When Wall Street Conflicts with Main Street...
–The Divergent Movements of Taiwan's Leading Indicators–

Shyh-Wei Chen
Department of Economics
Tunghai University
Taichung 407, Taiwan, ROC
TEL: 886-4-23590121 ext. 2922
FAX: 886-4-23590702
E-mail: schen@mail.thu.edu.tw

Chung-Hua Shen
Department of Money and Banking
National Chengchi University
Mucha, Taipei 116 Taiwan, ROC
TEL: 886-2-29393091 ext. 81020
Fax: 886-2-29398004
E-mail: chshen@cc.nccu.edu.tw

May 31, 2004

Abstract

This paper argues that the simultaneous use of all leading indicators may result in the blending of two different sets of information, which could lead to less accurate predictions of a future recession. We divide six of Taiwan's leading indicators into two different sectors, the real and financial sectors, and distinctly demonstrate that the two sectors may very well reveal different information. Three inconsistent, or even divergent, movements are found for 1988, 1991 and 1994, implying that the factor extracted from the real side may be different from that from the financial side. Thus, in contrast to the one-factor model typically used, we suggest a two-factor model. We compare four Markov Switching models, and it is evident the predicted recessions based on the two-factor one-state model seem to outperform other models. The second best is the one-factor model which is only based on the real side variables, and it is followed by the one-factor model with four variables. The worst model is that which simply uses financial variables. The results support our argument to use the two-factor model.

Keywords: Wall Street, Main Street, business cycle, Markov Switching model

JEL classification: C22, E32

1 Introduction

Dating a business cycle's turning point has long been not only in the interest of the public, but also in that of both academic circles and government. The first approach can be traced back to 1920 when the National Bureau of Economic Research (hereafter NBER) identified business cycle chronology in the United States. Since then, the stylized fact about asymmetric adjustment that a recovery takes up more time than a recession has often been reported. Hamilton (1989) recently applied a Markov Switching model to the U.S. GNP to date the business cycle turning point and found a remarkable consistent pattern in the generated recessionary and recovery periods to that of the NBER-defined chronology of the business cycle. He also confirmed the characteristic asymmetric adjustment.

The use of a single univariate process, either GNP or an industrial production index, was soon found to be too narrow to capture broad fluctuations in economic activity even when using a time-varying transition probability was used (see Filardo, 1994). The fact is highly possible that a univariate process may ignore extraneous non-trivial information compelled researchers to begin to employ more macro time series. To cite one example, Stock and Watson (1989, 1991), for instance, assumed that the co-movements of four coincidence indicators share a common element that can be captured by a signal unobservable latent variable and that represents an economy's state. After combining the concepts of Hamilton (1989) and Stock and Watson (1991), Diebold and Rudebusch (1996) proposed a dynamic factor Markov Switching model to capture asymmetry and the co-movement features of business cycles. Chauvet (1998) and Kim and Nelson (1998) meanwhile applied the Markov Switching factor model to investigate coincident indicators to predict turning points in the U.S. business cycle. In addition, Chauvet and Potter (2000) applied the model to construct coincident and leading indicators of the U.S. stock market.[1]

[1] Kim and Yoo (1995) also assumed that an unobserved common factor is driven by a Markov Switching process with

A single unobservable common factor extracted from various multivariate processes indeed captures more information than that of a univariate process. The use of "a single" or "one" common factor, however, implicitly assumes that inconsistent movements among coincident and leading indicators are random and can be averaged out. Such an assumption ignores the fact that the coincident and leading indicators typically contain two distinct groups of variables, i.e., financial and non-financial ones. While the two groups often do reveal the same information regarding the dating of a business cycle, which justifies the use of a single factor, certainly some cases provide conflicting information. The often-heard asset bubble, where asset prices exceed the intrinsic value of the fundamentals, may be one case which exemplifies that different information may be contained in the two types of variables. With this in mind, Chauvet (1998/1999) proposed a two-factor regime switching model to identify the U.S. business cycle. Kim and Murray (2002) and Kim and Piger (2002) also used a similar model to extract common permanent and transitory factors within the U.S. business cycle.

The major purpose of this paper is in line with that of Chauvet (1998/1999) which is to extract the two unobservable common factors from two groups of leading indicators. The first common factor, which is referred to as the "Wall Street Factor" (hereafter WSF), extracts information from the financial variable group. The second common factor, on the other hand, is referred to as the "Main Street Factor" (hereafter MSF) and it extracts information from real variables. The co-movements of these two sectors are often seen, yet, as we argue above, the inconsistent movements do also occur.

The implications of our two-street factor hypothesis are crucial from two strands. For one, if the Wall Street factor suggests a boom but the Main Street sector does not, then the asset price

---

a time-varying transition probability. They found that both the composite index of leading indicators and disaggregate coincident indicators are informative in identifying the state since they reduce the idiosyncratic noise in the business cycle.

may be over-valued, and an asset bubble may form. Pricking the bubble or letting the economy land softly may then become an urgent concern for authorities. Secondly, if Wall Street suggests a recession but Main Street does not, then though the financial market may be pessimistic about the future, the manufacturing market is not endangered. Restoring the confidence of investors by adopting a credible and transparent policy may, at the point, become necessary.

In this paper, we use Taiwan's leading indicators as our example since Taiwan has ever experienced inconsistent episodes. To illustrate this, financial deregulations starting in 1987 stimulated asset prices to a historically unprecedented high level in 1989, whereas manufacturing production remained relatively stable. The "missile tests" of mainland China over Taiwan island on 1995 and on 1996 led to the reverse effect. The missile tests frightened investors, causing stock prices to drop substantially. All the while, the manufacturing industry, however, was only mildly hurt. Even though we use Taiwan as our an example here, the application of our study to other economies is immediate.

The rest of this paper is organized as follows. Taiwan's leading indicators are discussed in Section 2. The model specifications are described in Section 3, and the empirical results and implications are presented in Section 4. Our comparisons of forecasting performance are described in Section 5, while our concluding remarks are summarized in Section 6.

2 Taiwan Business Cycle Indicators

Taiwan's leading indicators include six variables, with the first three being non-financial (real) ones, containing new manufacturing orders (ORDER), exports through customs (EXPORT) and floor area permitted for building in Taiwan (BUILD). The latter three are financial variables, containing the stock price index (SP), narrowed money supply (M1B) and the wholesale price index (WPI). The two types of variables, real and financial, are also referred to as Main Street and Wall Street variables, respectively. These leading indicators are regularly published by a Taiwan authority, the Council for Economic Planning and Development (hereafter CEPD), which also publishes the dates of a business cycle whenever deemed necessary.

Figure 1 plots the annual growth rates of the six indicators. They are calculated from the formula $y_t = 100 \times (Y_t - Y_{t-12})$, where Y_t is the logarithm of the empirical series and y_t is the corresponding annual growth rate. The Main and Wall Street variables are plotted in the left- and right-hand panels, respectively. All series are taken from the monthly journal *Business Indicators* distributed by the CEPD. The variables are monthly data from 1983:m1 to 2001:m4 and total 220 observations.

The conventional aggregate leading indicator is the simple sum of the percentage changes in the six indicators. The aggregate leading indicator assumes the information contained in each indicator is the same except for a few random variations. This indicator distinguishes neither between the use of the real sector and that of the financial sector, nor the leading economic activities up to the three months and six months..

Of particular interest would therefore be to create two sub-aggregate leading indicators and investigate the differences between the two sectors, and this is the central to this study. For expositional purposes, we combine the three real variables into one Main Street indicator (or real sector indicator) and the three financial variables into one Wall Street indicator (or financial sector indicator). In the upper and lower panels of Figure 2, we respectively plot the simple sub-aggregate leading indicators and their spread between them.

In Figure 2, three inconsistent movements are noted between the two sectors. The first inconsistent and divergent movement appears in 1988. As shown, the Wall Street indicator increases substantially, while the Main Street indicator drops slightly. In other words, the financial sector indicates that the economy is "too hot", whereas the real sector shows a "mild cool", meaning such a combination of both phenomena is typical of an asset bubble. The reasons for this asset bubble include the effects of financial deregulation that started in 1987, a slow but steady appreciation of the exchange rate and a lax monetary policy among others. That is, during that period, both stock prices and real estate prices reached historically unprecedented high levels. The exchange rate also appreciated from 38 New Taiwan dollars (NTD) to around 25 NTD per US dollar. The central bank did not adopt any active monetary policy to prick the bubble, causing the growth rate of M1B to also reaches 51% in annual percentage terms. The real side, however, was not affected by this financial boom and to a certain extent, even dropped.

The next inconsistent movement occurred in 1991 once the central bank had finally decided to prick the asset bubble at the end of 1990. The Minister of Finance also announced their intention to tax capital gains earned from the stock market.[2] This simultaneous tightening monetary and fiscal news shattered the confidence of investors. Stock prices plunged from 9,800 to 5,400 over a period of nineteen consecutive days.[3] At the same time, the growth rate of M1B decreased. The real side, however, was hurt relatively less, meaning the degree of the response from the two sectors was somewhat different. Thus, from the perspective of the financial side, it was suggested that a recession was approaching, whereas from the standpoint of the real side, it was indicated that no change in the economic condition would occur. The aggregate index reflected more of a change in the financial side, also indicative of a recession.

The third inconsistent movement appears in 1994 (Figure 2). In contrast to the above two episodes, we observe that not only do the two sectors move toward different directions, but that there are divergent movements inside the real sector (see Figure 1). As observed in both Figures 1 and 2, while the first two real components EXPORT and ORDER display an upward trend, their aggregation is pulled down by a strong declining index of the third component BUILD. The Main Street indicator thus decreases even though they both increase. Unlike the conflicting information

[2]Previously, there had been no tax on profit gains in Taiwan.

[3]Because Taiwan's stock market has daily price limits, the impact of the bad news often spills over to consecutive days. See Shen and Wang (1998) for a description of price limits in Taiwan's stock market.

inside the real sector, the three financial variables overwhelmingly show a "consistent" prediction of the business cycle. That is, because the growth rate of WPI is countercyclical but the stock return and M1B growth rate are procyclical, the opposite movements between the wage rate and the other two lead to the same prediction vis-á-vis business cycle movements. Hence, based on the prediction of the business cycle direction, all of the three are deemed "consistent" in our paper. By contrast, the inside divergent movements in the real sector lead to the opposite prediction of the business cycle and affects the estimation of the Markov switching model (which will be discussed shortly).

Because the two sectors may not signal the same directions of future recessions, the conventional aggregate leading indicator which combines the six variables, may result in the annulment of important information. The resulting prediction of the business cycle may, as a result, be imprecise.

3 Econometric Model

3.1 The One-Factor Model

The one-factor Markov Switching model (hereafter the one-factor model) is based on Kim and Yoo (1995) and Kim and Nelson (1998). Let $\boldsymbol{y}_t = [y_{1t}, y_{2t}, \ldots, y_{6t}]'$ be a function of a common unobserved dynamic factor F_t and idiosyncratic noises $\boldsymbol{z}_t = [z_{1t}, z_{2t}, \ldots, z_{6t}]'$.[4] The terms y_{it}, where $i = 1, \ldots, 6$ are the six Taiwan leading indices described above, and $\boldsymbol{z}_{it}$ is a vector stationary series with a mean of zero and variance Σ. All variables are transformed into annual growth rates and deviate from their respective means. Factor F_t captures market-wide co-movements underlying

[4] The ranking of the variables in descending order are ORDER, EXPORT, BUILD, M1B, WPI and SP.

the six leading indices. Thus, the model is:

$$\begin{aligned} \boldsymbol{y}_t &= \gamma(L)F_t + \boldsymbol{z}_t, &(1)\\ \phi(L)F_t &= \beta(S_t) + \eta_t, \;\; \eta_t \sim \text{i.i.d.N}(0, \sigma_\eta^2), &(2)\\ \beta(S_t) &= \beta_0(1-S_t) + \beta_1 S_t, \;\; S_t = 0, 1, &(3)\\ \theta(L)\boldsymbol{z}_t &= \boldsymbol{e}_t, \;\; \boldsymbol{e}_t \sim \text{MVN}(\mathbf{0}, \Sigma), &(4) \end{aligned}$$

where the boldface variables denote the vectors. Function $\phi(L) = (1 - \phi_1 L - \cdots - \phi_k L^k)$ is a scalar lag polynomial; and $\gamma(L)$ and $\theta(L)$ are vector polynomials, as follows: $\gamma(L) = \gamma_0 + \gamma_1 L + \cdots + \gamma_q L^q$ and $\theta(L) = 1 - \theta_1 L - \cdots - \theta_r L^r$, with L denoting the lag operator, and k, q and r being lag lengths. Σ is the diagonal matrix with diagonal elements equal to $\sigma_1^2, ..., \sigma_6^2$. Since the factors summarize information common to different variables and cannot be directly observed, a scale must be provided to allow for their interpretation. This is done by setting one of their factor loadings or the factor variance at unity. We adopt the latter approach by setting the factor variance σ_η^2 at unity.

Term S_t is an unobserved latent variable which takes on the value 1 when the economic state is in expansion and 0 when the economic state is in contraction. It is assumed that it follows the following first-order Markov chain:[5]

$$\begin{aligned} \Pr[S_t = 0 | S_{t-1} = 0] &= p_{00}, \qquad \Pr[S_t = 1 | S_{t-1} = 1] = p_{11},\\ \Pr[S_t = 1 | S_{t-1} = 0] &= 1 - p_{00}, \;\; \Pr[S_t = 0 | S_{t-1} = 1] = 1 - p_{11}. \qquad (5) \end{aligned}$$

3.2 The Two One-Factor Models

The real and financial variables are separately specified to follow their respective one-factor model since the two types of variables may share various common factors. The first specification involves

[5]Upon submitting this paper, we find Bandholz and Funke's (2003) paper which is similar to ours.

only real variables, i.e., $\boldsymbol{y}_t$ is an $N_1 \times 1$ vector of monthly real variables containing {ORDER, EXPORT and BUILD}.[6] On the other hand, the second specification involves only financial variables, and $\boldsymbol{y}_t$ is an $N_2 \times 1$ vector of monthly financial variables containing {M1B, WPI and SP}. Models (1) $\sim$ (4) are then repeatedly used by replacing $\boldsymbol{y}_t$ with the real and financial vectors. The common factor F_t resulting from the financial variables is the Wall Street Factor (WSF), whereas when the real variables are used; it is the Main Street Factor (MSF) . All variables are transformed into annual growth rates and deviate from their respective means.

