JU CHING CHU SECONDARY SCHOOL (T.M.)
F. 5 School Leaving Certificate Examination
Marking Scheme for Mathematics (I)

1. $\sqrt{32}-\sqrt{50}=4\sqrt{2}-5\sqrt{2}$ ◄ 1A

$=-\sqrt{2}$ ◄ 1A

(2)

2. Remainder $=(-1)^{2047}+5$ ◄ 1M

$=4$ ◄ 1A

(2)

3. $(s-a)b=c(4s+d)$

$sb-ab=4cs+cd$

$sb-4cs=ab+cd$ ◄ 1M

$s(b-4c)=ab+cd$

$s=\dfrac{ab+cd}{b-4c}$ ◄ 1A

(2)

4. In $\triangle ABC$,

$AC=\sqrt{6^2+8^2}$

$=10$ ◄ 1A

In $\triangle ACD$, by the sine formula,

$\dfrac{AD}{\sin 50^\circ}=\dfrac{10}{\sin 60^\circ}$ ◄ 1M

$AD=\dfrac{10}{\sin 60^\circ}\times\sin 50^\circ$

$=8.85$, *cor. to 3 sig. fig.* ◄ 1A

(3)

5. $\dfrac{(x^2y^{-3})^2}{x^{-1}y^2}=\dfrac{x^4y^{-6}}{x^{-1}y^2}$ ◄ 1M❶

$=\dfrac{x^{4+1}}{y^{2+6}}$ ◄ 1M❷

$=\dfrac{x^5}{y^8}$ ◄ 1A

(3)

Remarks:
❶ 1M *for applying* $(a^pb^q)^n=a^{pn}b^{qn}$
❷ 1M *for applying* $a^{-n}=\dfrac{1}{a^n}$

6. Area of cross-section $=\dfrac{(6+4)\times 3}{2}$ (cm²) 1M

$=15$ (cm²)

Volume of the prism $=15\times 8$ (cm³) 1M

$=120$ cm³ 1A

(3)

7. $\angle BDA=\angle DAB$

$=x$ ◄ 1A

$\angle DBC=\angle DAB+\angle BDA$

$=x+x$

$=2x$ ◄ 1A

$\angle DCB=\angle DBC$

$=2x$

$\angle DAB+\angle DCB=111^\circ$

$x+2x=111^\circ$ ◄ 1M

$3x=111^\circ$

$x=37^\circ$ ◄ 1A

(4)

8. (a) From the graph, when $x=0$, $y=3$.

$\therefore\ 3=0^2-4(0)+k$

$k=3$ ◄ 1A

(1)

(b) The equation of the graph is

$y=x^2-4x+3$.

When $y=0$, $x^2-4x+3=0$ ◄ 1M

$(x-1)(x-3)=0$

$x=1$ *or* 3

$\therefore$ The coordinates of A and B are $(1, 0)$ and $(3, 0)$ respectively. ◄ 1A + 1A

(3)

9. (a) (i) $4x-9<0$

$x<\dfrac{9}{4}$ ◄ 1A

(1)

(ii) $x^2-3x-54>0$

$(x+6)(x-9)>0$ ◄ 1A❶

$x<-6$ *or* $x>9$ ◄ 2A

(3)

(b) The range of values of x which satisfy both the inequalities in (a) is

$x<-6$. ◄ 1A

(1)

Remark:
❶ 1A *for factorization*

10. (a) $(a+2)(a-4)=0$ ◄ 1A

$a=-2$ or $a=4$

(1)

(b) $(b^2-3b)^2-2(b^2-3b)-8=0$

$[(b^2-3b)+2][(b^2-3b)-4]=0$ ◄ 1M + 1A

$[b^2-3b+2][b^2-3b-4]=0$

$(b-1)(b-2)(b+1)(b-4)=0$ ◄ 1A + 1A❶

b=1, b=2, b=4 or b=-1

(4)

11. (a) $y = k_1x + k_2x^2$ where k_1, k_2 are non-zero constants ◄ 1A

$5 = k_1(1) + k_2(1)^2$

$5 = k_1 + k_2$ .... (i)

$1 = k_1(-1) + k_2(-1)^2$

$1 = -k_1 + k_2$ .... (ii) ◄ 1A

(i) – (ii), $4 = 2k_1$

$k_1 = 2$ ◄ 1A

(i) + (ii), $6 = 2k_2$

$k_2 = 3$ ◄ 1A

$\therefore$ $\underline{y = 2x + 3x^2}$

(4)

