7. Vectors

7.1 The Concept and Application of Vectors (Key Stage 4 and beyond)

7.1.1 The Concept of Vector

7.1.2 Application of Vectors

7.1.1 The Concept of Vector

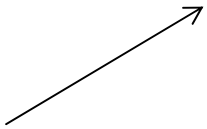
Vector is a quantity with magnitude and direction. Can you think of any daily life applications of vectors?

The following are measurements that we often use:
1. The mass of an object is 150 kg.
2. The height of Jack is 175 cm.
3. The area of the earth is 96 000 000 km^2.
4. A small boat travels 15 km in the northwest direction.

These four cases can be classified into two types:
For cases 1, 2 and 3, the measurements involve magnitudes only.
For case 4, the measurement involves both direction and magnitude.

The Notation of Vector

A vector can be denoted by a directed line segment. The magnitude is the length of the line segment and the direction is the direction of the line segment.



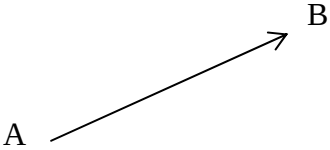
Vector can also be represented in the following two ways:

1. Using $\vec{a}$, $\vec{b}$, $\vec{c}$, etc to represent vectors.

   The magnitude of $\vec{a}$ is denoted by $|\vec{a}|$.

2. Using directed line segments to represent vectors.
   The initial and terminal points of a directed line segment are marked with alphabets.

   For example: $\overrightarrow{AB}$ represents a vector from A to B.

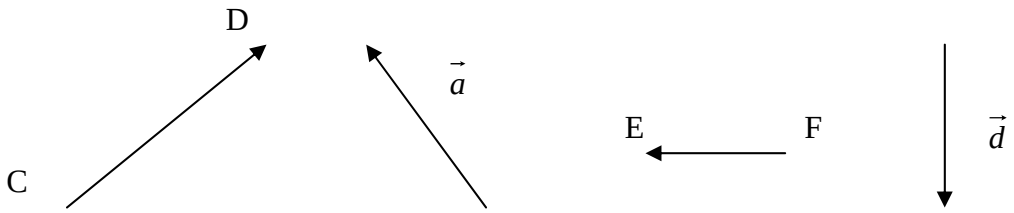


The magnitude of vector $\overrightarrow{AB}$ is denoted by $|\overrightarrow{AB}|$.

Vector with magnitude 0 is known as a zero vector, denoted by $\vec{0}$.
Vector with magnitude of 1 is known as a unit vector.

Example 1

Write down the notation for the following vectors:



Answer

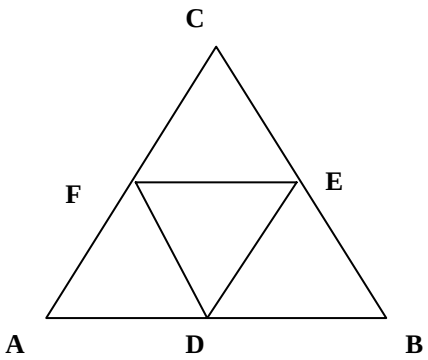
$\overrightarrow{CD}$	$\vec{a}$	$\overrightarrow{FE}$	$\vec{d}$

Equal Vectors

Equal vectors: vectors with the same magnitude and direction.
If vector $\vec{a}$ equals vector $\vec{b}$, it is denoted by $\vec{a}=\vec{b}$.

Example 2

1. D, E and F are the midpoints of the three sides of ΔABC as shown below. Write down the vectors which are equal to $\overrightarrow{DE}$, $\overrightarrow{EF}$ and $\overrightarrow{FD}$.



