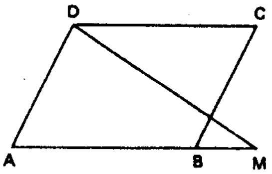
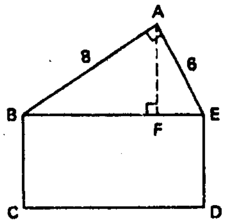
Mensuration

1. A side of a rhombus is 10 cm, and one of its angles is 30°. What is its area? $10 \times 10 \times \sin 30^\circ$
   - A. 25 cm²
   - B. 50 cm²
   - C. 100 cm²
   - D. 150 cm²
   - E. 175 cm²

2. In the figure, ABCD is a parallelogram. AB is produced to M such that BM = $\frac{1}{3}$ AB. Find the ratio Area of triangle AMD : Area of parallelogram ABCD.
   - A. 2 : 3
   - B. 1 : 3
   - C. 1 : 2
   - D. 3 : 4
   - E. 3 : 2



3. With reference to the diagram, AB = 8 cm, AE = 6 cm, AF ⊥ BE. BCDE is a rectangle with AF = ED. What is the area of rectangle BCDE?
   - A. cannot be determined
   - B. 24 cm²
   - C. 48 cm²
   - D. 80 cm²
   - E. 96 cm²



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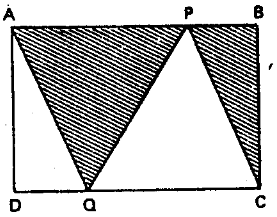
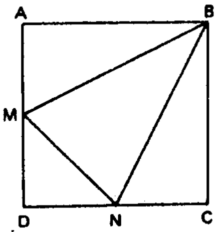
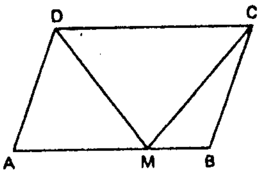


In the figure, ABCD is a rectangle in which AB = 10 cm, BC = 8 cm. P and Q are 2 points on AB and CD respectively. Find the area of the shaded region.
- A. 20 cm²
- B. 40 cm²
- C. 60 cm²
- D. 80 cm²
- E. inadequate data

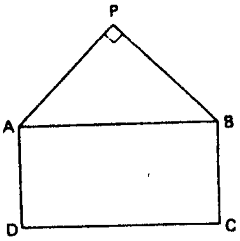
5. In the figure, ABCD is a square of side 2a. M and N are the mid-points of AD and CD respectively. Find the area of ΔMNB.
   - A. $\frac{1}{2}a^2$
   - B. a^2
   - C. $\frac{3}{2}a^2$
   - D. $2a^2$
   - E. $\frac{5}{2}a^2$



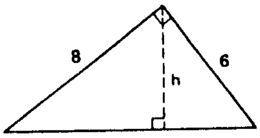
6. In the figure, ABCD is a parallelogram. M is a point of trisection of AB. If the area of ΔBCM = a, then the area of ΔADM is
   - A. $\frac{3}{2}a$
   - B. 2a
   - C. $\frac{5}{2}a$
   - D. $\frac{4}{3}a$
   - E. 3a



7. In the figure, ABCD is a rectangle. AB = a, BC = b. ABP is an isosceles triangle right-angled at P. Find the area of ΔAPB.
   - A. $\frac{1}{2}ab$
   - B. $\frac{2}{3}ab$
   - C. $\frac{1}{2}a^2$
   - D. $\frac{3}{2}a^2$
   - E. $\frac{1}{4}a^2$

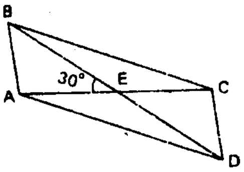


8. In the figure, find the length of h.
   - A. 1.2
   - B. 2.4
   - C. 3.6
   - D. 4.8
   - E. 6.0

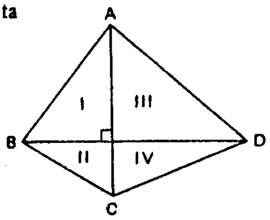


9. Which of the following statements is/are true for a rhombus?
   - (1) The diagonals are perpendicular to each other.
   - (2) The diagonals are equal in length.
   - (3) All the sides are equal in length.
   - A. (1) only
   - B. (3) only
   - C. (1) and (2) only
   - D. (1) and (3) only
   - E. (2) and (3) only

