad \begin{cases} x = x_i \text{ and } z = z_i \text{ under illuminated conditions} \\ x = x_d \text{ and } z = z_d \text{ under dark conditions} \end{cases} \tag{11}$$

or

$$\left.\begin{matrix} \exp[x] \\ z - xy \end{matrix}\right\} = yW_0\left[\frac{\exp\left[{}^{z}\!/_{y}\right]}{y}\right]. \qquad \begin{cases} x = x_i \text{ and } z = z_i \text{ under illuminated conditions} \\ x = x_d \text{ and } z = z_d \text{ under dark conditions} \end{cases} \tag{12}$$

so that

$$x = \begin{cases} \log_e\left[yW_0\left[\dfrac{\exp\left[{}^{z}\!/_{y}\right]}{y}\right]\right] \\ \dfrac{z}{y} - W_0\left[\dfrac{\exp\left[{}^{z}\!/_{y}\right]}{y}\right] \end{cases} \qquad \begin{cases} x = x_i \text{ and } z = z_i \text{ under illuminated conditions} \\ x = x_d \text{ and } z = z_d \text{ under dark conditions} \end{cases} \tag{13}$$

Physically, we must have $x > 0$. This condition is valid only when $y > 0$ and $z > 1$. Clearly, the condition $y = 0$ leads to a division by zero in Eq. (13), and thus analytically not acceptable for the closed form solutions of x .

3 RESULTS AND DISCUSION

3.1 Closed form solution for output voltage for a solar cell under illumination

Replacing the corresponding expressions for x , y and z for the solar cell equation under illumination into Eq. (13) leads to

$$\frac{V + IR_s}{AV_{th}} = \begin{cases} \log_e\left[\dfrac{AV_{th}}{I_oR_{sh}} W_0\left[I_oR_{sh}\dfrac{\exp\left[\dfrac{R_{sh}\left(I_{ph} + I_o - I\right)}{AV_{th}}\right]}{AV_{th}}\right]\right] \\ \dfrac{R_{sh}\left(I_{ph} + I_o - I\right)}{AV_{th}} - W_0\left[I_oR_{sh}\dfrac{\exp\left[\dfrac{R_{sh}\left(I_{ph} + I_o - I\right)}{AV_{th}}\right]}{AV_{th}}\right] \end{cases} \tag{14}$$

which upon solving for V gives

$$V = \begin{cases} \log_e\left[\dfrac{AV_{th}}{I_oR_{sh}} W_0\left[I_oR_{sh}\dfrac{\exp\left[\dfrac{R_{sh}\left(I_{ph} + I_o - I\right)}{AV_{th}}\right]}{AV_{th}}\right]\right] - IR_s \\ R_{sh}\left(I_{ph} + I_o\right) - I\left(R_s + R_{sh}\right) - AV_{th}W_0\left[I_oR_{sh}\dfrac{\exp\left[\dfrac{R_{sh}\left(I_{ph} + I_o - I\right)}{AV_{th}}\right]}{AV_{th}}\right] \end{cases} \tag{15}$$

which is a closed form solution for V in the case of an illuminated solar cell. The lower expression of Eq. (15) has been reported in [14]. However the upper expression, which tells us that V is strictly logarithmic, is at least to the author's knowledge unreported.

3.2 Closed form solution for output voltage for a solar cell under dark conditions

Similarly, replacing the corresponding expressions for x , y and z for the solar cell equation under dark conditions into Eq. (13) , then solving for V results in

$$V = \begin{cases} \log_e\left[\dfrac{AV_{th}}{I_oR_{sh}} W_0\left[I_oR_{sh}\dfrac{\exp\left[\dfrac{R_{sh}\left(I + I_o\right)}{AV_{th}}\right]}{AV_{th}}\right]\right] - IR_s \\ R_{sh}\left(I_{ph} + I_o\right) - I\left(R_s + R_{sh}\right) - AV_{th}W_0\left[I_oR_{sh}\dfrac{\exp\left[\dfrac{R_{sh}\left(I + I_o\right)}{AV_{th}}\right]}{AV_{th}}\right] \end{cases} . \tag{16}$$

The lower expression of Eq. (16) has been reported in [15] in the analysis of non-ideal diodes while the upper equivalence is, to the author's knowledge, undocumented. Again, we see that V is strictly logarithmic.

