

F.5 MATHEMATICS TEST (96 - 97)
Ch. 6 Approximate Solution of Equations

Section A (30%)

- 1 Given an equation $f(x) = 0$. It is found that $f(x_1) > 0$ and $f(x_2) < 0$. If $f(x_3) > 0$ where $x_3 = \frac{x_1 + x_2}{2}$, then there **must** be a root between
- x_1 and x_3 .
 - x_2 and x_3 .
 - $f(x_1)$ and $f(x_3)$.
 - $f(x_2)$ and $f(x_3)$.
 - x_2 and $f(x_3)$.

- 2 From the given table, a root of $f(x) = 0$ must be

x	sign of $f(x)$
3.95	+
3.94	-
3.945	+
3.9425	-

- 4.0, correct to 2 significant figures.
- 3.94, correct to 3 significant figures.
- 3.95, correct to 3 significant figures.
- 3.943, correct to 4 significant figures.
- 3.945, correct to 4 significant figures.

- 3 In the table, the values of the function $f(x)$ for $x = 0.5, 0.6, \dots, 0.9$ are shown. Which of the following intervals **must** contain a root of the equation $f(x) + 4x = 0$?

x	$f(x)$
0.5	-3
0.6	-2
0.7	1
0.8	-4
0.9	-5

- $0.5 < x < 0.6$
 - $0.6 < x < 0.7$
 - $0.7 < x < 0.8$
 - $0.8 < x < 0.9$
- I and III only
 - II and III only
 - I, II and IV only
 - II, III and IV only
 - I, II, III and IV

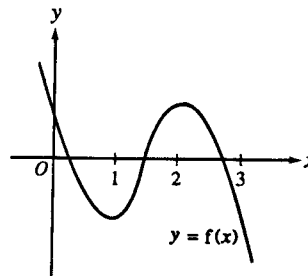
- 4 The equation $x^3 - x - 3 = 0$ has a real root lying between
- 2 and -1.
 - 1 and 0.
 - 0 and 1.
 - 1 and 2.
 - 2 and 3.

- 5 If the interval $a \leq x \leq b$ contains exactly one root of $f(x) = 0$, then under which of the following conditions the method of bisection could be used to solve the equation $f(x) = 0$?

- $f(a) < 0$ and $f(b) > 0$
- $f(a) > 0$ and $f(b) > 0$
- $f(a) < 0$ and $f(b) < 0$
- $f(a) > 0$ and $f(b) < 0$

- I only
- II and III only
- I and III only
- III and IV only
- I and IV only

- 6 The figure shows the graph of a function $y = f(x)$. Which of the following intervals can be taken as the initial interval(s) when applying the method of bisection in solving the equation $f(x) = 0$?



- $0 < x < 1$
 - $1 < x < 3$
 - $0 < x < 3$
- I only
 - II only
 - I and II only
 - I and III only
 - II and III only

Section B (70%)

7. (a) Show that the equation $3x - \cos x = 0$ has a root between 0 and 1. (10%)
 (b) Find the root of the equation in (a) correct to 1 decimal place. (20%)

8 The figure shows the graph of

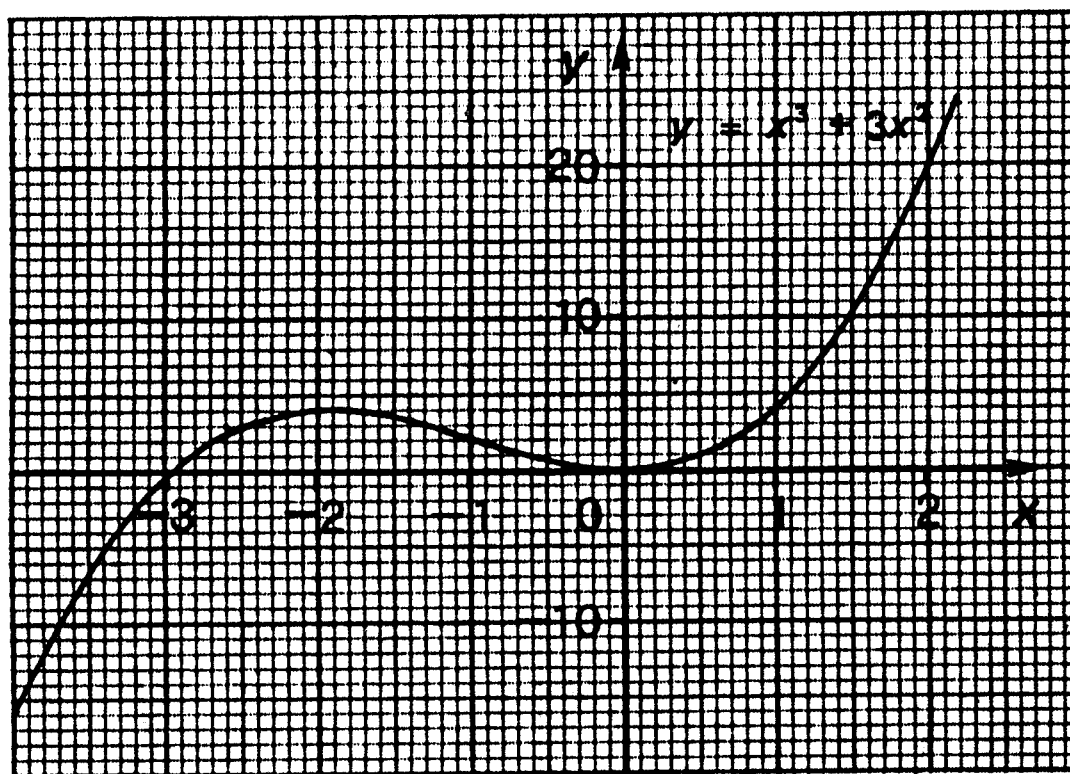
$$y = x^3 + 3x^2, \quad -4 \leq x \leq 2.$$

(a) By adding a suitable straight line on the graph, solve

$$4x^3 + 12x^2 - 5x - 10 = 0.$$

(Give the answer correct to 1 decimal place.) (20%)

(b) Using the method of bisection, find the smallest value obtained in (a) correct to 3 decimal places. (20%)



END

Numerical Answers

1. B 2. B 3. A 4. D 5. E 6. D
7. (b) 0.3 8. (a) -3.1, -0.8, 1.0 (b) -3.145