10. In the figure, ABCD is a quadrilateral. The diagonals AC and BD meet at E. ∠AEB = 30°. AC = 6 cm, BD = 8 cm. Find the area of the quadrilateral.
    - A. 12 cm²
    - B. 18 cm²
    - C. 24 cm²
    - D. 32 cm²
    - E. 36 cm²



11. In the quadrilateral ABCD, the 2 diagonals AC and BD intersect at right angles and divide the whole region into 4 parts I, II, III, IV as shown. The areas of regions I, II, and III are 12, 6 and 10 respectively. Find the area of region IV.
    - A. not enough data
    - B. 8
    - C. 14
    - D. 16
    - E. 20

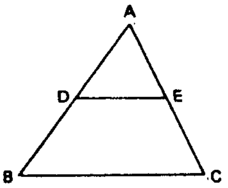


12. The length of the diagonal of a rectangle is m and its area is equal to n. Find the perimeter of the rectangle.

A. $\sqrt{m^2 + n}$
B. $2\sqrt{m^2 + 2n}$
C. $\sqrt{m^2 + 2n}$
D. $m^2 + 2n$
E. $\sqrt{\frac{n}{m}} + 2n$

13. In the figure, D and E are 2 points on AB and AC such that DE is parallel to BC. If the area of BCED = 2 X area of ΔADE, and DE = 3 cm, find the length of BC.

A. $\sqrt{3}$
B. $2\sqrt{3}$
C. $3\sqrt{3}$
D. $\frac{\sqrt{3}}{2}$
E. $\frac{3}{2}$



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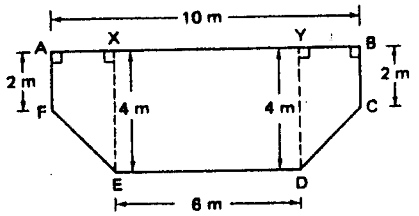


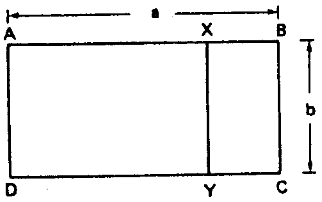
The figure shows a polygon ABCDEF in which AB = 10 m, DE = 6 m, XE = YD = 4 m, AF = BC = 2m. Find the area of the polygon.

A. cannot be determined
B. 36 m^2
C. 40 m^2
D. 48 m^2
E. 52 m^2

15. Find the area of a regular hexagon with the length of each side equal to a.

A. $6a^2$
B. $3a^2$
C. $3\sqrt{3}a^2$
D. $\frac{3\sqrt{3}}{2}a^2$
E. $\sqrt{6}a^2$

16.

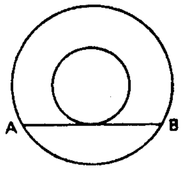


In the figure, ABCD is a rectangle. AB = a, BC = b. X and Y are 2 points on AB and CD such that rectangles ABCD and BCYX are similar. Find the ratio of the area of rectangle ABCD to that of BCYX.

A. $a : b$
B. $\sqrt{3}a : b$
C. $\sqrt{3}a^2 : \sqrt{2}b^2$
D. $2a^2 : 3b^2$
E. $a^2 : b^2$

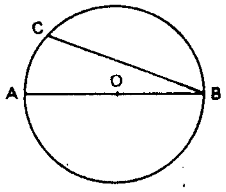
17. The figure shows 2 concentric circles with radii a and b where $b > a$. The line AB is a chord of the greater circle and is a tangent to the smaller circle. What is the length of AB?