3.3 The Two-Factor Model with Regime Switching

The two-factor model with regime switching (hereafter, referred to as just the two-factor model) is a straightforward generalization of the one-factor regime switching model. Let $\boldsymbol{Y}_t = [\boldsymbol{y}_t \ \boldsymbol{y}_t^*]'$ be the 6×1 vector of the leading indices, where the first three terms are the real (or Main Street) sector, i.e., $\boldsymbol{y}_t = [y_{1t} \ y_{2t} \ y_{3t}]'$, and the last three terms are the financial (or Wall Street) sector, i.e., $\boldsymbol{y}_t^* = [y_{1t}^* \ y_{2t}^* \ y_{3t}^*]'$. The asterisk (*) denotes the Wall Street sector. The two-factor Markov Switching model is specified as follows:

$$\boldsymbol{Y}_t = \begin{bmatrix} \boldsymbol{y_t} \\ \boldsymbol{y_t^*} \end{bmatrix} = \Gamma(L) \begin{bmatrix} F_t \\ F_t^* \end{bmatrix} + \begin{bmatrix} \boldsymbol{z_t} \\ \boldsymbol{z_t^*} \end{bmatrix} \tag{6}$$

$$\Phi(L) \begin{bmatrix} F_t \\ F_t^* \end{bmatrix} = \begin{bmatrix} \beta_0(1 - S_t) + \beta_1 S_t \\ \beta_0^*(1 - S_t^*) + \beta_1^* S_t^* \end{bmatrix} + \begin{bmatrix} \eta_t \\ \eta_t^* \end{bmatrix} \tag{7}$$

$$\Theta(L) \begin{bmatrix} \boldsymbol{z_t} \\ \boldsymbol{z_t^*} \end{bmatrix} = \begin{bmatrix} \boldsymbol{e_t} \\ \boldsymbol{e_t^*} \end{bmatrix}. \tag{8}$$

[6]Term N_1 is the number of real variables, and N_2 is the number of financial variables.

where

$$\boldsymbol{F}_t = \begin{bmatrix} F_t \\ F_t^* \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} \beta_0(1-S_t)+\beta_1 S_t \\ \beta_0^*(1-S_t^*)+\beta_1^* S_t^* \end{bmatrix}, \boldsymbol{Z}_t = \begin{bmatrix} \boldsymbol{z_t} \\ \boldsymbol{z_t^*} \end{bmatrix}, \boldsymbol{N}_t = \begin{bmatrix} \eta_t \\ \eta_t^* \end{bmatrix}, \boldsymbol{E}_t = \begin{bmatrix} \boldsymbol{e_t} \\ \boldsymbol{e_t^*} \end{bmatrix},$$

with the latter two noise vectors distributed as:

$$\boldsymbol{N}_t \sim \boldsymbol{MVN}(\boldsymbol{0}, \Omega); \quad \boldsymbol{E}_t \sim \boldsymbol{MVN}(\boldsymbol{0}, \Xi); \quad S_t,\ S_t^* = 0, 1,$$

where

$$\Omega = \mathrm{E}(\boldsymbol{N}_t \boldsymbol{N}_t^{'}) = \mathrm{E}([\eta_t\ \eta_t^*]'[\eta_t\ \eta_t^*]) = \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^{*2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, and$$

$$\Xi = \begin{bmatrix} \Sigma & 0 \\ 0 & \Sigma^* \end{bmatrix},$$

where Σ and Σ^* stand for the respective covariance matrix for the Main Street and Wall Street factors. The factor variances σ^2 and σ^{*2} are set at unity.

Equation (6) states that the vectors $\boldsymbol{Y}_t$ are composed of two sets of leading indices which can be expressed as the stochastic latent factors $\boldsymbol{F}_t$ and the two idiosyncratic terms $\boldsymbol{Z}_t$. The two latent factors are the Main Street and Wall Street factors, respectively. The factor loading matrix of the observable variables

$$\Gamma(L) = \begin{bmatrix} \gamma(L) & 0 \\ 0 & \gamma^*(L) \end{bmatrix}$$

is a 6×2 diagonal matrix with two vector polynomials. The terms $\gamma(L)$ and $\gamma^*(L)$ are the 3×1 polynomial vectors of loading.

Equation (7) describes the movement of the latent variable $\boldsymbol{F_t}$, which consists of an intercept vector $\boldsymbol{B}$ and a white noise vector $\boldsymbol{N}_t$. The intercept vector is a function of the two different state variables S_t and S_t^*, both of which are unobserved latent variables, taking on 1 when the real and financial factors are in expansion and 0 when they are in contraction. The variance of $\boldsymbol{N}_t$, which is also the variance of the dynamic factors, $\boldsymbol{F}_t$, consists of the two variances σ^2 and σ^{*2}. The matrix

$$\Phi(L) = \begin{bmatrix} \phi(L) & 0 \\ 0 & \phi^*(L) \end{bmatrix}$$

is the autoregressive term for the factors.

The states that affect the intercepts and variances are governed by the transition probabilities of the first-order two-state Markov process, $p_{ij} = \text{Prob}(S_t = j | S_{t-1} = i)$ and $p^*_{ij} = \text{Prob}(S^*_t = j | S^*_{t-1} = i)$, with $\sum_{j=0}^{1} p_{ij} = \sum_{j=0}^{1} p^*_{ij} = 1$, $i, j = 0, 1$. It should be noted that if regime switching in the Wall Street and Main Street types of asymmetry is driven by the same state variable, i.e., $S_t = S^*_t$, then in essence, this assumption would mean all recessions switch synchronously. Besides this, we allow the Main Street and Wall Street factors to switch non-synchronously over time, that is $S_t \neq S^*_t$. Under this assumption, the two-factor model can be estimated separately, and in so doing, the log likelihood function corresponds to the sum of the function for each factor derived from the one-factor model (see Chauvet, 1998/1999).

Finally, equation (8) specifies the error term of the leading indicators. Term $\Theta(B) = [\theta(B)\ \ \theta^*(B)]$ makes up 1×6 vector polynomials, and $\boldsymbol{E}_t = [\boldsymbol{e}_t\ \ \boldsymbol{e}^*_t]'$ represents 6×1 measurement errors with the covariance matrix Σ and Σ^* for the main street and Wall Street factors, respectively.

3.4 Estimation Procedure

The estimation procedure is based on Kim's (1994) approximate maximum likelihood method, which requires converting equations (6) through (8) into a state-space representation. We report the state-space in the appendix due to constraints with respective to space.

Performing the estimation consists of the following steps. First, the ergodic probability must be calculated as the initial value and then the Kalman filter and the Hamilton filter must be applied to this model. The most innovative aspect of the Hamilton filter is in its ability to objectively date the state of the economy using the so-called filtered and smoothed probabilities. The filtered probabilities (collected in a $(T \times 1)$ vector denoted as $\xi_{t|t}$, i.e., $\xi_{t|t} = p(S_t = j|\Psi_t)$; $t = 1, \ldots, T,\ j = 0,\ 1$; and Ψ_t which make up the information set) denote the conditional probability that the analyst's inference about the value of S_t is based on information obtained through date t. It is also indeed possible to calculate smoothed probabilities, $\xi_{t|T} = p(S_t = j|\Psi_T)$, $t = 1, \ldots, T,\ j = 0,\ 1$, which are based on the full sample.[7] Finally, as proposed by Kim (1994), an approximation must be made in order to be able to record the log-likelihood function as follows:

$$\text{logL} = \ln f(\boldsymbol{Y}_T, \boldsymbol{Y}_{T-1}, ...|\Psi_0) = \sum_{t=1}^{T} \ln f(\boldsymbol{Y}_t|\Psi_{t-1}). \tag{9}$$

The unknown parameter estimates of the model can then be obtained by maximizing the log-likelihood with respect to the unknown parameters by using the numerical method.

3.5 Prediction Criteria

We suggest two criteria to evaluate prediction failures. The first is the missed signal failure, viz, when there is a recession, but the model fails to predict it. The other is the false signal failure, namely, when the model predicts there is a recession, but one does not actually occur.

Once the conditional regime probabilities are generated, the issue pertaining to a decision rule to translate these probabilities into binary regime predictions remains. Birchenhall et al. (1999) suggested using two rules to convert a predicted probability into a predicted classification. One is the 0.5 rule and the other is the sample rule. Following the 0.5 rule, a recession is expected if the predicted probability exceeds 0.5 . On the other hand, based on the sample rule, a future recession is plausible if the predictive probability exceeds $\hat{p}$, where $\hat{p}$ is the sample proportion of the recession periods. Conflicting predictions arise when the predictive probability falls between

---

[7] Hamilton (1989) describes the manner in which to make an inference as to the particular state of the economy at date t.

0.5 and $\hat{p}$. The probability signals a contraction, but this signal is not sufficiently strong enough to overturn the overall population information in 0.5. This region delineates a period of market uncertainty. For simplicity, we adopt only the $\hat{p}$ rule for regime prediction.

4 Empirical Results

Our empirical results can be categorized into two parts depending on the number of leading indicators used. The first part involves the use of the three Main Street leading indicators, the three Wall Street leading indicators and the six leading indicators. They are all one-factor models.

Bear in mind that our objective is to adopt a two-factor model to extract two possible information sets from our six variables. Hence, a two-factor, six-variable model is expected. We find, however, that such a model is difficult to implement because of the problem of non-convergence.[8] Also, as we will explain in the next section, one real as well as the financial variables are insignificant in the generation of the unobservable factor. Omitting these two variables from the six-variable model does not affect the estimated results significantly but does allow us to adopt the two-factor model. From this reason, we downsize the scale of the model from six to four variables. This forms is the second part of our estimated results.

The next part is the use of four variables to extract the factor. The three models in this part are the one-factor (with only four-variable) model, the two-factor one-state model and the two-factor two-state model, where the *state* here denotes the state variable, S. Hence, one-state means the one

[8] All computations of the unknown parameters are implemented by using the OPTMUM module of GAUSS 3.2 combined with the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm. We impose no constraint on any of the transition probabilities p_{00} and p_{11} other than the conditions that $0 \leq p_{00} \leq 1$ and $0 \leq p_{11} \leq 1$, and that σ is constrained to be positive. For the two-factor model with six variables, we find that the model is extremely difficult to converge, and is sensitive to initial values. When it converges, the results are not reliable; for example, they have very large standard errors. Thus, we reduce the number of variables used from six to four.

state variable with two regimes, while two-state denotes two state variables with four regimes.

4.1 Three and Six Leading Indicators

Three one-factor models are estimated in this section, with the first two using only three variables and the last one using the six variables discussed earlier. All of them are one-factor models. The former two are also respectively referred to as the Main Street one-factor (MSF) and the Wall Street one-factor (WSF) model depending on whether the real or financial variables are used.

The lag lengths of the factor loading $\gamma(L)$, k, the factor autoregressive terms $\phi(L)$, q, and the idiosyncratic terms $\theta(L)$, r, are determined based on the Schwarts Bayesian Criterion (SBC). The maximum length is 2 to maintain the degree of freedom.

For all models, we first examine whether the unobserved factor indeed switches between two regimes or not. To this end, we estimate the linear dynamic factor model (hereafter DF) of Stock and Watson (1989, 1991, 1993), a model which resembles the MS model but which does not have the switching coefficients, β_0 and β_1. For simplicity, our model is referred to as the dynamic factor Markov Switching model (DFMS). In other words, both models assume that there is one unobservable factor among the variables, but which the DFMS assumes that the factor switches between two regimes the DF does not. Hence, by imposing zero on the switching coefficients in the DFMS model, we obtain the DF model. Since the DF is nested in the current DFMS, a significant likelihood ratio (LR) implies the rejection of the null of no switching.[9]

There is one econometric issue when LR is used. Because the parameters are not identified under the null, the conventional LR test does not yield the standard asymptotic distribution.[10]

---

[9]See Kim and Yoo (1995), Kim and Nelson (1998), and Chauvet (1998).

[10]The problem comes from two sources: under the null hypothesis, some parameters are not identified, and the values are identified as zero. Hansen (1992, 1996) proposed a bound test that addressed these problems, but its computational difficulty has limited its applicability. See Hansen (1992, 1996) and Garcia (1998) for a detailed explanation of these

Most researchers, however, still use LR as a useful supporting evidence. LR itself, however, may not be suitable to serve as the sole evidence for the rejection or non-rejection of the null hypothesis. Throughout the paper, our LR tests are considered in this way.

4.1.1 The Main Street One-Factor Model (MSF)

Table 1 presents the estimated results of the MSF model using both DF and DFMS. The three variables from the real sector are {ORDER, EXPORT and BUILD}. The lag lengths of k, q and r are selected as 1, 2 and 2. The switching coefficients β_0 and β_1 in the two regimes are first examined in the DFMS. While the intercept of regime 0, $\beta_0 = -0.670$, is insignificant, the intercept of regime 1, i.e., $\beta_1 = 0.404$, is significant at the 5% level. Also, the null of no switching coefficients, i.e., $\beta_0 = \beta_1 = 0$, is rejected by the LR test (= 19.032). The DFMS model is thus the preferable one, and the discussion below is based on this model.

There are three factor loading vectors, γ_0, γ_1 and γ_2 on F_t, F_{t-1} and F_{t-2}, respectively, in generating the unobserved factor. The factor loadings, γ_{10}, γ_{11} and γ_{12} of ORDER_t, ORDER_{t-1} and ORDER_{t-2} are 0.523, –0.293 and 0.020, respectively, with the first two significant at the 5% level but the last one insignificant. The factor loadings, γ_{20}, γ_{21} and γ_{22} are 0.168, 0.018 and 0.063 of EXPORT_t, EXPORT_{t-1} and EXPORT_{t-2}, with only the first one significant but the last two insignificant. In contrast to the above significant coefficients, none of the BUILD is significant. Thus, ORDER has the strongest effect in generating the factor, followed by EXPORT. BUILD has no effect on generating the factor whatsoever. BUILD is thus the candidate that could be removed from the model for the following four-variable model.

