## **Homework Set 1** (Due on Feb. 26)

Read Chapters 1, 2, 5, 6 of the textbook.

1.

A stock price is currently \$40. Over each of the next two 3-month periods it is expected to go up by 10% or down by 10%. The risk free interest rate is 12% per annum with continuous compounding.
(a). What is the value of a 6-month European put option with a strike price of \$42?
(b). What is the value of a 6-month American put option with a strike price of \$42?

2.

A stock price is currently \$30. During each 2-month period for the next 4 months it will increase by 8% or reduce by 10%. The risk free interest rate is 5%. Use a two-step tree to calculate the value of a derivative that pays off $\max[(30 - S_T), 0]^2$, where $S_T$ is the stock price in 4 months. If the derivative is American-style, should it exercised early?

3.

The price of a stock is \$40. The price of an 1-year European put option on the stock with a strike price of \$30 is quoted as \$7 and the price of an 1-year European call option on the stock with a strike price of \$50 is quoted as \$5. Suppose that an investor buys 100 shares, shorts 100 call options and buys 100 put options. Draw a diagram illustrating how the investor's profit or loss varies with the stock price over the next year. How does your answer change if the investor buys 100 shares, shorts 200 call options and buys 200 put options?

4.

Suppose that $c_1, c_2$ and $c_3$ are the prices of European call options with strike price $K_1, K_2$ and $K_3$, respectively, where $K_3 > K_2 > K_1$ and $K_3 - K_2 = K_2 - K_1$. All options have the same maturity. Show that

$$c_2 \leq 0.5(c_1 + c_3).$$

(**Hint**: Consider a portfolio that is long one option with strike price $K_1$, long one option with strike price $K_3$ and short two options with strike price $K_2$. Show that if $c_2 > 0.5(c_1 + c_3)$, then the portfolio can be constructed without any initial investment and it will provide non-negative profit when it matures. That is, one may have an arbitrage opportunity if $c_2 > 0.5(c_1 + c_3)$. )

5.

A stock is expected to pay a dividend of \$1 per share in 2 months and in 5 months. The stock price is \$50, and the risk free rate of interest is 8% per annum with continuous compounding for all maturities. An investor has just taken a short position in a 6-month forward contract on the stock.
(a). What are the fair delivery price on this stock this investor should pay?
(b). Three months later, the price of the stock is \$48 and the risk free interest rate is still 8% per annum. What is the value of the investor's short position then?