A. $2(b - a)$
B. $\sqrt{a^2 + b^2}$
C. $2\sqrt{a^2 + b^2}$
D. $\sqrt{b^2 - a^2}$
E. $2\sqrt{b^2 - a^2}$



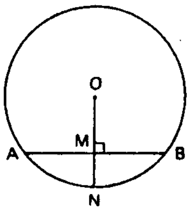
18. In the figure, O is the centre of the circle. If AB = 10 cm and BC = 8 cm, how far is the chord BC from O.

A. 6 cm
B. $5\sqrt{2}$ cm
C. $3\sqrt{3}$ cm
D. 3 cm
E. $2\sqrt{3}$ cm



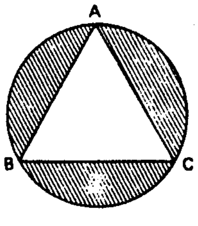
19. In the figure, AB is a chord of the circle, M is the midpoint of AB. If AM = a, MN = b, what is the diameter of the circle?

A. $2(a^2 + b^2)$
B. $\frac{a^2 + b^2}{2}$
C. $\frac{a^2 + b^2}{a}$
D. $\frac{a^2 + b^2}{b}$
E. $\frac{a^2 - b^2}{2a}$



20. In the figure, ABC is an equilateral triangle inscribed in a circle. If the length of each side of ΔABC is S, find the area of the shaded region.

A. $\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)S^2$
B. $\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{4}\right)S^2$
C. $\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)S^2$
D. $\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)S^2$
E. None of the above

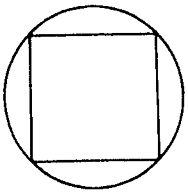


21. The numerical value of the area of a circle is equal to the circumference. Find the radius of the circle.

A. 1
B. 2
C. $\sqrt{\pi}$
D. $\sqrt{2}\pi$
E. $\sqrt{2\pi}$

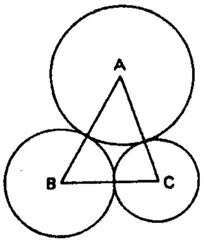
22. A square is inscribed in a circle. Find the ratio the area of the square : the area of the circle.

A. $1 : \pi$
B. $2 : \pi$
C. $2 : 3\pi$
D. $3 : 2\pi$
E. $3 : \pi$



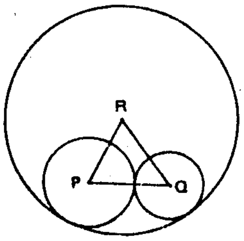
23. In the figure, the circles with centres A, B and C touch each other. If AB = 9, BC = 7, AC = 8, find the radius of the circle with centre A.

A. 3
B. $4\frac{1}{2}$
C. 4
D. 5
E. $5\frac{1}{2}$



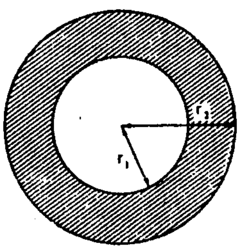
24. In the figure, the circles with centres P and Q touch each other externally. They both touch the circle with centre R internally. If PQ = 14, QR = 10, RP = 12, find the radius of the circle with centre R.

A. 12
B. 14
C. 16
D. 18
E. 20



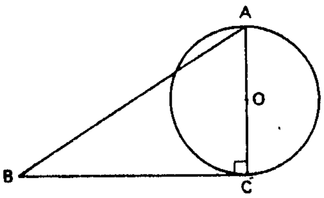
25. In the figure, the circle with radius r_2 is concentric with the circle with radius r_1. If the area of the shaded region is the same as the area of the inner circle, find $r_2 : r_1$.

A. $\sqrt{2} : 1$
B. 2 : 1
C. 3 : 2
D. $\sqrt{3} : 1$
E. $\sqrt{3} : \sqrt{2}$



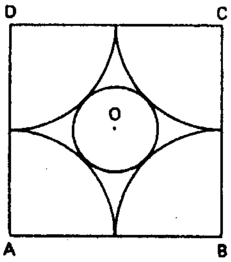
26. In the figure, if the circle with centre O rolls along BC, the point A will coincide with the point B. If the area of the circle is A, find the area of ΔABC.