3.3 Comparative analysis of closed form and approximated solar cell equations

The contemporary view on the extraction of solar cell solar cell optoelectronic device parameters highly accepts the approximation $R_{sh} \approx \infty$. This approximation translates Eq. (1) into

$$I \approx \begin{cases} I_{ph} - I_o\left(\exp\left[\dfrac{q(V+IR_s)}{AkT}\right]-1\right) & \text{under illuminated conditions} \\ I_o\left(\exp\left[\dfrac{q(V-IR_s)}{AkT}\right]-1\right) & \text{under dark conditions} \end{cases} \tag{17}$$

and can be expressed in dimensionless form as

$$z_{ap} = \exp[x_{ap}] \qquad \begin{cases} x_{ap} = x_i \text{ and } z_{ap} = z_i \text{ under illuminated conditions} \\ x_{ap} = x_d \text{ and } z_{ap} = z_d \text{ under dark conditions} \end{cases} \tag{18}$$

whose solution for x_{ap} is

$$x_{ap} = \log_e|z_{ap}| \qquad \begin{cases} x_{ap} = x_i \text{ and } z_{ap} = z_i \text{ under illuminated conditions} \\ x_{ap} = x_d \text{ and } z_{ap} = z_d \text{ under dark conditions} \end{cases} \tag{19}$$

Notice that the condition $R_{sh} = \infty$ results in $y = \dfrac{AkT}{qI_o \times \infty} = 0 = \text{constant}$. Hence conventional literature takes y to be constant, namely zero. However, as seen earlier, the condition $y = 0$ results in a singularity in Eq. (13) and analytically unacceptable for a closed form solution of x. Our argument in this paper is that, while R_{sh} can take very large values, these values should be such that $0 << R_{sh} < \infty$ (which is a physical requirement) $0 < y << \infty$ (which is a mathematical requirement).

It is important for us to understand the basic difference between the solution of equation x and x_{ap} with respect to z and/or η. Defining $\Psi(y,\eta)$ as the difference between Eqs. (13) and (19) (i.e. $\Psi(y,\eta) = x_{ap} - x$), results in

$$\Psi(y,\eta) = \log_e\left|\frac{1+\eta}{yW_0\left[\dfrac{\exp\left[\dfrac{1+\eta}{y}\right]}{y}\right]}\right| \qquad \begin{cases} x = x_i \text{ and } \eta = \eta_i \text{ under illuminated conditions} \\ \\ x_{ap} = x_d \text{ and } \eta = \eta_d \text{ under dark conditions} \end{cases} \tag{20}$$

The graphs of $\Psi(y,\eta)$ for $y = 1$, 5, 10, 50, 100 and 500 with $0 \le \eta \le 700$ are presented in fig 1.

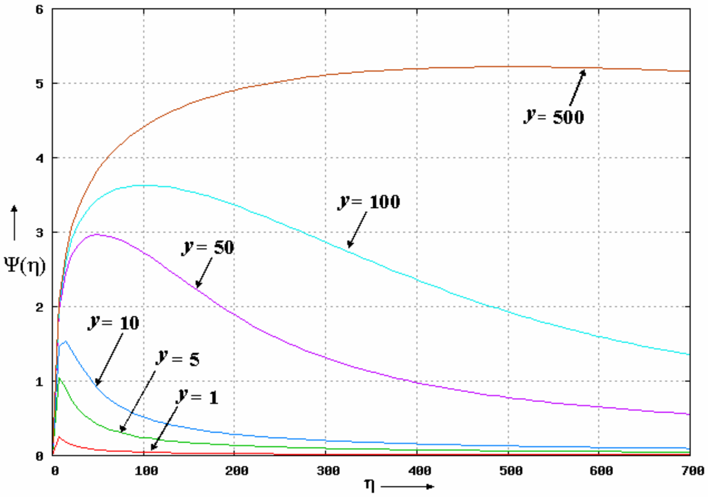

Figure 1 *Graphs of* $\Psi(y,\eta)$ *for* $y = 1$*, 5, 10, 50, 100 and 500 with* $0 \le \eta \le 700$.