The three plots in the panel on the right of Figure 3 are consistent with the results that indicate ORDER and EXPORT respectively have the strongest and second strongest influence on the factor and that BUILD has no influence. The pair-wise scatter plots of the generated factor against either

---

problems.

ORDER or EXPORT are clearly shown to be positively sloped. By comparison, the scatter plot of the factor against BUILD is less clear. The respective correlation coefficients of the three pair wise plots are 0.904, 0.761 and –0.032, respectively.

The panel on the left in Figure 3 presents the graphs of the filtered and smoothed probabilities of the MSF model, and the results are mainly analyzed on the basis of the smoothed probability. The shaded areas are the contraction periods, as determined by the CEPD, and serve as the benchmarks of the comparisons. Based on the $\hat{p}$ prediction criterion, two "strong" false signals occurred in 1987 and 1992, when the smoothed probabilities are larger than 0.8. Aside from this, two "mild" false signals occur in 1993 and 1996, when the smoothed probabilities are higher than $\hat{p}$ but lower than 0.65. There is no missed signal. This model is aggressive in the sense that it generates only type II errors (false signals) but no type I errors (missed signals). Also calculated is the average lead time in predicting troughs.[11] The average lead time for correct signals is five months. The transition probability estimates $p_{11} = 0.914$ are greater than $p_{00} = 0.882$, showing that the duration periods for the booms are longer than the period during which the economy is in contraction.

4.1.2 The One-Factor Wall Street Model (WFS)

Table 2 presents the estimated results of the WFS model. It repeats the calculations for Table 1 but uses the three Wall Street variables {M1B, WPI and SP} to substitute for the three Main Street variables. The lag lengths k, q and r change slightly and are 2, 1 and 2, respectively. The two switching coefficients, β_0 and β_1, are –4.078 and 3.267, respectively. The null of the no switching restriction is significantly rejected by the LR test (LR statistic is 38.141), again suggesting that the factor taken from the Wall Street variables switches. The DFMS is thus preferable to the DF and

[11] We do not calculate the average lead time in predicting peaks because our smoothed probabilities is related to the probability of a recession.

the following report is based on the DFMS. It is worth noting that the difference between the two switching coefficients is much larger here than that in Table 1, suggesting that fluctuations in the factor from Wall Street are larger than those from Main Street.

All factor loading estimates are significant except for γ_{20} which is the coefficient of the current WPI, but although the coefficients are significant, their values are much smaller than those reported in the case of Main Street. Most of the values here are less than 0.1, in contrast to an average of 0.5 in the Main Street case. Put briefly, though the generated factor is also affected by the financial variables, their relative influence is small.

The panel on the right in Figure 4 presents the pairwised scatter plots of the generated factor against the three Wall Street variables. From the naked eye, except for the M1B, the scatter plots are less clear in direction on account of the above reported low values of the factor loadings. The corresponding correlation coefficients of the three pairs of {M1B, WPI and SP} against factor are 0.742, -0.272 and 0.548, respectively. Because the WPI has the lowest correlation with the factor and due to its insignificant factor loading, the WPI is the other candidate which should be removed when the four-variable model is attempted.

The panels on the left in Figure 4 show the graphs of the filtered and smoothed probabilities. Based on the latter probabilities, the WSP model characterizes the entire post-1995 period as a recession. This should not seem surprising because, in Taiwan, the financial market performed relatively badly after 1995 for the following reasons. First, two banking runs and one bill company scandal occurred in 1995,[12] During the same period, at the end of 1995 and 1996, China launched two "missile tests" against Taiwan. Third, the Asian crisis broke out in mid 1997 lasting

[12]The two banking runs are KauChi and DonChi Small and Medium Enterprise Banks, while the bill company is the International Bill Company. The former two arose from the relationship lending and the proxy fights for the controlling shares, respectively, whereas the latter is attributed to a scam of more than 10 billion New Taiwan Dollars by one employee.

to 1999 and was followed by the worldwide recession in 2000 largely owing to the burst of the bubble within the Internet industry. The continuing bad news and other incidents destroyed shattered investors' confidence, making them pessimistic about the economy. Accordingly, though the economy had not dipped into a recession throughout the whole-1995 period, the factor extracted from the financial leading indicators falsely shows such a downturn in the economy. Because of this, the average lead time in predicting a recession is difficult to determine. Another "mild" false signal is seen in 1983 as evidenced by the smoothed probabilities which are just higher than $\hat{p}$, but less than 0.6. This together with the fact no missed signals are found, suggests that the model is also too aggressive. The continuing errors in dating the recession for the entire 1995-period indicates that the financial variables may over-react to any negative incidents. Thus, the Wall Street sector alone is likely not an ideal leading indicator.

Moreover, unlike the case of Main Street, the transition probability estimates predict that the periods of duration are longer during contraction ($p_{11} = 0.981$) than in expansion ($p_{00} = 0.97$).

4.1.3 The One-Factor Six-Variable Model

This subsection puts the six leading indicators into the one-factor model without distinguishing Wall Street from Main Street. Table 3 summarizes the estimated results. Both the switching coefficients β_0 and β_1 are insignificantly different from zero, strongly suggesting that there is no switching when six variables are used simultaneously. This is confirmed by the LR test which does not reject the null of no switching. Thus, the DF model as opposed to the DFMS is the most acceptable. This evidence may reflect some inconsistent, or even divergent, movement between two sets of variables. To some extent here, information content for the six variables is inconsistent, if not contradictory. The predicted business cycle chronologies may also be affected.

In the DF model, the loading factors are mostly significant with the exception of γ_{30} and γ_{40}, implying that a common factor might exist but does not switch. In the DFMS model, the factor loadings are insignificant.

Even though no switching occurs in this model, we summarize the filtered and smoothed probabilities but do not report them here. This shows that several false signals occur in mid-1987, throughout 1988, from 1992–1994 and at the beginning of 1999. Nevertheless, we still plot the pair-wised scatter plot of the generated factor against the six leading indicators, as shown in Figure 5. Not surprisingly the previous clearly-sloped patterns of ORDER, EXPORT and the M1B disappear because the factor is not significantly affected by any of the variables. The correlation between the estimated factor and ORDER, EXPORT, BUILD, the M1B, WPI and the SP are 0.253, 0.308, 0.765, 0.389, -0.224 and 0.223, respectively, values which are much lower than their respective three-variable counterparts. This further motivates us to adopt the two-factor model.

4.2 The Four Leading Indicators

The discussion above shows that BUILD and WPI might help a little to generate the common factor. These results, along with the two-factor, six-variable model are not easy to converge, providing us with an even greater incentive to adopt a four-variable model. In this section, therefore, we repeat the above exercises, but we simultaneously employ the four variables, {ORDER, EXPORT, M1B and Stock}. We also plot the simple weighted index of the two Main Street and two Wall Street variables, and the spread between them. Their patterns are similar to those shown in Figure 1, and hence, are not reproduced here.

4.2.1 The One-Factor Model

Table 4 reports the estimated results from using the one-factor model with four variables. Compared with the case with six variables, it is most interesting to note that the previously insignificant switching coefficients (in Table 3) end up being significant. The two switching coefficients $\beta_0 = -1.112$ and $\beta_1 = 0.463$ are significant at the 5% level here. Unfortunately, the LR test

(=4.642) cannot reject the null of no switching. As mentioned earlier, the findings obtained from the LR should be considered along with those from other tests ln light of the LR's non-standard distribution. Since the finding of the individual t is significant and the LR test is indecisive, we tend to rely more on the results from the individual t tests. Hence, we said that "on the margin" the DFMS is acceptable in predicting the turning points in the Taiwan business cycle.

The graphs of the filtered and smoothed probabilities are plotted in the panel on the left in Figure 6. The smoothed probabilities here predict two "strong" false signals in 1987 and 1992 and two "mild" false signals in 1993 and 1996. Mild here means that these two false signals are around 0.4 and seem like replications of those of the MSF (Figure 3), which are around 0.6. In this sense, removing BUILD and WPI from the model improves the prediction to some extent. As a general rule, the correlation estimates between the generated factors increase and are 0.878, 0.798, 0.197 and 0.489 for {ORDER, EXPORT, the M1B, and the SP}, respectively.[13] Based on these coefficients, the generated factor seems to be affected more by the real than by the Wall Street variables. The average lead time in predicting troughs is around 3 months.

4.2.2 The Two-Factor One-State Variable

There are two designs in our two-factor model depending on whether one or two state variables are used. Table 5 reports the estimated results of using one state variable, and assume that the state variable, S_t, generated by the Main Street is equal to the state variable, S_t^*, generated by Wall Street. Hence, there is only one smoothed probability, making all recessions obtained by the two types of variables switch synchronously. In the table, the two switching coefficients of the real sector, β_0 and β_1, are found to be significantly different from zero, whereas the two coefficients of the financial sector, β_0^* and β_1^*, are insignificant. These findings are consistent with those above which state that Main Street variables have a greater effect on the generated factor. These results

[13]The scatter plots are not shown due to space considerations but available from the authors upon request.

also suggest that the smoothed probabilities are impacted more by the real than by the financial sector. The LR test, however, rejects the null of no switching. Our discussion is based on the DFMS.

The loadings estimates of the variables are also improved. The loadings of ORDER and EXPORT at $t, t-1$ and $t-2$ are significant at least at the 10% level, those of the M1B at t and $t-2$ are significant, while the loading of only the current SP is significant. This evidence again suggests that the Main Street variables dominate Wall Street ones.

Figure 7 plots the posterior probabilities of the model. In contrast to the two strong and the two mild false signals using the Main Street variables, along with the false signals for the whole post-1995 period using the Wall Street variables, the new model has only one strong false signal, namely the one in 1987. Noteworthily is that the predicted probabilities are around 0.4 in 1995 and 1998, values which are only slightly higher than $\hat{p}$. Recessions are predicted if the $\hat{p}$ rule is followed but not if the 0.5 rule is followed. Because the former is adopted here, we feel it is reasonable to suggest that there were recessions in the two periods. Thus, it correctly forecast these two recessions in the margin. This could be explained by uncertainty in our predictions, as we mention above. Thus, in sharp contrast to the aggressive nature of the previous models, for the most part, using the two-factor with the one-state variable model mitigates the aggressive nature. The average lead time in predicting troughs is around 4 months.

4.2.3 The Two-Factor Two-State Variables

The estimates of the two-factor with two-state variables, S_t and S_t^*, are summarized in Table 6. The smoothed probabilities of the Main Street and Wall Street factors in this model are allowed to switch non-synchronously over time. Some factor loading estimates for the Wall Street variables are, again, still not significant, but most of those are of the Main Street variables. The regime estimates for both Main and Wall Street are significant at 5%. The null hypothesis of no switching by the LR is also rejected by the LR test (The LR statistic is 40.590).

Figure 8 represents the plots of the filtered and smoothed probabilities with $S_t \neq S_t^*$ for the Main and Wall Street variables. The smoothed probabilities of Main Street here are similar to the MSF model (Figure 3), and the smoothed probabilities of Wall Street are similar to the WSF model (Figure 4). The resulting false signals here are therefore almost the same as theirs.

In sum, except for the two-factor one-state variable model, all the remaining models produce more than four false signals and have no missed signals. Simply put, the model seems to lessen the "aggressiveness" to some extent.

5 Out-of-Sample Forecasting

This section conducts out-of-sample forecasting to determine exactly which model is superior. Apart from this, and an important point, a good in-sample may have a bad out-of-sample forecast because of over-fitting (Clements and Krolzig, 1998). We employ the quadratic probability score (QPS), as defined by Hamilton and Perez-Quiros (1996), to evaluate the forecasting performances of in-sample and out-of-sample.[14] The QPS is:

$$\text{QPS} = K^{-1} \sum_{t=1}^{K} \{\text{prob}(S_t = 0|\Psi_T) - d_t\}^2, \tag{10}$$

where $d_t = 1$ if dated as a period within the CEPD-defined contraction. The closer the QPS is to zero, the more consistent the model-generated regime is with the chronology of an official business cycle.

Only four models, the MSF, WSF, one-factor four-variable and the one-factor one-state variable

[14]The QPS was first suggested by Diebold and Rudebusch (1989), who used the QPS as a measure of correspondence between turning point probabilities and actual turning points. By contrast, Filardo (1994) and Hamilton and Perez-Quiros (1996) used it in conjunction with the actual NBER phase dates and the model-generated regime probabilities for each data point in the series.

are compared. We do not consider the one-factor six-variable since it rejects the switching hypothesis. The two-factor two-state model is also not considered in that it produces almost the same results as the first two models.

The QPS of the in-sample and out-of-sample comparisons is calculated in the following. The in-sample QPS covers the period from 1983:m1 to 2001:m4 and totals 220 observations. The calculation of the out-of-sample QPS requires more works as it necessitates the defining estimation periods first and then making an out-sample forecast. We conduct one- and two-period-ahead forecasts but only report the former. The explanation below is based on the one-step-ahead forecast. Also, given the eruption of the 1997 Asian crisis, we consider two starting periods of the prediction, namely, 1997:m11 and 1999:m1, for our out-of-sample forecast.

The results of the in-sample forecasting (Table 7) show that the (four-variable) two-factor one-state model leads to the best recession predictions and is followed by the MSF model and the one-factor six-variable model. This should not come as a surprise since the two-factor one-state model is less aggressive. That the WSF model has the worst in-sample forecasting performance is also not astonishing as the Wall Street sector alone (Figure 4) produces continuous false signals for the whole post-1995 period. Thus very simply staked, this is not expected.

As stated above, we conduct two out-of-sample forecasts using both 1997:m11 and 1999:m1 as the starting periods. When 1997:m11 is used as the starting period of the forecast, the initial estimation covers 1983:m1 to 1997:m10. Upon obtaining the estimates, we conduct a one-step-ahead forecast. Once these estimate-forecast steps are performed, we move the one-period forward and re-estimate the model. We then make new one-step-ahead forecast using the new estimates. In the next stage, we repeat these steps until we reach the last sample. In this way, we recursively estimate the parameters throughout the out-of-sample period. There are $K = 42$ out-of-sample forecasts, ranging from 1997:m11 to 2001:m4. In Table 7, the resulting QPSs are almost the same as those in the four models. This is probably because the abruptly burst Asian crisis changed the stable relation between the leading indicators and the business cycle. That is, the leading indicators may have predicted a boom before a crisis but this is inaccurate since the suddenly happened crisis change the relations. It is obvious then that the four models perform similarly when 1997:m11 is used as the starting period of the prediction.