(b) When $x = 2$,

$y = 2(2) + 3(2)^2$ ◄ 1M❷

$= \underline{\underline{16}}$ ◄ 1A

(2)

12. (a) P(the same value)

$= P$(two \$10 notes) + P(two \$20 notes)

$= \frac{3}{6} \times \frac{2}{5} + \frac{2}{6} \times \frac{1}{5}$ ◄ 1M

$= \frac{4}{15}$ *(or 0.267, cor. to 3 sig. fig.)* ◄ 1A

(b) P(total value of \$60)

$= P$(1st \$50 note and 2nd \$10 note) + P(1st \$10 note and 2nd \$50 note)

$= \frac{1}{6} \times \frac{3}{5} + \frac{3}{6} \times \frac{1}{5}$

$= \frac{1}{5}$ ◄ 1A

P(total value of \$70)

$= P$(1st \$50 note and 2nd \$20 note) + P(1st \$20 note and 2nd \$50 note)

$= \frac{1}{6} \times \frac{2}{5} + \frac{2}{6} \times \frac{1}{5}$

$= \frac{2}{15}$ ◄ 1A

$\therefore$ P(total value less than \$60)

$= 1 - \frac{1}{5} - \frac{2}{15}$ ◄ 1M

$= \underline{\underline{\frac{2}{3}}}$ *(or 0.667, cor. to 3 sig. fig.)* ◄ 1A

(4)

$\frac{3}{6} \times \frac{2}{5} + \frac{3}{6} \times \frac{2}{5} + \frac{3}{6} \times \frac{1}{5} + \frac{2}{6} \times \frac{3}{5}$

13. (a) First term = 598

Common difference = 592 – 598

$= -6$ ◄ 1A

$\therefore$ $T(n) = 598 + (n-1)(-6)$

$= \underline{\underline{604 - 6n}}$ ◄ 1A

(2)

(b) $T(k) > 0$

$604 - 6k > 0$

$k < 100\frac{2}{3}$

$\because$ $T(k)$ is the last positive term,

$\therefore$ $k = \underline{\underline{100}}$ ◄ 1A

$T(k) = T(100)$

$= 604 - 6(100)$

$= \underline{\underline{4}}$ ◄ 1A

(2)

(c) Sum of all positive terms $= \frac{100}{2}(598 + 4)$ ◄ 1M

$= \underline{\underline{30\ 100}}$ ◄ 1A

(2)

14. (a) The median of the distribution

$= \underline{\underline{50}}$ ◄ 1A

The inter-quartile range of the distribution

$= 65 - 40$ ◄ 1A❶

$= \underline{\underline{25}}$ ◄ 1A

(3)

(b)

Mark (x)	Class mark	Frequency
$20 < x \le 30$	25	20
$30 < x \le 40$	35	30
$40 < x \le 50$	45	50
$50 < x \le 60$	55	40
$60 < x \le 70$	65	20
$70 < x \le 80$	75	20
$80 < x \le 90$	85	10
$90 < x \le 100$	95	10

◄ 1A + 1A❷

The mean of the distribution

$= \underline{\underline{53}}$ ◄ 1A

The standard deviation of the distribution

$= \underline{\underline{18.6}}$, *cor. to 3 sig. fig.* ◄ 1A

(4)

Remarks:
❶ 1A *for either 65 or 40*
❷ 1A *for any two answers being correct*

15. (a) Consider $\triangle OO'P$ and $\triangle OO'Q$.

$OO' = OO'$	*common side*
$OP = OQ$	*radii*
$O'P = O'Q$	*radii*
$\therefore \triangle OO'P \cong \triangle OO'Q$	SSS

Marking Scheme

Case 1	*Any correct proof with correct reasons*	[3]
Case 2	*Any correct proof without correct reasons*	[2]
Case 3	*Any relevant correct argument with correct reason*	[1]❶

(3)

(b) (i)

$\angle POQ = 2\angle PRQ$
$= 2 \times 75°$
$= 150°$ ◄ 1A

$\because \triangle OO'P \cong \triangle OO'Q$
$\therefore \angle POO' = \angle QOO'$
$\therefore \angle POO' = \underline{\underline{75°}}$ ◄ 1A

(2)

(ii) In $\triangle POO'$,

$\because O'O = O'P$
$\therefore \angle OPO' = \angle POO'$ ◄ 1M
$= 75°$
$\angle PO'O = 180° - 75° - 75°$
$= 30°$ ◄ 1A

$\because \triangle OO'P \cong \triangle OO'Q$
$\therefore \angle QO'O = \angle PO'O$
$= 30°$
$\angle PO'Q = 60°$

