Homework Set 2

1.

Consider an option on a non-dividend-paying stock when the stock price \$30, the exercise price is \$29, the risk-free interest rate is 5%, the volatility is 25% per annum, and the time to maturity is 4 months.
(a). What is the price of the option if it is a European call?
(b). What is the price of the option if it is a European put?
(c). Verify that put-call parity holds.

2.

Assume that the stock in Question 2 is due to ex-dividend in 1.5 months. The expected dividend is 50 cents.
(a). What is the price of the option if it is a European call?
(b). What is the price of the option if it is a European put?

3. (**Hard, Optional**) The formula of Greek Delta.
(a). What is $N'(x)$?
(b). Show that $SN'(d_1) = Ke^{-r(T-t)}N'(d_2)$, where S is the stock price at time t and

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}};$$
$$d_2 = \frac{\ln(S/K) + (r - \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$$

(c). Calculate $\partial d_1/\partial S$ and $\partial d_1/\partial S$.
(d). Show that when

$$c = SN(d_1) - Ke^{-r(T-t)}N(d_2)$$

it follows that

$$\frac{\partial c}{\partial t} = -rKe^{-r(T-t)}N(d_2) - SN'(d_1)\frac{\sigma}{2\sqrt{T-t}}$$

where c is the price of a call option on a non-dividend-paying stock.
(e). Show that $\partial c/\partial S = N(d_1)$.

4.

Suppose that the spot price of the Canadian dollar is US\$0.75 and that the Canadian dollar/US dollar exchange rate has a volatility of 4% per annum. The risk-free rates of interest in Canada and the United State are 9% and 7% per annum, respectively. Calculate the value of a European call option to buy one Canadian dollar for US \$0.75 in 9 months.

5.

Use the put-call parity relationship to derive, for a non-dividend-paying stock, the relationship between:
(a). The delta of a European call and the delta of a European put;
(b). The gamma of a European call and the gamma of a European put;
(c). The vega of a European call and the vega of a European put;
(d). The theta of a European call and the theta of a European put.