A. $\frac{3}{4}$A
B. $\frac{A}{2}$
C. A
D. 2A
E. $\frac{3}{2}$A



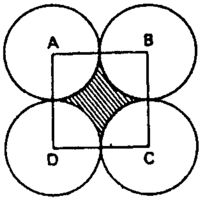
27. ABCD is a square of side 2r. Four quadrants of radius r are drawn with A, B, C and D as centres. If the circle with centre O touches all of the four quadrants, what is the radius of the central circle?

A. $\frac{r}{2}$
B. $\frac{\sqrt{2}}{2}r$
C. $(\pi - \sqrt{2})r$
D. $(\sqrt{2} - 1)r$
E. $\sqrt{2r}$



28. In the figure, the circles with centres ABCD touch each other externally and the centres form a square. If the radius of each circle is r, find the area of the shaded region.

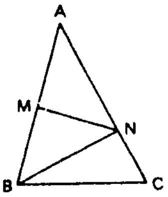
A. $(4 - \pi)r^2$
B. $4(\pi - 1)r^2$
C. $4(\pi - 2)r^2$
D. $\left(4 - \frac{\pi}{2}\right)r^2$
E. $(4 - \pi)\frac{r^2}{2}$



29. In the figure, M is the mid-point of AB. N is a point on AC such that AN : NC = 2 : 1. Which of the following statements is/are true?

(1) ΔAMN and ΔBMN are equal in area.
(2) Area of ΔABN = 2 × Area of ΔCBN
(3) Area of ΔBMN = Area of ΔCBN

A. (1) only
B. (1) and (2) only
C. (1) and (3) only
D. (2) and (3) only
E. (1), (2), and (3)



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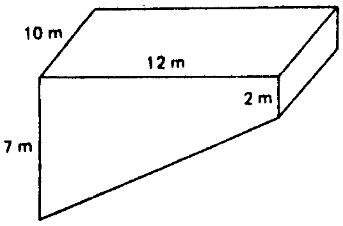
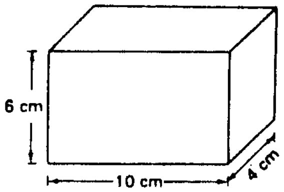


The figure represents a 10 m × 12 m swimming pool. The cross-section of the pool is in the shape of a trapezium with the botton at increasing depths. The dimensions of the pool are as shown in the figure. What is the capacity of the pool in m^3?

A. 490 m^3
B. 500 m^3
C. 510 m^3
D. 530 m^3
E. 540 m^3

31. The dimension of a rectangular block is as shown in the figure. If this block is immersed into water and 3/4 of its volume is under water, what is the total surface area of the block under water?

A. 93 cm^2
B. 144 cm^2
C. 156 cm^2
D. 166 cm^2
E. 174 cm^2

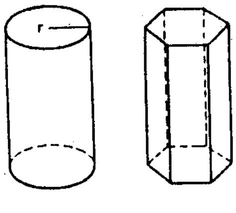


32. The areas of the surfaces of a cuboid are 2, 8 and 16 cm^2. Find the volume of the cuboid.

A. 12 cm^3
B. 16 cm^3
C. 18 cm^3
D. 24 cm^3
E. not enough data

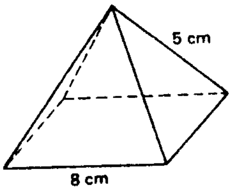
33. The cylinder and the prism have the same volume and the same height. Find the cross-sectional area of the prism.

A. πr^2
B. πrh
C. $\frac{3}{2}\pi r^2$
D. $\frac{3}{2}\pi rh$
E. $2\pi r^2$



34. The figure shows a right pyramid with a square base. Find the total surface area of the pyramid.

A. 72 cm²
B. 84 cm²
C. 88 cm²
D. 96 cm²
E. 112 cm²



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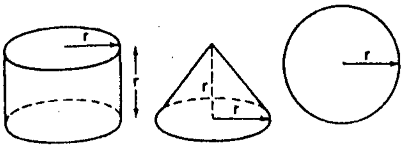


The figures above show a cylinder with radius r and height r, a cone with base radius r and height r and a sphere with radius r. Let the volumes of the 3 solids be V_1, V_2, V_3. Which of the following statements is true?