The use of the second starting period of the forecast helps us to determine the forecasting ability of the models. The estimation period ranges from 1983:m1 to 1998:m12, yielding 28 out-of-sample forecasts. We find that the out-of-sample forecasts in this case produce a similar ranking of forecasts to that from those in-sample forecasts. Not unlike what we observe with other models, the two-factor one-state model has the best predicting performance, reaching the minimum out-of-sample QPS ($= 0.099$). Furthermore, its QPS is far less than the second best model, the MSF (0.138). Once more, the WSF model shows the worst performance. Figure 10 summarizes the corresponding out-of-sample filtered probabilities for the recession regime. These plots clearly demonstrate the superiority of using the two-factor one-state model if the samples are run from 1999:m1 to 2001:m4.

6 Concluding Remarks

This paper argues that the simple sum of all the leading indicators may combine conflicting information and, therefore, provide less than accurate predictions. We highly suggest that, based on their inherent characteristics, the leading indicators be divided into two sectors, real and financial sectors. The real sector contains new manufacturing orders, exports through customs, and floor space available for building in Taiwan. The financial sector contains the stock price index, a narrowed money supply and the wholesale price index. The rational for this is that the two sectors may not share the same information with respect to future recessions.

Six models are considered in this paper. The first set of the group includes either three or six variables in our models, which comprise the one-factor Main Street model (three variables). The second set of the group contains only four variables for three models, i.e., the one-factor, two-factor one-state and the two-factor two-state models.

The in- and out-of-sample forecasts are not conducted for either the one-factor six-variable or the two-factor two-state models because the former rejects the switching hypothesis, as mentioned above, and the datings from the latter are very similar to the results as of the MSF and WSF models, respectively. As a consequence only four models are repeated in the prediction comparisons. The ranking of the forecasting ability of the remaining four models are explained below.

First, the two-factor one-state model performs the best in both in- and out-of-sample forecasts. It produces fewer false signals than do the other models. The model is, in fact, the least aggressive as it only produces one false signal. The runner-up for both forecasts is the one-factor Main-Street model, followed by the one-factor four-variable model. The former is aggressive in the sense that there are no missed signals, but it produces two strong and two mild false signals, whereas the latter has only two strong and two evil milder false signals. The worst performing model is the Wall Street model since it suggests the whole post-1995 period was in a as recession caused by continuous bad news in the financial markets studied here.

Our results have three central implications. First, the concept of the two-factor model is important because the two sectors may have conflicting information, which somewhat lessens the degree of forecast aggressiveness. The aggressiveness may be due to overfitting, but that is another issue. Next, the Wall Street sector tends to over-react to news, especially that continuing bad news, which makes investors overly pessimistic, and causes the financial leading indicators to perform badly. Third, we can downsize the model if it is too difficult to converge because of the complexities inherent in the model. Removing those variables which have insignificant factor loadings changes the results very little.

Appendix: State-Space Representation and Algorithm

In this appendix, we briefly describe how to restate the two-factor model with regime switching, i.e., equations (6) to (8), into a state-space representation and then apply the Kim's (1994) algorithm with regard to the approximate maximum likelihood method to calculate unknown parameter estimates. Basically, Kim's algorithm is a synthesis of Hamilton's and Kalman's filters. Equations (6) to (8) can be transformed into the measurement equation (11) and the transition equation (12) as follows:

$$\boldsymbol{Y}_t = \boldsymbol{H}_t \boldsymbol{\xi}_t; \tag{11}$$

$$\boldsymbol{\xi}_t = \boldsymbol{T}_t \boldsymbol{\xi}_{t-1}; + \boldsymbol{\beta}_{S_t} + \boldsymbol{u}_t \tag{12}$$

with

$$\boldsymbol{H}_t = \begin{bmatrix} \gamma_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \gamma_2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \gamma_3 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_1^* & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_2^* & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_3^* & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

$$\boldsymbol{T}_t = \begin{bmatrix} \phi_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \theta_{11} & \theta_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \theta_{21} & \theta_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \theta_{31} & \theta_{32} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \phi_1^* & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_{11}^* & \theta_{12}^* & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_{21}^* & \theta_{22}^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_{31}^* & \theta_{32}^* \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

$$\boldsymbol{\xi}_t = \begin{bmatrix} F_t & z_{1,t} & z_{1,t-1} & z_{2,t} & z_{2,t-1} & z_{3,t} & z_{3,t-1} & F_t^* & z_{1,t}^* & z_{1,t-1}^* & z_{2,t}^* & z_{2,t-1}^* & z_{3,t}^* & z_{3,t-1}^* \end{bmatrix}',$$

$$\boldsymbol{\beta}_{S_t} = \begin{bmatrix} \beta(S_t) & 0 & 0 & 0 & 0 & 0 & 0 & \beta^*(S_t{}^*) & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}',$$

$$\boldsymbol{u}_t = \begin{bmatrix} \eta_t & \varepsilon_{1t} & 0 & \varepsilon_{2t} & 0 & \varepsilon_{3t} & 0 & \eta_t^* & \varepsilon_{1t}^* & 0 & \varepsilon_{2t}^* & 0 & \varepsilon_{3t}^* & 0 \end{bmatrix}',$$

$$\boldsymbol{Q}_t = \begin{bmatrix}
\sigma_\eta^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \sigma_{\varepsilon_1}^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \sigma_{\varepsilon_2}^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sigma_{\varepsilon_3}^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{\eta^*}^2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{\varepsilon_1{}^*}^2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{\varepsilon_2{}^*}^2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{\varepsilon_3{}^*}^2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},$$

where $\boldsymbol{Q} = \mathrm{E}(\boldsymbol{u}_t \boldsymbol{u}_t')$. Under the restriction of $S_t = S_t^*$ (given the realization of the state variables at times t and $t-1$ ($S_t = j$ and $S_{t-1} = i$, where i, j =0 or 1)) and using the notation $\mathcal{Z}_{t|t-1}^{(i,j)}$ to denote the variable $\mathcal{Z}$ conditional on the information available up to $t-1$ and the realized states j and i, the Kalman filter can then be represented as:

$$\boldsymbol{\xi}_{t|t-1}^{(i,j)} = \boldsymbol{T}_t \boldsymbol{\xi}_{t-1|t-1}^{(i)} + \boldsymbol{\beta}_{S_t}^{(j)}; \tag{13}$$

$$\boldsymbol{P}_{t|t-1}^{(i,j)} = \boldsymbol{T}_t \boldsymbol{P}_{t-1|t-1}^{(i)} \boldsymbol{T}_t' + \boldsymbol{Q}_t; \tag{14}$$

$$\boldsymbol{\xi}_{t|t}^{(i,j)} = \boldsymbol{\xi}_{t|t-1}^{(i,j)} + \boldsymbol{K}_t^{(i,j)} \boldsymbol{\eta}_{t|t-1}^{(i,j)}; \tag{15}$$

$$\boldsymbol{P}_{t|t}^{(i,j)} = (\boldsymbol{I} - \boldsymbol{K}_t^{(i,j)} \boldsymbol{H}_t) \boldsymbol{P}_{t|t-1}^{(i,j)}; \tag{16}$$

$$\boldsymbol{\eta}_{t|t-1}^{(i,j)} = \boldsymbol{Y}_t - \boldsymbol{H}_t \boldsymbol{\xi}_{t|t-1}^{(i,j)}; \tag{17}$$

$$\boldsymbol{W}_{t|t-1}^{(i,j)} = \boldsymbol{H}_t \boldsymbol{P}_{t|t-1}^{(i,j)} \boldsymbol{H}_t'; \tag{18}$$

$$\boldsymbol{K}_t^{(i,j)} = \boldsymbol{P}_{t|t-1}^{(i,j)} \boldsymbol{H}_t' (\boldsymbol{W}_{t|t-1}^{(i,j)})^{-1}, \tag{19}$$

where equations (13) and (14) are the prediction formulae, equations (15) and (16) are the updating formulae, and equation (19) is the Kalman gain. Term $\boldsymbol{\eta}^{(i,j)}_{t|t-1}$ is the conditional forecast error of $\boldsymbol{Y}_t$ based on information up to $t-1$, and $\boldsymbol{W}^{(i,j)}_{t|t-1}$ is the conditional variance of the forecast error $\boldsymbol{\eta}^{(i,j)}_{t|t-1}$.

As noted by Harrison and Stevens (1976), each iteration of the above Kalman filtering produces a two-fold increase in the number of cases to consider. Kim (1994) provides a fast approximation algorithm which can be applied to this problem. The crux of the issue is to collapse the dimensions of the (2×2) posteriors ($\boldsymbol{\xi}^{(i,j)}_{t|t}$ and $\boldsymbol{P}^{(i,j)}_{t|t}$) to two posteriors ($\boldsymbol{\xi}^{(j)}_{t|t}$ and $\boldsymbol{P}^{(j)}_{t|t}$) by taking the weighted averages over the states at $t-1$. That is,

$$\boldsymbol{\xi}^{(j)}_{t|t} = \frac{\sum_{S_{t-1}=0}^{1}\Pr[S_t=j,S_{t-1}=i|\Psi_t]\times\boldsymbol{\xi}^{(i,j)}_{t|t}}{\Pr[S_t=j|\Psi_t]}, \tag{20}$$

$$\boldsymbol{P}^{(j)}_{t|t} = \frac{\sum_{S_{t-1}=0}^{1}\Pr[S_t=j,S_{t-1}=i|\Psi_t]\times\{\boldsymbol{P}^{(i,j)}_{t|t}+(\boldsymbol{\xi}^{(j)}_{t|t}-\boldsymbol{\xi}^{(i,j)}_{t|t})(\boldsymbol{\xi}^{(j)}_{t|t}-\boldsymbol{\xi}^{(i,j)}_{t|t})'\}}{\Pr[S_t=j|\Psi_t]} \tag{21}$$

where Ψ_t refers to the information available at time t. Following Hamilton (1989), the filter can be obtained by Bayes's theorem. That is:

$$\begin{aligned}\Pr[S_t=j,S_{t-1}=i|\Psi_t] &= \frac{\Pr[\boldsymbol{Y}_t,S_t=j,S_{t-1}=i|\Psi_{t-1}]}{\Pr[\boldsymbol{Y}_t|\Psi_{t-1}]}\\ &= \frac{f[\boldsymbol{Y}_t|S_t=j,S_{t-1}=i,\Psi_{t-1}]\times\Pr[S_t=j,S_{t-1}=i|\Psi_{t-1}]}{\Pr[\boldsymbol{Y}_t|\Psi_{t-1}]}\end{aligned} \tag{22}$$

where

$$f[\boldsymbol{Y}_t|S_t=j,S_{t-1}=i,\Psi_{t-1}] = (2\pi)^{-N/2}|\boldsymbol{W}^{(i,j)}_{t|t-1}|^{-1/2}\times\exp\{-\frac{1}{2}\boldsymbol{\eta}^{(i,j)\prime}_{t|t-1}(\boldsymbol{W}^{(i,j)}_{t|t-1})^{-1}\boldsymbol{\eta}^{(i,j)}_{t|t-1}\} \tag{23}$$

The smoothed probabilities, $p(S_t|\Psi_T)$, on the other hand stand for the conditional probability based on data available through the whole sample at future date T, which amount to

$$\Pr[S_{t+1}=k,S_t=j|\Psi_T] \approx \frac{\Pr[S_{t+1}=k|\Psi_T]\times\Pr[S_t=j|\Psi_t]\times\Pr[S_{t+1}=k|S_t=j]}{\Pr[S_{t+1}=k|\Psi_t]} \tag{24}$$

$$\Pr[S_t=j|\Psi_T] = \sum_{S_{t+1}=0}^{1}\Pr[S_{t+1}=k,S_t=j|\Psi_T]. \tag{25}$$

The approximate sample conditional log-likelihood is:

$$\text{logL} = \ln f(\boldsymbol{Y}_T,\boldsymbol{Y}_{T-1},...|\Psi_0) = \sum_{t=1}^{T}\ln f(\boldsymbol{Y}_t|\Psi_{t-1}). \tag{26}$$

The approximate maximum likelihood estimates of the model can be obtained by maximizing the log-likelihood with respect to the unknown parameters.

References

Bandholz, H. and M. Funke (2003), In Search of Leading Indicators of Economic Activity in Germany, *Journal of Forecasting*, **22**, 277–297.

Birchenhall, C. R., H. Jessen, D. R. Osborn and P. Simpson (1999), Predicting U.S. business cycle regime, *Journal of Business and Economic Statistics*, **17**, 313–323.

Chauvet, M. (1998), An econometric characterization of business cycle dynamics with factor structure and regime switching, *International Economic Review*, **39**, 969–996.

Chauvet, M. (1998/1999), Stock market fluctuations and the business cycle, *Journal of Economic and Social Measurement*, **3, 4**, 235–258.

Chauvet, M. and S. Potter (2000), Coincident and leading indicators of the stock market, *Journal of Empirical Finance*, **7**, 87–111.

Clements, M.P. and H.-M. Krolzig (1998), A Comparison of the Forecast Performance of Markov-Switching and Threshold Autoregressive Models of US GNP, *Econometrics Journal*, **1**, C47–C75.

Diebold, F. X. and G. D. Rudebusch (1989), Scoring the leading indicator, *Journal of Business*, **62**, 369–391.

Diebold, F. X. and G. D. Rudebusch (1996), Measuring business cycles: A modern perspective, *The Review of Economics and Statistics*, **78**, 67–77.

Filardo, A. J. (1994), Business-cycle phases and their transitional dynamics, *Journal of Business and Economic Statistics*, **12**, 299–308.

Garcia, R. (1998), Asymptotic null distribution of the likelihood ratio test in Markov-switching model, *International Economic Review*, **39**, 763–788.

Hamilton, J. D. (1989), A new approach to the economic analysis of nonstationary time series and the business cycle, *Econometrica*, **57**, 357–384.