$\therefore \angle QSR = \frac{1}{2}\angle PO'Q$
$= \frac{1}{2} \times 60°$
$= \underline{\underline{30°}}$ ◄ 1A

(3)

Remark :
❶ *maximum 1 mark*

16. (a)

$\frac{AD}{AB} = \sin 30°$

$AB = \frac{h}{\sin 30°}$ m ◄ 1M
$= 2h$ m ◄ 1A

$\frac{AD}{AC} = \sin 45°$

$AC = \frac{h}{\sin 45°}$ m
$= \sqrt{2}h$ m ◄ 1A

$BC^2 = AB^2 + AC^2 - 2 \times AB \times AC \times \cos \angle BAC$ ◄ 1M

$(2\sqrt{2})^2 = (2h)^2 + (\sqrt{2}h)^2 - 2(2h)(\sqrt{2}h)\cos 45°$
$8 = 4h^2 + 2h^2 - 4h^2$
$h^2 = 4$
$h = \underline{\underline{2}}$ ◄ 1A

(5)

(b)

$\frac{AD}{BD} = \tan 30°$

$BD = \frac{2}{\tan 30°}$ m
$= 2\sqrt{3}$ m ◄ 1A

16. (b)

$\frac{AD}{CD} = \tan 45°$

$CD = \frac{2}{\tan 45°}$ m
$= 2$ m ◄ 1A

$\cos \angle BDC = \frac{BD^2 + CD^2 - BC^2}{2(BD)(CD)}$ ◄ 1M
$= \frac{(2\sqrt{3})^2 + 2^2 - (2\sqrt{2})^2}{2(2\sqrt{3})(2)}$

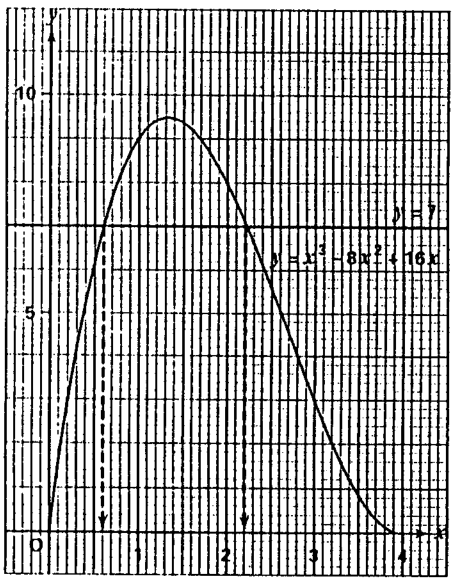
$\angle BDC = 54.735\ 6°$ ◄ 1A

Area of $\triangle BCD$

$= \frac{1}{2} \times BD \times CD \times \sin \angle BDC$ ◄ 1M
$= \left(\frac{1}{2} \times 2\sqrt{3} \times 2 \times \sin 54.735\ 6°\right)$ m^2
$= 2.83$ m^2, *cor. to 3 sig. fig.* ◄ 1A

$\therefore$ The area of the shadow cast by $\triangle ABC$ is 2.83 m^2.

17. (a) (i) Add the line $y = 7$ to the graph.



◄ 1A

From the points of intersection, we have

$x = \underline{\underline{0.6}}$, *cor. to 1 d.p.* ◄ 1A

or $x = \underline{\underline{2.2}}$, *cor. to 1 d.p.* ◄ 1A

(ii) When $y = 7$,

$7 = x^3 - 8x^2 + 16x$

Let $f(x) = x^3 - 8x^2 + 16x - 7$ and the larger value of x in **(i)** be x_0.
From the graph, x_0 lies between 2.2 and 2.3.

$f(2.2) = 0.128 > 0$
$f(2.3) = -0.353 < 0$ ◄ 1M❶

Bracketing interval	Mid-value (x_m)	$f(x_m)$
$2.2 < x_0 < 2.3$	2.25	−ve
$2.2 < x_0 < 2.25$	2.225	+ve
$2.225 < x_0 < 2.25$	2.238	−ve
$2.225 < x_0 < 2.238$	2.232	−ve
$2.225 < x_0 < 2.232$		

◄ 1M + 1A❷
◄ 1M❸

$\therefore$ The larger value of x is 2.23, *cor. to 2 d.p.* ◄ 1❹

(8)

17. (b) $28 = x(8-2x)^2$ ◄ 1A

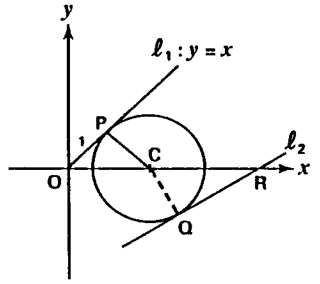
$28 = 4x(16 - 8x + x^2)$

$7 = x^3 - 8x^2 + 16x$ ◄ 1A

Using the above results and the fact that $x > 2$, we have $x = \underline{\underline{2.23}}$, *cor. to 2 d.p.* ◄ 1A

(3)

18.