Answer

$\overrightarrow{DE} = \overrightarrow{AF} = \overrightarrow{FC}$
$\overrightarrow{EF} = \overrightarrow{BD} = \overrightarrow{DA}$
$\overrightarrow{FD} = \overrightarrow{CE} = \overrightarrow{EB}$

Practice 7.1A

7.1.2 Applications of Vectors

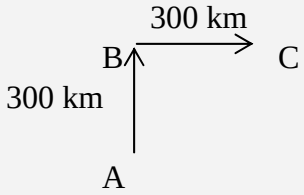
Example 1

An aeroplane flies 300 km due north from A to B. Afterwards, it changes its direction to the east and flies 300 km to C.

1. Draw the route of the aeroplane.
2. Find the distance between A and C. (Correct to 3 significant figures)
3. Find the compass bearing from A to C.

Answer

1.



2.

$\angle ABC = 90°$

By Pythagoras Theorem,

$AC^2 = AB^2 + BC^2$

$= (300^2 + 300^2)\ km^2$

$\therefore AC = 424\ km$

3.

$AB = BC$

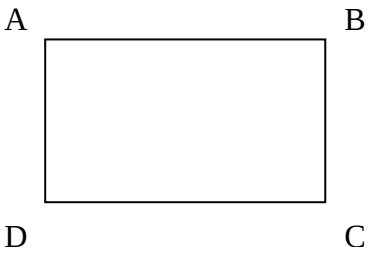
$\therefore \angle BAC = 45°$

The compass bearing of C from A is N45°E.

Practice 7.1B

Practice 7.1A

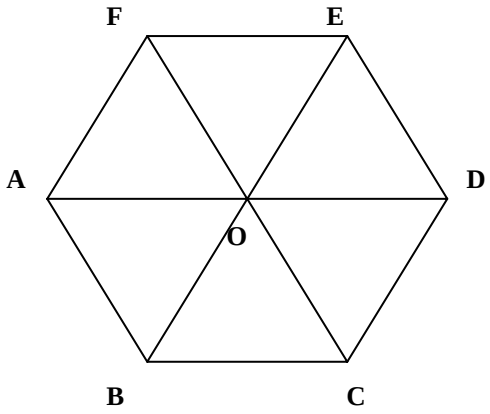
1. Given rectangle ABCD, write down vectors that is equal to $\overrightarrow{AB}$ and $\overrightarrow{BC}$.



Answer

$\overrightarrow{AB} = \overrightarrow{DC}$
$\overrightarrow{BC} = \overrightarrow{AD}$

2. Let O be the center of hexagon ABCDEF, write down vectors that is equal to $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$.



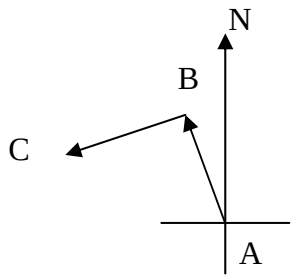
Answer

$\overrightarrow{OA} = \overrightarrow{CB} = \overrightarrow{EF} = \overrightarrow{DO}$
$\overrightarrow{OB} = \overrightarrow{DC} = \overrightarrow{FA} = \overrightarrow{EO}$
$\overrightarrow{OC} = \overrightarrow{AB} = \overrightarrow{ED} = \overrightarrow{FO}$

Practice 7.1B

An aeroplane flies 1400 km in the direction of N15°W from A to B, and then it changes its direction to S75°W and flies 1400km to C.

1. Find the distance between A and C (Correct to 3 significant figures)
2. Find the compass bearing from A to C.



Answer

1.

$\angle ABC = 90°$

$AB = BC = 1400\text{km}$

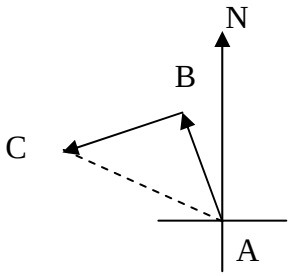
From the Pythagoras theorem,

$AC = \sqrt{AB^2 + BC^2}$

$= \sqrt{1400^2 + 1400^2}$ km

$= 1980$ km

2.



$\because AB = BC$

$\therefore \angle BAC = 45°$

$\angle NAB = 15°$

$\angle NAC = 45° + 15° = 60°$

The compass bearing from A to C is N60°W.