Hamilton, J. D. and G. Perez-Quiros (1996), What do leading indicators lead? *Journal of Business*, **69**, 27–49.

Hamilton, J. D. and G. Lin (1996), Stock market volatility and the business cycle, *Journal of Applied Econometrics*, 11, 573–593.

Hansen, B. E. (1992), The likelihood ratio test under nonstandard conditions: Testing the Markov-switching model of GNP, *Journal of Applied Econometrics*, **7**, S61–S82.

Hansen, B. E. (1996), Erratum: The likelihood ratio test under nonstandard conditions: Testing the Markov-switching model of GNP, *Journal of Applied Econometrics*, **11**, 195–198.

Kim, C.-J. (1994), Dynamic linear models with Markov-switching, *Journal of Econometrics*, **60**, 1–22.

Kim, C.-J. and C. J. Murray (2002), Permanent and transitory components of recessions, *Empirical Economics*, **27**, 163–183.

Kim, C.-J. and C. R. Nelson (1998), Business cycle turning points, a new coincident index, and tests of duration dependence based on a dynamic factor model with regime switching, *The Review of Economics and Statistics*, **80**, 188–201.

Kim, C.-J. and J. Piger (2002), Common stochastic trends, common cycles, and asymmetry in economic fluctuations, *Journal of Monetary Economics*, **49**, 1189–1211.

Kim, M.-J. and J.-S. Yoo (1995), New index of coincident indicators: A multivariate Markov-switching factor model approach, *Journal of Monetary Economics*, **36**, 607–630.

Shen, C.-H. and L.-R. Wang (1998), Daily serial correlation, trading volume, and price limit, *Pacific Basin Finance Journal*, **6**, 251–274.

Stock, J. H. and M. W. Watson (1989), New indexes of coincident and leading economic indicators, in O. Blanchard and S. Fischer (eds.), *NBER Macroeconomics Annual*, Cambridge, MA: MIT Press.

Stock, J. H. and M. W. Watson (1991), A probability model of the coincident economic indicators, in K. Lahiri and G. H. Moore (eds.), *Leading Economics Indicators: New Approach and Forecasting Records*, Cambridge, Cambridge University Press.

Stock, J. H. and M. W. Watson (1993), A procedure for predicting regressions with leading indicators: econometric issues and recent experience, in J. H. Stock and M. W. Watson (eds.), *Business Cycle, Indicators and Forecasting*, Chicago: University of Chicago Press for NBER, 255–284.

Table 1: Three Main Street Variables of the One-Factor Model

Parameter	DF	Std. Err.	Parameter	DFMS	Std. Err.
ϕ_1	0.897	0.035	ϕ_1	0.879	0.061
θ_{11}	1.301	0.005	θ_{11}	1.848	0.056
θ_{12}	−1.000	0.001	θ_{12}	−0.896	0.052
θ_{21}	−0.069	0.068	θ_{21}	−0.391	0.079
θ_{22}	0.195	0.069	θ_{22}	−0.038	0.015
θ_{31}	0.402	0.063	θ_{31}	0.393	0.063
θ_{32}	0.433	0.064	θ_{32}	0.444	0.063
σ_1	0.001	0.001	σ_1	0.044	0.012
σ_2	0.742	0.035	σ_2	0.627	0.033
σ_3	0.655	0.031	σ_3	0.653	0.031
γ_{10}	0.125	0.043	γ_{10}	0.523	0.035
γ_{11}	−0.267	0.046	γ_{11}	−0.293	0.050
γ_{12}	0.574	0.027	γ_{12}	0.020	0.038
γ_{20}	0.057	0.054	γ_{20}	0.168	0.045
γ_{21}	0.053	0.068	γ_{21}	0.018	0.068
γ_{22}	0.185	0.051	γ_{22}	0.063	0.051
γ_{30}	0.036	0.044	γ_{30}	−0.010	0.039
γ_{31}	−0.057	0.047	γ_{31}	−0.016	0.047
γ_{32}	0.004	0.045	γ_{32}	0.062	0.041
			β_0	−0.670	0.313
			β_1	0.404	0.272
			p_{00}	0.882	0.080
			p_{11}	0.914	0.050
logL	−53.286		logL	−43.770	
H_0: DF vs DFMS	19.032				

The ordering of the variables are ORDER, EXPORT and BUILD, respectively.
The subscript index 1 of parameter γ_{1i} denotes the factor loading estimate of ORDER.
The subscript index 1 of parameter θ_{1i} denotes the idiosyncratic loading estimate of ORDER.
The model is estimated with $k = 1$, $q = 2$ and $r = 2$, which are selected using the SBC criterion.
The terms k, q, and r are the lag parameters of polynomial $\phi(L)$, $\gamma(L)$, and $\theta(L)$, respectively.
The term DF denotes the linear dynamic one-factor model.
The term DFMS the denotes the dynamic one-factor model with regime switching.
The factor variance σ_η is set to unity for interpretation purpose.
The test for H_0: DF vs DFMS is rejected when compared to $\chi^2_{0.05}(4) = 9.488$.
The correlation between the estimated factor and ORDER, EXPORT and BUILD
are 0.904, 0.761 and −0.032, respectively.

Table 2: Three Wall Street Variables with the One-Factor Model

Parameter	DF	Std. Err.	Parameter	DFMS	Std. Err.
ϕ_1	0.484	0.172	ϕ_1	0.925	0.156
ϕ_2	-0.058	0.041	ϕ_2	-0.214	0.072
θ_{11}	1.805	0.042	θ_{11}	1.788	0.044
θ_{12}	-0.815	0.038	θ_{12}	-0.800	0.039
θ_{21}	1.401	0.059	θ_{21}	1.398	0.059
θ_{22}	-0.465	0.059	θ_{22}	-0.467	0.059
θ_{31}	1.314	0.063	θ_{31}	1.279	0.064
θ_{32}	-0.374	0.063	θ_{32}	-0.354	0.064
σ_1	0.065	0.011	σ_1	0.064	0.009
σ_2	0.253	0.012	σ_2	0.251	0.012
σ_3	0.272	0.013	σ_3	0.268	0.012
γ_{10}	-0.121	0.009	γ_{10}	-0.092	0.008
γ_{11}	-0.055	0.013	γ_{11}	0.028	0.015
γ_{20}	-0.030	0.015	γ_{20}	-0.013	0.010
γ_{21}	0.022	0.017	γ_{21}	0.041	0.011
γ_{30}	-0.036	0.015	γ_{30}	-0.038	0.011
			β_0	-4.078	1.203
			β_1	3.267	1.161
			p_{00}	0.970	0.017
			p_{11}	0.981	0.011
logL	650.492		logL	669.563	
H_0: DF vs DFMS	38.142				

The ordering of the variables are M1B, WPI, and SP, respectively.

The subscript index 1 of parameter γ_{1i} denotes the factor loading estimate of the M1B.

The subscript index 1 of parameter θ_{1i} denotes the idiosyncratic loading estimate of M1B.

The model is estimated with $k = 2$, $q = 1$ and $r = 2$ with $\gamma_{31} = 0$, which is selected using the SBC criterion.

The terms k, q, and r are the lag parameters of the polynomials $\phi(L)$, $\gamma(L)$ and $\theta(L)$, respectively.

The term DF denotes the linear dynamic one-factor model.

The term DFMS denotes the dynamic one-factor model with regime switching.

The factor variance σ_η is set to unity for the purpose of interpretation.

The test for H_0: DF vs DFMS is rejected when compared to $\chi^2_{0.05}(4) = 9.488$.

The correlation between the estimated factor and the M1B, WPI, and SP are 0.742, -0.272, and 0.548, respectively.

Table 3: Six Variables of the One-Factor Model

Parameter	DF	Std. Err.	Parameter	DFMS	Std. Err.
ϕ_1	0.925	0.026	ϕ_1	-0.469	0.103
ϕ_2	0.074	0.026	ϕ_2	-0.055	0.024
θ_{11}	0.345	0.073	θ_{11}	0.573	0.065
θ_{12}	0.208	0.072	θ_{12}	0.266	0.065
θ_{21}	-0.472	0.084	θ_{21}	0.195	0.062
θ_{22}	-0.055	0.020	θ_{22}	0.441	0.062
θ_{31}	0.400	0.062	θ_{31}	0.646	0.319
θ_{32}	0.433	0.062	θ_{32}	0.229	0.290
θ_{41}	1.399	0.060	θ_{41}	1.399	0.060
θ_{42}	-0.426	0.061	θ_{42}	-0.425	0.061
θ_{51}	1.500	0.120	θ_{51}	1.388	0.060
θ_{52}	-0.546	0.114	θ_{52}	-0.453	0.060
θ_{61}	1.336	0.063	θ_{61}	1.310	0.062
θ_{62}	-0.393	0.063	θ_{62}	-0.371	0.063
σ_1	0.525	0.028	σ_1	0.597	0.028
σ_2	0.595	0.035	σ_2	0.823	0.040
σ_3	0.657	0.031	σ_3	0.459	0.123
σ_4	0.169	0.008	σ_4	0.171	0.008
σ_5	0.175	0.033	σ_5	0.258	0.012
σ_6	0.269	0.013	σ_6	0.276	0.013
γ_{10}	0.263	0.033	γ_{10}	0.018	0.024
γ_{20}	0.274	0.024	γ_{20}	0.047	0.031
γ_{30}	-0.010	0.017	γ_{30}	0.197	0.125
γ_{40}	0.023	0.014	γ_{40}	-0.002	0.004
γ_{50}	0.183	0.032	γ_{50}	0.007	0.008
γ_{60}	-0.061	0.022	γ_{60}	-0.001	0.006
			β_0	-2.749	2.378
			β_1	5.686	4.371
			p_{11}	0.960	0.020
			p_{00}	0.902	0.043
logL	591.625		logL	555.469	
H_0: DF vs DFMS	-72.312				

The ordering of the variables are ORDER, EXPORT, BUILD, M1B, WPI and SP, respectively.

The subscript index 1 of parameter γ_{1i} denotes the factor loading estimate of ORDER.

The subscript index 1 of parameter θ_{1i} denotes the idiosyncratic loading estimate of ORDER.

The model is estimated with $k = 2$, $q = 0$ and $r = 2$, which are selected using the SBC criterion.

The terms k, q and r are the lag parameters of the polynomials $\phi(L)$, $\gamma(L)$ and $\theta(L)$, respectively.

The term DF denotes linear dynamic one factor model.

The term DFMS denotes the dynamic one-factor model with regime switching.

The factor variance σ_η is set to unity for interpretation purpose.

The test for H_0: DF vs DFMS is not rejected when compared to $\chi^2_{0.05}(4) = 9.488$.

The correlation between the estimated factor and ORDER, EXPORT, BUILD, M1B, WPI and SP are 0.253, 0.308, 0.765, 0.389, -0.224 and 0.223, respectively.

Table 4: Four Variables of the One-Factor Model

Parameter	DF	Std. Err.	Parameter	DFMS	Std. Err.
ϕ_1	1.556	0.081	ϕ_1	1.141	0.167
ϕ_2	-0.605	0.063	ϕ_2	-0.226	0.155
θ_{11}	0.325	0.074	θ_{11}	0.339	0.073
θ_{12}	0.206	0.071	θ_{12}	0.209	0.071
θ_{21}	-0.381	0.080	θ_{21}	-0.387	0.080
θ_{22}	-0.036	0.015	θ_{22}	-0.037	0.015
θ_{31}	1.752	0.049	θ_{31}	1.755	0.049
θ_{32}	-0.767	0.043	θ_{32}	-0.770	0.043
θ_{41}	1.316	0.063	θ_{41}	1.317	0.063
θ_{42}	0.375	0.063	θ_{42}	-0.376	0.063
σ_1	0.512	0.026	σ_1	0.517	0.026
σ_2	0.628	0.034	σ_2	0.625	0.033
σ_3	0.086	0.014	σ_3	0.084	0.014
σ_4	0.270	0.013	σ_4	0.271	0.013
γ_{10}	0.114	0.045	γ_{10}	0.105	0.039
γ_{11}	0.083	0.061	γ_{11}	0.085	0.049
γ_{12}	-0.035	0.042	γ_{12}	-0.027	0.035
γ_{20}	0.125	0.055	γ_{20}	0.107	0.048
γ_{21}	-0.049	0.094	γ_{21}	-0.022	0.083
γ_{22}	0.078	0.061	γ_{22}	0.072	0.051
γ_{30}	-0.110	0.012	γ_{30}	-0.088	0.012
γ_{31}	0.073	0.016	γ_{31}	0.051	0.016
γ_{32}	0.060	0.012	γ_{32}	0.059	0.012
γ_{40}	-0.047	0.020	γ_{40}	-0.039	0.017
γ_{41}	0.029	0.021	γ_{41}	0.018	0.018
γ_{42}	0.021	0.020	γ_{42}	0.019	0.017
			β_0	-1.122	0.358
			β_1	0.463	0.184
			p_{00}	0.843	0.075
			p_{11}	0.920	0.033
logL	432.051		logL	434.372	
H_0: DF vs DFMS	4.642				

The ordering of the variables are ORDER, EXPORT, M1B, and SP, respectively.
The subscript index 1 of parameter γ_{1i} denotes the factor loading estimate of ORDER.
The subscript index 1 of parameter θ_{1i} denotes the idiosyncratic loading estimate of ORDER.
The model is estimated with $k = 2$, $q = 2$ and $r = 2$, which are selected using the SBC criterion.
The terms $k,\ q,\ r$ are the lag parameters of the polynomials $\phi(L)$, $\gamma(L)$, and $\theta(L)$, respectively.
The term DF denotes the linear dynamic one-factor model.
The term DFMS denotes the dynamic one factor model with regime switching.
The factor variance σ_η is set to unity for interpretation purpose.
The test for H_0: DF vs DFMS is not rejected when compared to $\chi^2_{0.05}(4) = 9.488$.
The correlation between the estimated factor and ORDER, EXPORT, M1B, and SP
are 0.878, 0.798, 0.197, and 0.489, respectively.

