Seg7570
Assignment 2
Suggested Solution
Q1. In this case $S_0 = 30$, $K = 29$, $r = 0.05$, $\sigma = 0.25$ and $T = 4/12 = 0.3333$

$$d_1 = \frac{ln(30/29) + (0.05 + 0.25^2/2) \times 0.3333}{0.25\sqrt{0.333}} = 0.4225$$

$$d_2 = \frac{ln(30/29) + (0.05 - 0.25^2/2) \times 0.3333}{0.25\sqrt{0.333}} = 0.2782$$

$$N(0.4225) = 0.6637, \quad N(0.2782) = 0.6096$$

$$N(-0.4225) = 0.3363, \quad N(-0.2782) = 0.3904$$

(a) The European call price is

$$30 \times 0.6637 - 29e^{-0.05\times0.3333} \times 0.6096 = 2.52$$

(b) The European put price is

$$29e^{-0.05\times0.3333} \times 0.3904 - 30 \times 0.3363 = 1.05$$

(c) Put-call parity states that:

$$p + S_0 = c + Ke^{-rT}$$

In this case $c = 2.52$, $S_0 = 30$, $K = 29$, $p = 1.05$ and $e^{-rT} = 0.9835$ and it is easy to verify that the relationship is satisfied.

Q2. (a) The present value of the dividend must be subtracted from the stock price. This gives a new stock price of:

$$30 - 0.5e^{-0.125\times0.05} = 29.5031$$

and

$$d_1 = \frac{ln(29.5031/29) + (0.05 + 0.25^2/2) \times 0.3333}{0.25\sqrt{0.333}} = 0.3068$$

$$d_2 = \frac{ln(29.5031/29) + (0.05 - 0.25^2/2) \times 0.3333}{0.25\sqrt{0.333}} = 0.1625$$

$$N(0.4225) = 0.6637, \quad N(0.2782) = 0.6096$$

The price of the option is therefore

$$29.5031 \times 0.6205 - 29e^{-0.333\times0.05} \times 0.5645 = 2.21$$

(b) Since

$$N(-d_1) = 0.3795, \quad N(-d_2) = 0.4355$$

the value of the option when it is a European put is

$$29e^{-0.3333\times 0.05} \times 0.4355 - 29.5031 \times 0.3795 = 1.22$$

Q3. (a)

$$N'(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

(b)

$$d_1 - d_2 = \sigma\sqrt{T-t}$$

$$\begin{aligned}
& SN'(d_1) - Ke^{-r(T-t)}N'(d_2) \\
= \;& \tfrac{1}{\sqrt{2\pi}}[Se^{-d_1^2/2} - Ke^{-d_2^2/2+r(T-t)}] \\
= \;& \tfrac{1}{\sqrt{2\pi}}[Se^{-(d_2+\sigma\sqrt{T-t})^2/2} - Ke^{-d_2^2/2+r(T-t)}] \\
= \;& \tfrac{e^{-d_2^2/2}}{\sqrt{2\pi}}[Se^{-(\sigma^2(T-t)/2+(r-\sigma^2/2)(T-t)+ln(S/K))} - Ke^{-r(T-t)}] \\
= \;& \tfrac{e^{-(d_2^2/2+r(T-t))}}{\sqrt{2\pi}}[Se^{-(\sigma^2(T-t)/2-\sigma^2(T-t)/2)}e^{-ln(S/K)} - K] \\
= \;& \tfrac{e^{-(d_2^2/2+r(T-t))}}{\sqrt{2\pi}}[K - K] \\
= \;& 0
\end{aligned}$$

(c)

$$\partial d_1/\partial S = \partial d_2/\partial S = \frac{1}{S\sigma\sqrt{T-t}}$$

(d)

$$\partial d_1/\partial t = -\frac{(r+\sigma^2/2)}{2\sigma\sqrt{T-t}}$$

$$\partial d_2/\partial t = -\frac{(r-\sigma^2/2)}{2\sigma\sqrt{T-t}}$$

$$\begin{aligned}
\partial c/\partial t &= SN'(d_1)\partial d_1/\partial t - rKe^{-r(T-t)}N(d_2) - Ke^{-r(T-t)}N'(d_2)\partial d_2/\partial t \\
&= -rKe^{-r(T-t)}N(d_2) - SN'(d_1)\tfrac{(r+\sigma^2/2)}{2\sigma\sqrt{T-t}} + Ke^{-r(T-t)}N'(d_2)\tfrac{(r-\sigma^2/2)}{2\sigma\sqrt{T-t}} \\
&= -rKe^{-r(T-t)}N(d_2) - [SN'(d_1) - Ke^{-r(T-t)}N'(d_2)]\tfrac{r}{2\sigma\sqrt{T-t}} \\
&\quad -(SN'(d_1) + Ke^{-r(T-t)}N'(d_2))\tfrac{\sigma^2/2}{2\sigma\sqrt{T-t}} \\
&= -rKe^{-r(T-t)}N(d_2) - (SN'(d_1) + Ke^{-r(T-t)}N'(d_2))\tfrac{\sigma^2/2}{2\sigma\sqrt{T-t}} \\
&= -rKe^{-r(T-t)}N(d_2) - SN'(d_1)\tfrac{\sigma^2}{2\sigma\sqrt{T-t}}
\end{aligned}$$

(e)

$$\begin{aligned}
\partial c/\partial S &= SN'(d_1)\partial d_1/\partial S + N(d_1) - Ke^{-r(T-t)}N'(d_2)\partial d_2/\partial d_2 \\
&= (SN'(d_1) - Ke^{-r(T-t)}N'(d_2))\frac{1}{S\sigma\sqrt{T-t}} + N(d_1) \\
&= N(d_1)
\end{aligned}$$