(a) (i) $\angle OPC = 90°$ ◄ 1A

$\because$ The slope of ℓ_1 is 1,

$\therefore$ $\angle POC = 45°$ ◄ 1A

$\tan\angle COP = \frac{PC}{OP}$

$\tan 45° = \frac{PC}{1}$

$\therefore$ $PC = \underline{\underline{1}}$ ◄ 1A

$\cos\angle COP = \frac{OP}{OC}$

$\cos 45° = \frac{1}{OC}$

$\therefore$ $OC = \underline{\underline{\sqrt{2}}}$ ◄ 1A

(ii) Coordinates of C = $(\sqrt{2}, 0)$

Radius = 1

$\therefore$ The equation of the circle is

$(x - \sqrt{2})^2 + (y - 0)^2 = 1^2$ ◄ 1M

or $\underline{x^2 + y^2 - 2\sqrt{2}x + 1 = 0.}$ ◄ 1A

(6)

(b) $OR = OC + CR$

$= \sqrt{2} + 2$

$\therefore$ The coordinates of R are $(\sqrt{2} + 2, 0)$. ◄ 1A

Join CQ.

In $\triangle CQR$,

$\sin\angle CRQ = \frac{CQ}{CR}$

$= \frac{1}{2}$

$\therefore$ $\angle CRQ = 30°$

Slope of $\ell_2 = \tan\angle CRQ$ ◄ 1M

$= \tan 30°$

$= \frac{1}{\sqrt{3}}$ ◄ 1A

$\therefore$ The equation of ℓ_2 is

$\frac{y - 0}{x - (\sqrt{2} + 2)} = \frac{1}{\sqrt{3}}$ ◄ 1M

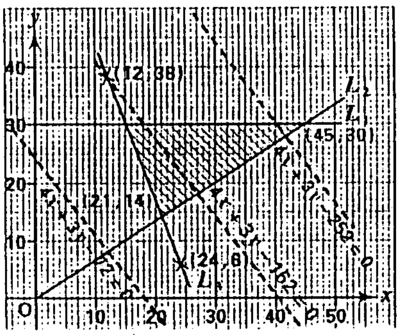
or $\underline{x - \sqrt{3}y - \sqrt{2} - 2 = 0.}$ ◄ 1A ❶

(5)

Remark :

❶ *or equivalent forms*

19.



(a) The equation of L_2 is

$\frac{y - 0}{x - 0} = \frac{2}{3}$

$3y = 2x$

or $2x - 3y = 0.$ ◄ 1A

The equation of L_3 is

$\frac{y - 6}{x - 24} = \frac{38 - 6}{12 - 24}$

$3y - 18 = -8x + 192$

or $8x + 3y - 210 = 0.$ ◄ 1A

The three inequalities are

$\underline{y \le 30,}$ ◄ 1A ❶

$\underline{2x - 3y \le 0,}$ ◄ 1A

$\underline{8x + 3y - 210 \ge 0.}$ ◄ 1A

(5)

(b) (i) Draw the line $4x + 3y - 72 = 0$ on the figure and shift it upwards to get the maximum and minimum values of P. From the graph, P is maximum at (45 , 30) and minimum at (21 , 14). ◄ 1M

$\therefore$ Maximum value of P

$= 4(45) + 3(30) - 72$

$= \underline{\underline{198}}$ ◄ 1A

Minimum value of P

$= 4(21) + 3(14) - 72$

$= \underline{\underline{54}}$ ◄ 1A

(ii) $90 \le P \le 180$

i.e. $90 \le 4x + 3y - 72 \le 180$

$162 \le 4x + 3y \le 252$

Draw the lines $4x + 3y - 162 = 0$ and $4x + 3y - 252 = 0$. ◄ 1M

(1) From the graph, the range of possible values of x is $\underline{18 \le x \le 42.}$ ◄ 1A

(2) From the graph, the range of possible values of y is $\underline{18 \le y \le 30.}$ ◄ 1A

(6)

Remark :

❶ *withhold 1 mark for any strict inequality*