Table 5: Four Variables for the Two-Factor Model and $S_t = S_t^*$

Parameter	TDF	Std. Err.	Parameter	TDFMS	Std. Err.
ϕ_1	0.762	0.133	ϕ_1	0.715	0.111
ϕ_2	0.146	0.120	ϕ_2	0.186	0.097
ϕ_1^*	1.812	0.051	ϕ_1^*	1.817	0.052
ϕ_2^*	−0.821	0.046	ϕ_2^*	−0.826	0.048
θ_{11}	0.101	0.090	θ_{11}	0.128	0.104
θ_{12}	−0.002	0.004	θ_{12}	0.015	0.088
θ_{21}	1.520	0.124	θ_{21}	1.574	0.104
θ_{22}	−0.578	0.094	θ_{22}	−0.620	0.082
θ_{31}	0.520	0.399	θ_{31}	0.499	0.408
θ_{32}	−0.067	0.103	θ_{32}	−0.062	0.102
θ_{41}	1.210	0.064	θ_{41}	1.208	0.064
θ_{42}	−0.330	0.065	θ_{42}	−0.328	0.064
σ_1	0.475	0.026	σ_1	0.481	0.029
σ_2	0.077	0.026	σ_2	0.062	0.020
σ_3	0.068	0.028	σ_3	0.066	0.029
σ_4	0.256	0.012	σ_4	0.256	0.012
γ_{10}	−0.149	0.041	γ_{10}	−0.108	0.035
γ_{11}	−0.130	0.045	γ_{11}	−0.074	0.038
γ_{12}	−0.133	0.043	γ_{12}	−0.076	0.037
γ_{20}	0.358	0.064	γ_{20}	0.316	0.054
γ_{21}	−0.876	0.059	γ_{21}	−0.786	0.054
γ_{22}	0.225	0.093	γ_{22}	0.284	0.060
γ_{30}	−0.126	0.040	γ_{30}	−0.127	0.043
γ_{31}	−0.003	0.062	γ_{31}	0.002	0.071
γ_{32}	0.072	0.028	γ_{32}	0.073	0.031
γ_{40}	−0.067	0.024	γ_{40}	−0.067	0.022
γ_{41}	−0.001	0.042	γ_{41}	0.002	0.036
γ_{42}	0.018	0.031	γ_{42}	0.019	0.028
			β_0	−0.364	0.108
			β_1	1.438	0.301
			β_0^*	−0.040	0.071
			β_1^*	0.263	0.169
			p_{00}	0.944	0.024
			p_{11}	0.813	0.071
logL	438.862		logL	446.444	
H_0: DF vs DFMS	15.164				

The ordering of the variables are ORDER, EXPORT, M1B, and SP, respectively.
The subscript index 1 of parameter γ_{1i} denotes the factor loading estimate of ORDER.
The subscript index 1 of parameter θ_{1i} denotes the idiosyncratic loading estimate of ORDER.
The model is estimated with $k = 2$, $q = 2$ and $r = 2$, which are selected using the SBC criterion.
The terms $k,\ q,\ r$ are the lag parameters of the polynomials $\phi(L)$, $\gamma(L)$, and $\theta(L)$, respectively.
$\phi(L)$ and $\phi^*(L)$ are autoregressive terms for the factors.
The term TDF denotes the linear dynamic two-factor model.
The term TDFMS denotes the dynamic two-factor model with regime switching.
The factor variance σ_η is set to unity for interpretation purpose.
The test for H_0: DF vs DFMS is rejected compared to $\chi^2_{0.05}(6) = 12.592$.

Table 6: Four Variables for the Two-Factor Model and $S_t \neq S_t^*$

Parameter	MSI	Std. Err.	Parameter	WSI	Std. Err.
ϕ_1	0.896	0.040	ϕ_1	0.664	0.124
θ_{11}	0.537	0.186	θ_{11}	1.766	0.050
θ_{12}	0.121	0.138	θ_{12}	−0.780	0.044
θ_{21}	−0.423	0.084	θ_{21}	1.338	0.050
θ_{22}	−0.044	0.017	θ_{22}	−0.448	0.033
σ_1	0.410	0.082	σ_1	0.067	0.012
σ_2	0.607	0.035	σ_2	0.267	0.012
γ_{10}	−0.341	0.106	γ_{10}	−0.095	0.010
γ_{11}	0.237	0.089	γ_{11}	−0.002	0.013
γ_{12}	−0.131	0.081	γ_{12}	0.012	0.012
γ_{20}	−0.127	0.082	γ_{20}	−0.045	0.013
γ_{21}	−0.057	0.101	γ_{21}	−0.015	0.014
γ_{22}	−0.029	0.070	γ_{22}	−0.019	0.015
β_0	−0.437	0.192	β_{00}	1.435	0.704
β_1	0.926	0.417	β_{11}	−4.495	1.013
p_{00}	0.921	0.037	p_{00}	0.983	0.009
p_{11}	0.863	0.068	p_{11}	0.932	0.037
logL	447.369				
H_0: DF vs DFMS	40.590				

The ordering of the variables are ORDER, EXPORT, M1B and SP, respectively.
The subscript index 1 of parameter γ_{1i} denotes the factor loading estimate of ORDER.
The subscript index 1 of parameter θ_{1i} denotes the idiosyncratic loading estimate of ORDER.
The model is estimated with $k = 1$, $q = 2$ and $r = 2$, which are selected using the SBC criterion.
The terms k, q and r are the lag parameters of the polynomials $\phi(L)$, $\gamma(L)$, and $\theta(L)$, respectively.
The term TDF denotes the linear dynamic two factor model.
The term TDFMS denotes the dynamic two factor model with regime switching.
The factor variance σ_η is set to unity for interpretation purpose.
The test for H_0: DF vs DFMS is rejected when compared to $\chi^2_{0.05}(6) = 12.592$.
The correlation between the estimated factor and ORDER, EXPORT, M1B, and SP are 0.894, 0.795, 0.424 and 0.549, respectively.

Table 7: Forecasting Performance: In-Sample and Out-of-Sample

Models	In-QPS[a]	Out-QPS[b]	Out-QPS[c]
MSF	0.185	0.234	0.138
WSF	0.524	0.324	0.448
One-Factor	0.200	0.312	0.222
Two-Factor	0.180	0.322	0.099

(a) The in-sample QPS runs from 1983:m1 to 2001:m4, totaling 220 observations.
(b) The out-of-sample QPS runs from 1997:m11 to 2001:m4, totaling 42 observations.
(c) The out-of-sample QPS runs from 1999:m1 to 2001:m4, totaling 28 observations.

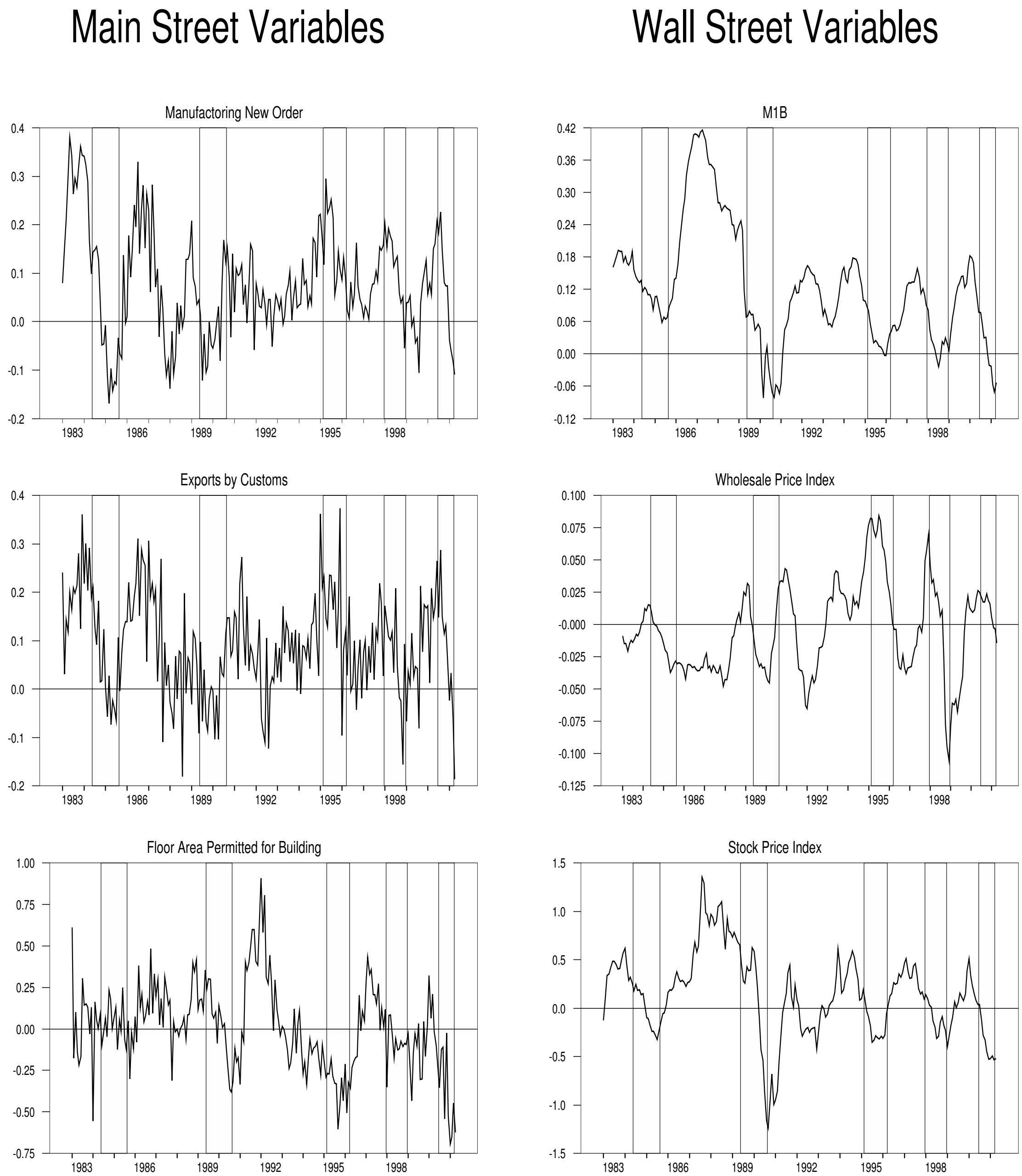

Figure 1: Scatter plots of the annual growth rates of ORDER, EXPORT, BUILD, M1B, WPI and SP. All series are seasonally-adjusted monthly data extending from 1983:m1 to 2001:m4 which amount to 220 observations.

Simple Weighted Index of Main Street and Wall Street Variables

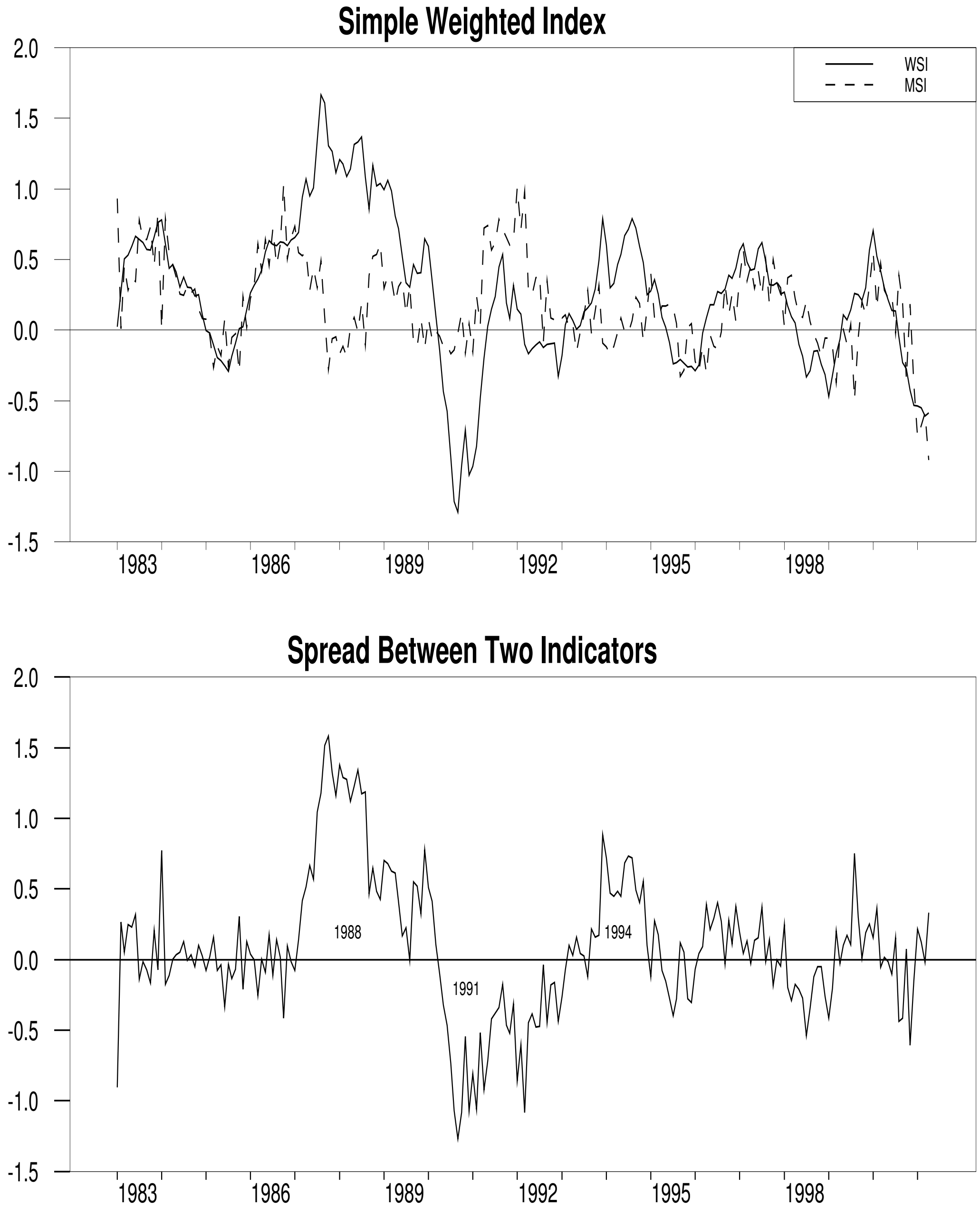

Figure 2: Simple weighted index of the Main Street and Wall Street variables.

