Problem 1:

How might your measurement be affected by the mass of the stringl?

REQUIREMENT: Now, you must treat the metal ball as a point mass and the string as a stiff rod. And, please show all your work numerically!

(0.5) *What is* $I_{total} = I_{point\_mass} + I_{rod}$, *the total moment of inertia of this system, with respect to the pivot?*

Answer: $I_{total} = \frac{1}{3} M_{string} L_{string}^2 + m_{sphere}\left(L_{string} + R_{sphere}\right)^2$

(0.5) *What is h, the distance from the pivot to the center of mass of this stiff-rod-and-point-mass system?*

Answer: $h = |X_{center\_of\_mass} - X_{pivot}|$, *if we define* $X_{pivot} = 0$, *then,*

$$h = X_{center\_of\_mass} = \frac{M_{string} X_{center\_of\_string} + m_{sphere} X_{sphere}}{M_{string} + m_{sphere}} = \frac{M_{string} \frac{L_{string}}{2} + m_{sphere}\left(L_{string} + R_{sphere}\right)}{M_{string} + m_{sphere}}.$$

(0.5) *What will be the new time period formula if you treat this system as a stiff-rod-and-point-mass physical pendulum, starting from* $T = 2\pi\sqrt{\frac{I_{total}}{m_{total} g h}}$? *Here* $m_{total} = m_{pointmass} + m_{rod}$ *is the total mass.*

Answer:

$$T_{new} = 2\pi\sqrt{\frac{I_{total}}{m_{total} g h}} = 2\pi\sqrt{\frac{\frac{1}{3} M_{string} L_{string}^2 + m_{sphere}\left(L_{string} + R_{sphere}\right)^2}{\left(M_{string} + m_{sphere}\right) g \frac{M_{string} \frac{L_{string}}{2} + m_{sphere}\left(L_{string} + R_{sphere}\right)}{M_{string} + m_{sphere}}}} = 2\pi\sqrt{\frac{\frac{1}{3} M_{string} L_{string}^2 + m_{sphere}\left(L_{string} + R_{sphere}\right)^2}{g\left[M_{string} \frac{L_{string}}{2} + m_{sphere}\left(L_{string} + R_{sphere}\right)\right]}}.$$

(0.5) *What is the percent difference of time period between the point-mass ideal pendulum and the stiff-rod physical pendulum?*

Answer:

$$\%ERROR \quad \left| \frac{T_{new} \quad T_0}{T_0} \right| \quad 100\%$$

$$\left| \frac{2 \quad \sqrt{\dfrac{\frac{1}{3}M_{string}L_{string}^2 \quad m_{sphere} \; L_{string} \quad R_{sphere} \quad ^2}{g\left[M_{string}\frac{L_{string}}{2} \quad m_{sphere} \; L_{string} \quad R_{sphere} \quad \right]}} \quad 2 \quad \sqrt{\dfrac{L_{string} \quad R_{sphere}}{g}}}{2 \quad \sqrt{\dfrac{L_{string} \quad R_{sphere}}{g}}} \right| \quad 100\%$$

$$\left| \frac{\sqrt{\dfrac{\frac{1}{3}M_{string}L_{string}^2 \quad m_{sphere} \; L_{string} \quad R_{sphere} \quad ^2}{g\left[M_{string}\frac{L_{string}}{2} \quad m_{sphere} \; L_{string} \quad R_{sphere} \quad \right]}}}{\sqrt{\dfrac{L_{string} \quad R_{sphere}}{g}}} \quad 1 \right| \quad 100\%$$

$$\left| \sqrt{\frac{\frac{1}{3}M_{string}L_{string}^2 \quad m_{sphere} \; L_{string} \quad R_{sphere} \quad ^2}{\left[M_{string}\frac{L_{string}}{2} \quad m_{sphere} \; L_{string} \quad R_{sphere} \quad \right] L_{string} \quad R_{sphere}}} \quad 1 \right| \quad 100\%$$

Example *How might your measurement be affected by the size of the ball?*

REQUIREMENT: Now, you must treat the metal ball as a sizable sphere and the string as massless. And, please show all your work numerically!

(0.5) *What is h, the distance from the pivot to the center of mass of this massless-string-and-huge-sphere system?*

Answer: $h = |X_{center\_of\_mass} - X_{pivot}|$, *if we define* $X_{pivot} = 0$, *then,* $h = X_{center\_of\_mass} = L_{string} + R_{sphere}$.

(0.5) *What is the new total moment of inertia of this massless-string-and-huge-sphere system, with respect to the pivot? Please apply* ***parallel-axis theorem***, $I_{total} = I_{axis\_at\_center\_of\_sphere} + m_{sphere}h^2$, *here!*

Answer:

$$
\begin{aligned}
I_{total} &= \frac{2}{5}m_{sphere}R_{sphere}^2 + m_{sphere}h^2 \\
&= \frac{2}{5}m_{sphere}R_{sphere}^2 + m_{sphere}\left(L_{string} + R_{sphere}\right)^2 \\
&= m_{sphere}\left[\frac{2}{5}R_{sphere}^2 + \left(L_{string} + R_{sphere}\right)^2\right].
\end{aligned}
$$

(0.5) *What will be the new time period formula if you treat this system as a massless-string-and-heavy-sphere physical pendulum, starting from* $T = 2\pi\sqrt{\frac{I_{total}}{m_{total}gh}}$? *Here* $m_{total} = m_{sphere}$ *if we neglect the mass of the massless string.*