As can be seen in fig 1, for a given value of y, $\Psi(y,\eta)$ increases rapidly to some maximum value as η increases then decreases and asymptotically approaching zero for much higher values of the current ration η. The graphs of x for $y = 1$, 5, 10, 50, 100, 1000, 5000 and x_{ap} against η are also plotted in fig 2. Here we see that differences in x and x_{ap}, for given value of y, are significant only when values of η are small and become negligible as η becomes large. Additionally, the graph of x against η for $y = 1$ is much closer to that of x_{ap} against η. Clearly, the condition $y = 1 = \text{constant}$ not only result in closed form analytic solutions for x and V, but also to solutions whose graph is closer to the conventionally highly acceptable values of x_{ap}. Most importantly, whenever we take $y = \text{constant}$, R_{sh} has completely no effect on either I or V, as can be seen in Eqs. (13) and (18).

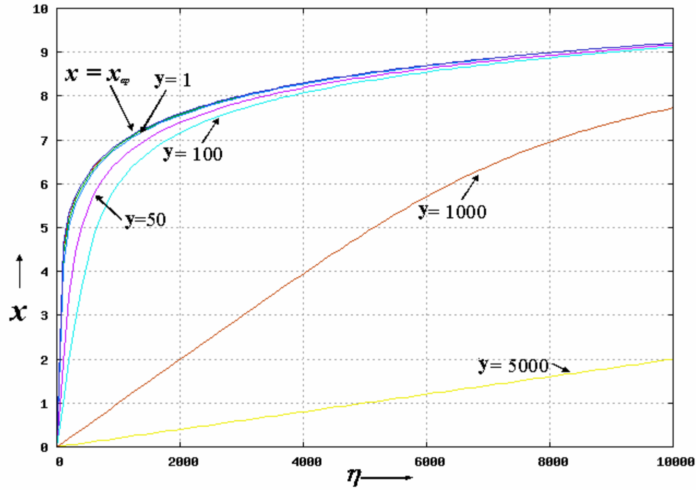

Figure 2: The graphs of x and x_{ap} against η for $y = 1$, 5, 10, 50, 100, 1000 and 5000 .

3.4 Implications of Wein's displacement Law to the standard solar cell equation

Wein's displacement law, as discussed in [16], is given by

$$n\lambda_m T = 2898\times10^{-6}\ \text{m.K} \tag{21}$$

where λ_m is the wavelength of maximum radiance and n is the average refractive index of the medium in which radiation is propagating. For the case where the medium is air, n is close to unity [17-18]. Solving for T in Eq. (3) gives

$$T = \frac{qI_oR_{sh}}{Ak}y \tag{22}$$

which upon substituting into Eq. (21) results in

$$\frac{I_oR_{sh}}{A}y = \wp\frac{\mathrm{E}_m}{n} \tag{23}$$

where

$$\wp = \frac{2898\times10^{-6}\times k}{q\times hc} = 1\,256\,674\,208\,144\,796\,380.09\ \text{V.J}^{-1} \tag{24}$$

with $h = 6.63\times10^{-34}$ J.s being the Plank's constant, $c \approx 3\times10^{8}$ m.s^{-1} is the speed of light in a vacuum and E_m is the maximum radiance of the solar cell at average operating temperature T. In the special case for which $y = 1$, Eq. (23) simplifies to

$$\frac{I_oR_{sh}}{A} = \wp\frac{\mathrm{E}_m}{n}. \tag{25}$$

where we observe that

$$I_oR_{sh} \propto \wp\,\mathrm{E}_m \tag{26}$$

and

$$A \propto n. \tag{27}$$

The proportionality constants in Eqs. (26) and (27) must be equal for Eq. (25) to be maintained as well as positive for the equations to be physically meaningful. Now taking unity as the proportionality constant transforms Eqs. (26) and (27) into

$$I_o R_{sh} = \wp \mathrm{E}_m \qquad (28)$$

and

$$A = n\,. \qquad (29)$$

In other words for the one-diode real solar cell equation with a single parasitic series and shunt resistance, the diode quality factor, A, of a solar cell working at an average operational temperature, T, is a measure of the average refractive index, n, of the solar cell. Since the refractive index of a solar cell may vary within the material, so too does the diode quality factor.

4. CONCLUSION

We have discussed two related standard problems of photovoltaic physics, in which the Lambert W function can be used. New closed form expressions of the output voltage under both dark and illuminated conditions have been presented. Analytic and graphical illustrations of the differences between our solutions and those in standard scientific literature have also been extensively presented. An important part of this paper is that, based on the one-diode model of a solar cell with a single shunt and parasitic series resistance, a solar cell operating at the same temperature has a mean diode quality factor directly proportional to its mean refractive index.

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