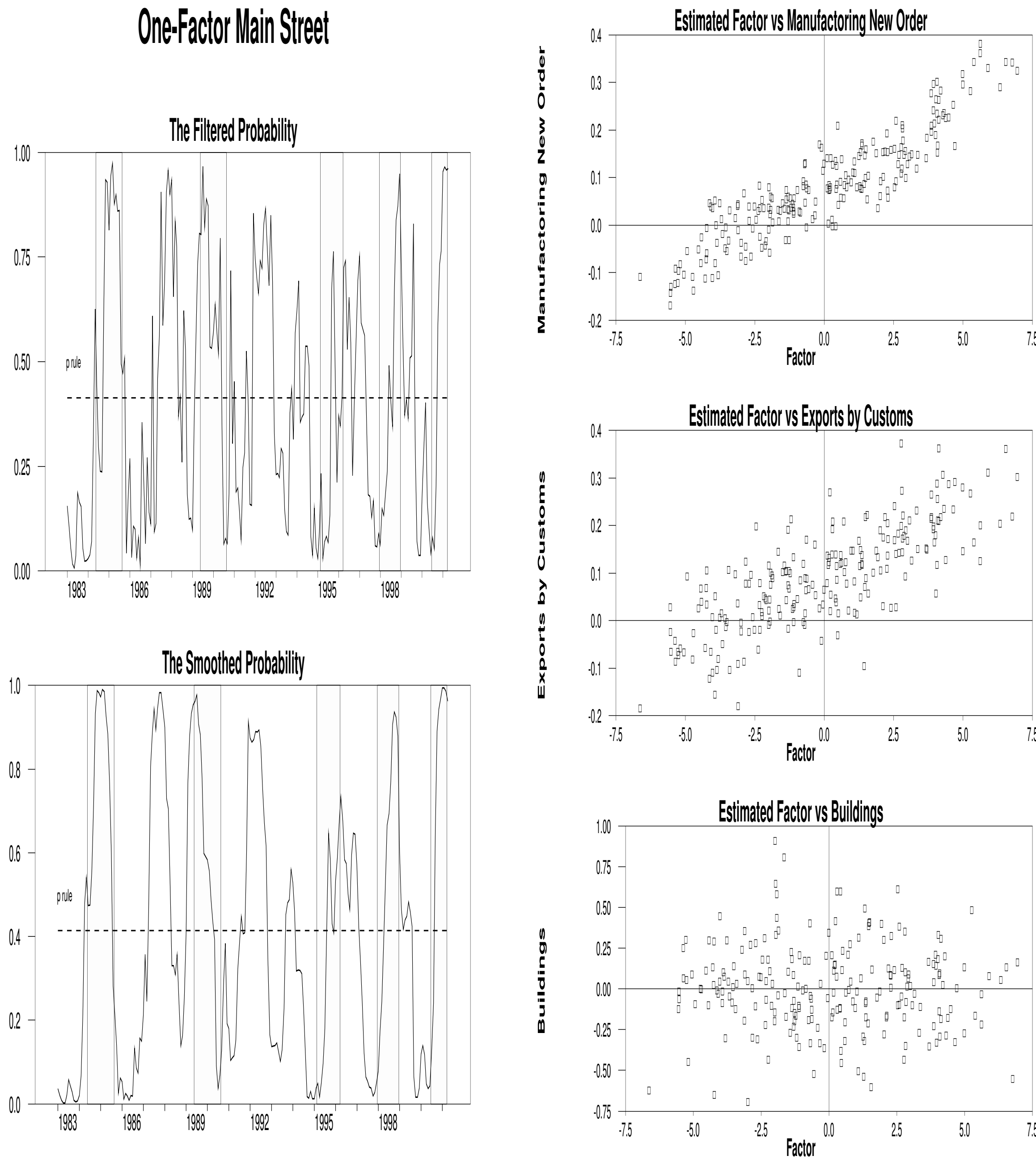

Figure 3: Panels on the left are the plots of the filtered and smoothed probabilities of the Main Street variables used in the tone-factor model. The shaded areas are the contraction periods as determined by the CEPD. The panels on the right are scatter plots of estimated factors and the three Main Street variables.

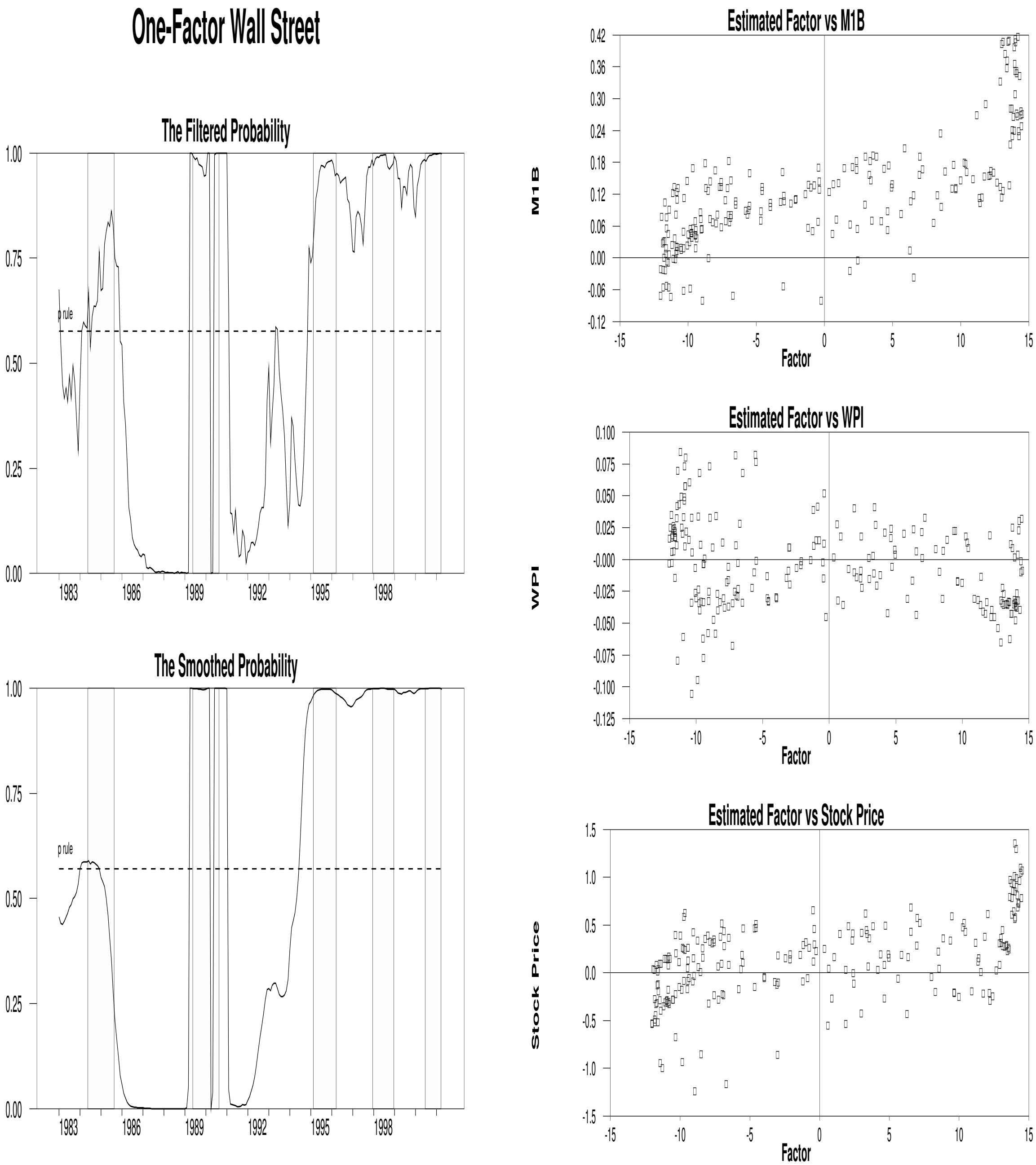

Figure 4: Panels on the left show the plots of the filtered and smoothed probabilities of the Wall Street variables used in one-factor model. The shaded areas are the contraction periods as determined by the CEPD. The panels on the right are scatter plots of the estimated factor and the three Wall Street variables.

Scatter Plots of the Factor and Variables

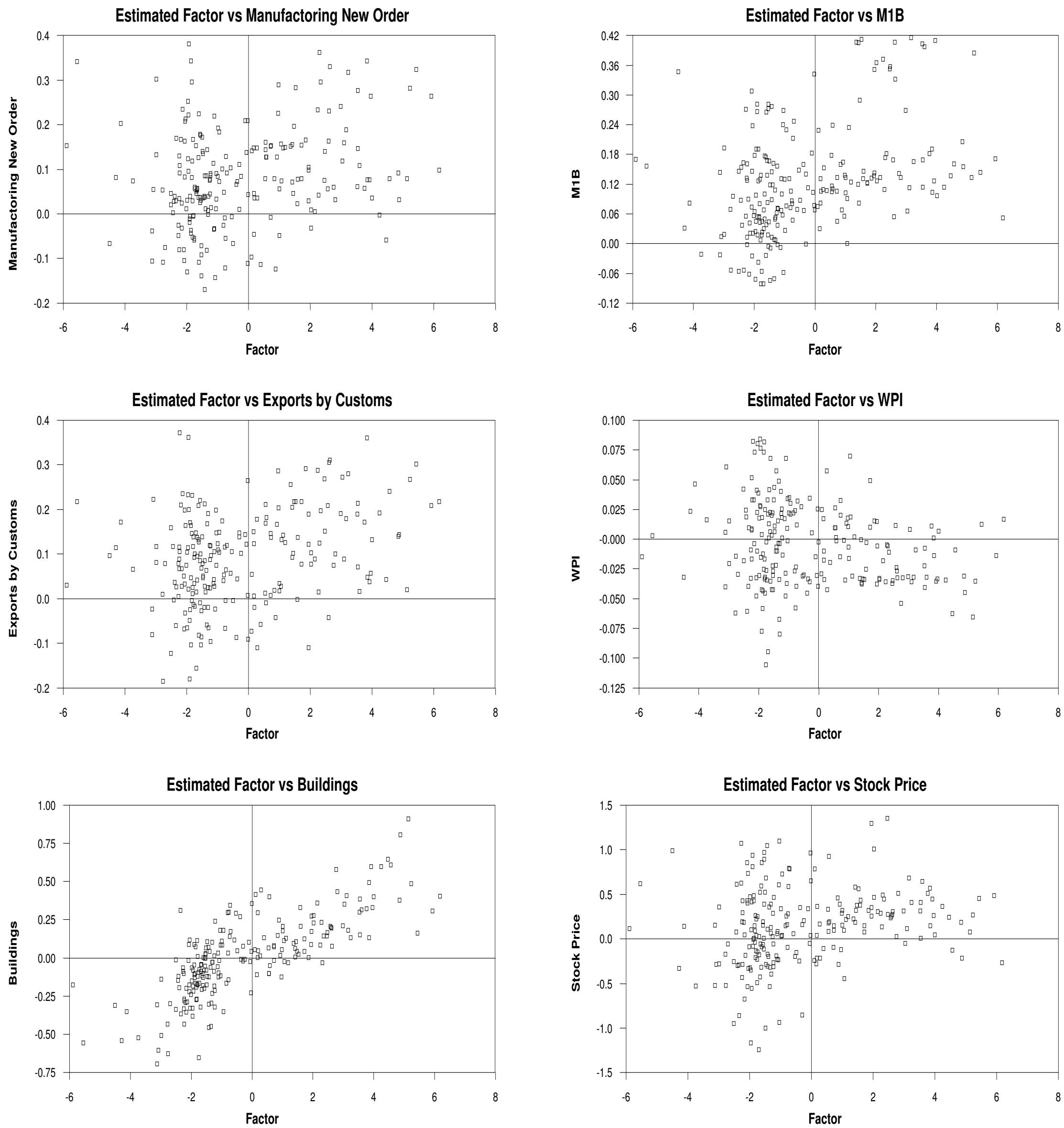

Figure 5: Panels on the left are scatter plots of the estimated factor and the three Main Street variables. The panels on the right are the scatter plots of the estimated factor and the three Wall Street variables.