Answer:

$$
\begin{aligned}
T_{new} &= 2\pi\sqrt{\frac{I_{total}}{m_{total}gh}} \\
&= 2\pi\sqrt{\frac{m_{sphere}h^2 + \frac{2}{5}m_{sphere}R_{sphere}^2}{m_{sphere}g\left(L_{string} + R_{sphere}\right)}} \\
&= 2\pi\sqrt{\frac{\left(L_{string} + R_{sphere}\right)^2 + \frac{2}{5}R_{sphere}^2}{g\left(L_{string} + R_{sphere}\right)}}.
\end{aligned}
$$

(0.5) *What is the percent difference of time period between the point-mass ideal pendulum and the sizeable-sphere physical pendulum?*

Answer:

$$\%ERROR \quad \left| \frac{T_{new} \quad T_0}{T_0} \right| \quad 100\%$$

$$\left| \frac{2 \quad \sqrt{\frac{L_{string} \quad R_{sphere} \quad ^2 \quad \frac{2}{5}R_{sphere}^2}{g \; L_{string} \quad R_{sphere}}} \quad 2 \quad \sqrt{\frac{L_{string} \quad R_{sphere}}{g}}}{2 \quad \sqrt{\frac{L_{string} \quad R_{sphere}}{g}}} \right| \quad 100\%$$

$$\left| \frac{\sqrt{\frac{L_{string} \quad R_{sphere} \quad ^2 \quad \frac{2}{5}R_{sphere}^2}{g \; L_{string} \quad R_{sphere}}}}{\sqrt{\frac{L_{string} \quad R_{sphere}}{g}}} \quad 1 \right| \quad 100\%$$

$$\left| \sqrt{\frac{L_{string} \quad R_{sphere} \quad ^2 \quad \frac{2}{5}R_{sphere}^2}{L_{string} \quad R_{sphere} \quad ^2}} \quad 1 \right| \quad 100\%$$

$$\left| \sqrt{1 \quad \frac{2}{5}\left(\frac{R_{sphere}}{L_{string} \quad R_{sphere}} \right)^2} \quad 1 \right| \quad 100\%$$

Problem 2:

Suppose a clock has a swing amplitude of 0.1 *radians*. *How much increase* (*or decrease*) *in amplitude would make the clock be off by one second in one day* (86,400 *seconds*)?

REQUIREMENT: *Now*, *to simplify the above question*, *according to TA manual*, *the answer of this question is* $_f \quad 0.1009(radian)$ *or* $0.0991(radian)$.

However, *please show all your work and prove the given answer*!

(0.5) *Set the initial amplitude* $_i \quad 0.1000$ (*radian*) *and define the unknown final amplitude as* $_f$. *With* $T_i \quad T_0\left(1 \quad \frac{^2_i}{16}\right)$, *please express* $T \quad |T_f \quad T_i|$, *the difference of time period between* $_f$ *and* $_i$, T, *in terms of* T_0, $_f$ *and* $_i$!

ANSWER:

$$T_i \quad T_0\left(1 \quad \frac{^2_i}{16}\right)$$

$$T_f \quad T_0\left(1 \quad \frac{^2_f}{16}\right)$$

$$T \quad |T_f \quad T_i|$$

$$\left|T_0\left(1 \quad \frac{^2_f}{16}\right) \quad T_0\left(1 \quad \frac{^2_i}{16}\right)\right|$$

$$\frac{T_0}{16}|\ ^2_f \quad ^2_i\ |$$

(0.5) *From the above result*, *please express the final amplitude* $_f$ *in terms of* $\frac{T}{T_0}$ *and* $_i$!

ANSWER:

$$T \quad \frac{T_0}{16}|\ ^2_f \quad ^2_i\ |$$

$$|\ ^2_f \quad ^2_i\ | \quad 16\left(\frac{T}{T_0}\right)$$

$$^2_f \quad ^2_i \quad 16\left(\frac{T}{T_0}\right)$$

$$^2_f \quad 16\left(\frac{T}{T_0}\right) \quad ^2_i$$

$$_f \quad \sqrt{^2_i \quad 16\left(\frac{T}{T_0}\right)}$$

(0.5) *Read the phrase* ?*off by one second in one day*? *carefully*. *Then write down the numerical value of* $\frac{T}{T_0}$.

ANSWER: $\frac{T}{T_0} \quad \frac{one\ second}{one\ day} \quad \frac{1}{86400}$

(0.5) *Write down the numerical value of* ${}_f$ - ${}_i$. *Prove the percent error in amplitude is* 0.9%.

ANSWER:

$$ {}_f \quad \sqrt{{}_i^2 \quad 16\left(\frac{T}{T_0}\right)} $$

$$ \sqrt{0.1000 \;^2 \quad 16\left(\frac{1}{86400}\right)} $$

$$ \%ERROR \quad \left|\frac{{}_f \quad {}_i}{{}_i}\right| \quad 100\% $$

$$ \left|\frac{0.1009 \quad 0.1000}{0.1000}\right| \quad 100\% $$

$$ \left|\frac{0.0009}{0.1000}\right| \quad 100\% $$

$$ 0.009 \quad 100\% $$

$$ 0.9\% $$