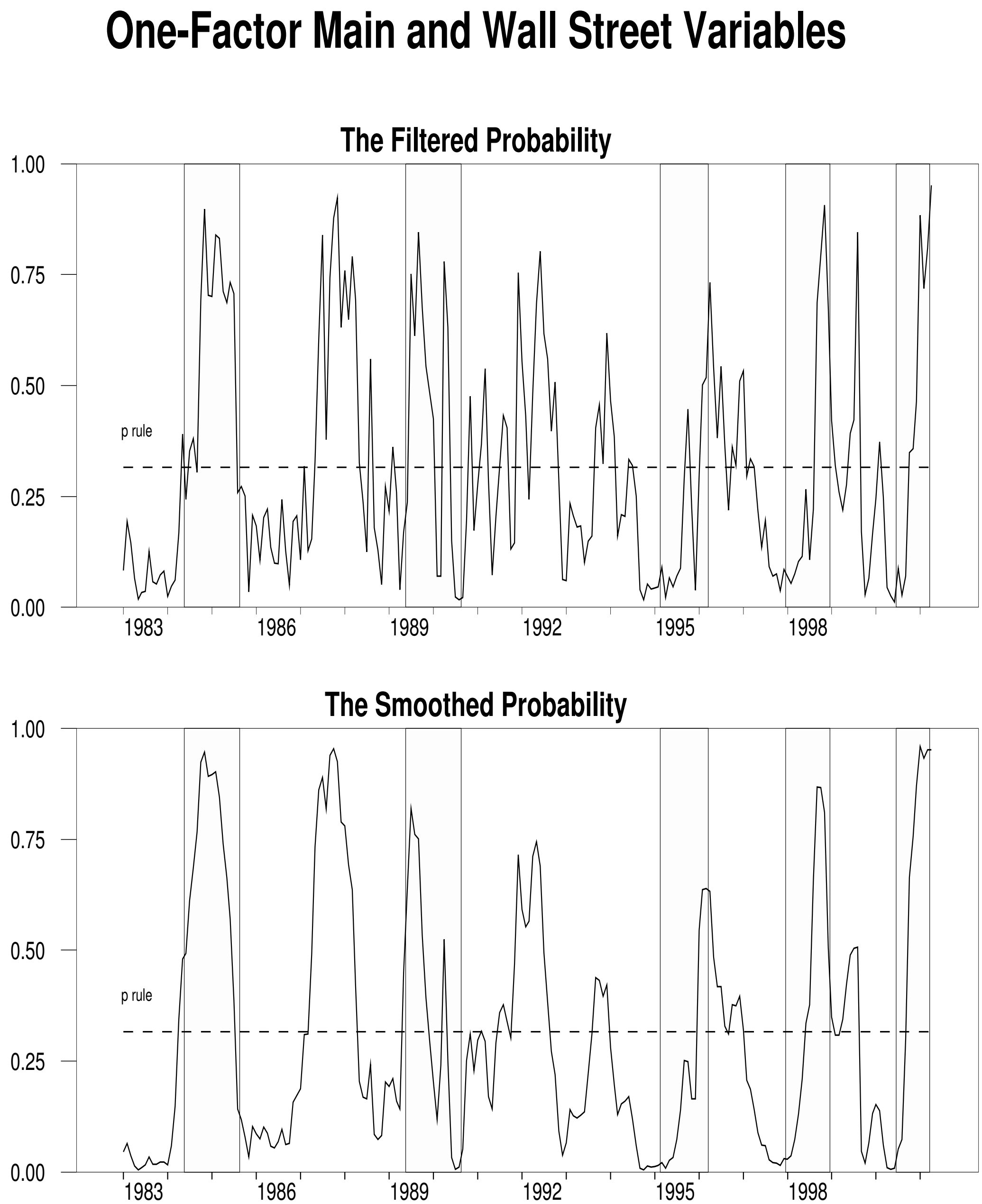

Figure 6: Plots of the filtered and smoothed probabilities of the four variables of the leading indicator, i.e., ORDER, EXPORT, M1B and SP, used in the one factor model. The shaded areas are the contraction periods as determined by the CEPD.

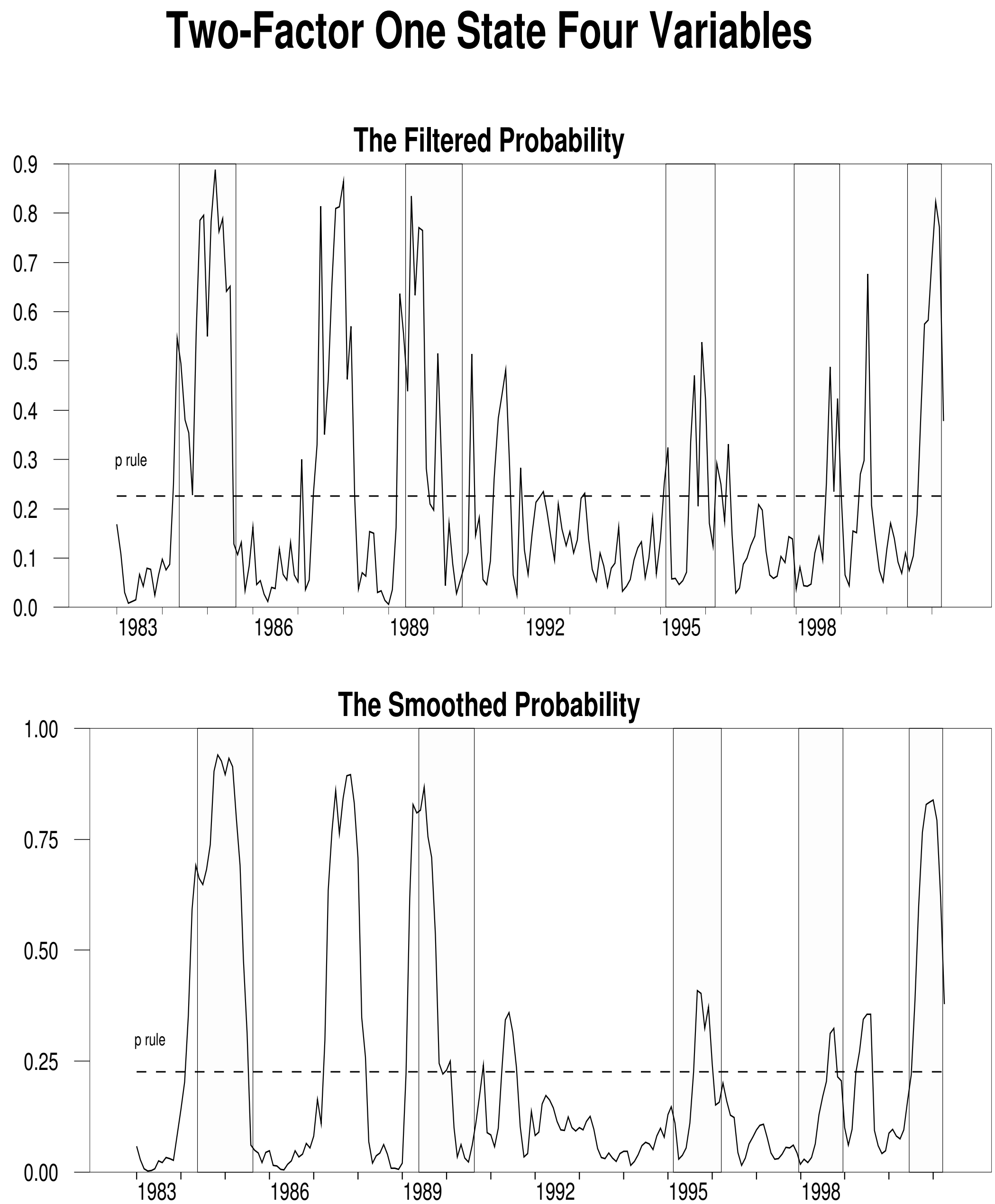

Figure 7: Plots of the filtered and smoothed probabilities of the four variables of leading indicator, i.e., ORDER, EXPORT, M1B and SP, used in the two-factor model with $S_t = S_t^*$. The shaded areas are the contraction periods, as determined by the CEPD.

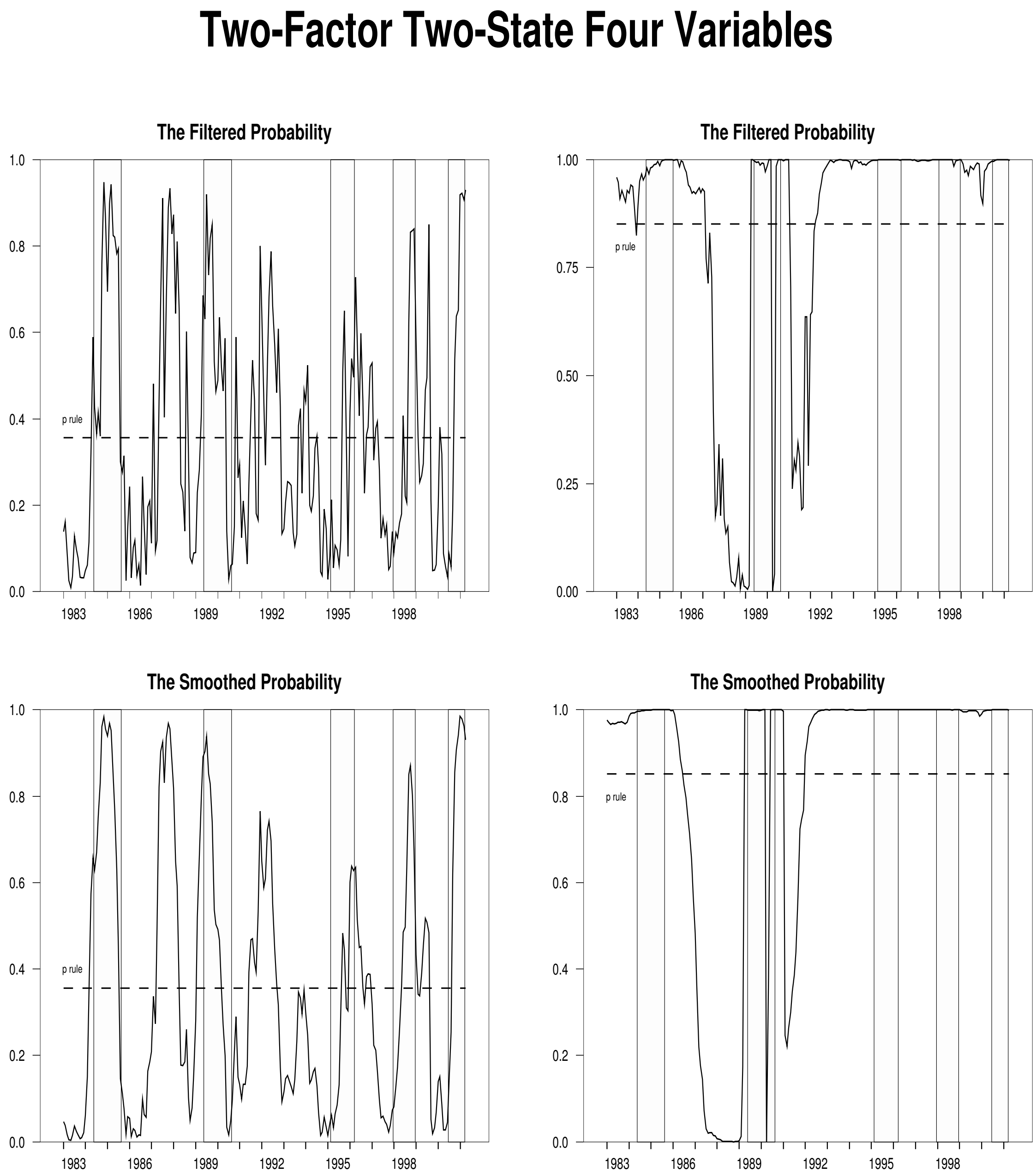

Figure 8: Plots of the filtered and smoothed probabilities of the four variables of the leading indicators, i.e., ORDER, EXPORT, M1B and SP, used in the two factor model with $S_t \neq S_t^*$. The shaded areas are the contraction periods as determined by the CEPD.

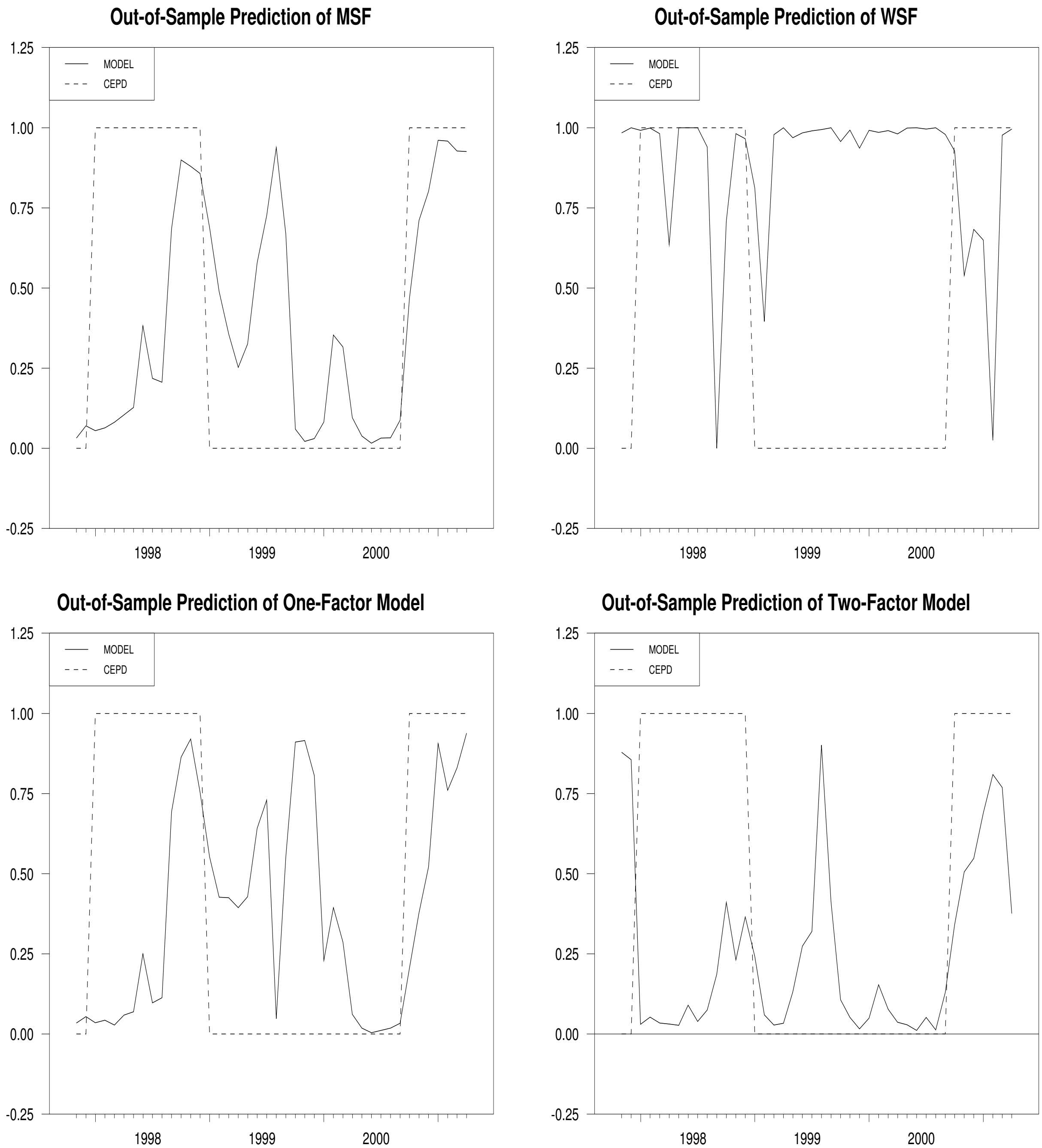

Figure 9: The out-of-sample posterior probabilities for various models. The solid and the posterior probabilities as predicted by models. The dashed lines are the contraction dates as determined by the CEPD.