A. $V_1 > V_2 > V_3$
B. $V_1 > V_3 > V_2$
C. $V_2 > V_3 > V_1$
D. $V_3 > V_2 > V_1$
E. $V_3 > V_1 > V_2$

36. A metal cylinder with base radius 2r and height $\frac{3}{4}$ r and a cone with base radius $\frac{3}{2}$ r and height 2r are fused together to form a sphere. Find the radius of the sphere.

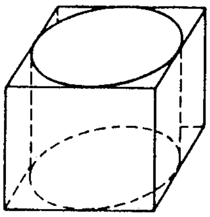
A. $\frac{1}{2}$r
B. r
C. $\frac{3}{2}$r
D. 2r
E. $\frac{5}{2}$r

37. Eight metal spheres, each of radius r, are melted and recast into one sphere. Which of the following statements concerning the original total surface area and the area of the new sphere is correct?

A. They remain the same.
B. There is an increase of 50% of the surface area.
C. There is a decrease of 25% of the surface area.
D. There is a decrease of 50% of the surface area.
E. There is a decrease of 75% of the surface area.

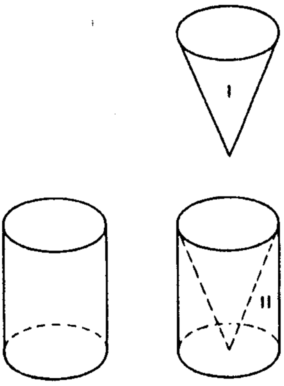
38. A cubical box of side a cm (internal measurement) just contains a solid cylinder of diameter a cm and height a cm. Find the empty space in the box.

A. $\left(1 - \frac{\pi}{4}\right)a^3$
B. $4(\pi - 1)a^3$
C. $\frac{(\pi - 1)a^3}{4}$
D. $(3\pi - 4)a^3$
E. $\left(1 - \frac{3\pi}{4}\right)a^3$



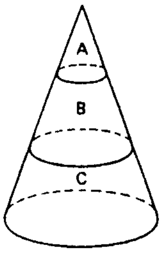
39. A cone is cut from a cylinder so that the latter is split into 2 solids I and II as shown. Find the ratio volume of I : volume of II.

A. 1 : 1
B. 2 : 1
C. 1 : 2
D. $\sqrt{2} : 1$
E. $1 : \sqrt{2}$



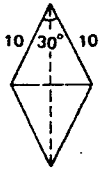
40. A right circular cone is divided into 3 portions A, B and C by planes parallel to the base as shown. The height of each portion is 1 unit. Calculate the ratio of the area of the curved surface of B to that of C.

A. 1 : 2
B. 1 : 3
C. 2 : 3
D. 3 : 4
E. 3 : 5



1.	B	2.	A	3.	C	4.	B	5.	C
6.	B	7.	E	8.	D	9.	D	10.	A
11.	E	12.	B	13.	C	14.	B	15.	D
16.	E	17.	E	18.	D	19.	D	20.	A
21.	B	22.	B	23.	D	24.	D	25.	A
26.	C	27.	D	28.	A	29.	E	30.	E
31.	D	32.	B	33.	A	34.	E	35.	E
36.	C	37.	D	38.	A	39.	C	40.	E

1. Area $= 2 \times \frac{1}{2} \times 10 \times 10 \times \sin 30° = 50$



2. Let h be the height

$$\text{The ratio} = \frac{\frac{1}{2} \times AM \times h}{AB \times h} = \frac{\frac{1}{2} \times (AB + BM)}{AB}$$

$$= \frac{\frac{1}{2} \times \left(AB + \frac{1}{3} AB\right)}{AB} = \frac{2}{3}$$

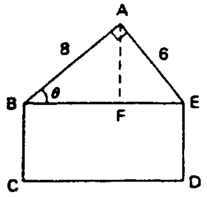
3. $BE^2 = 8^2 + 6^2 = 100$

$BE = 10$

$\tan \theta = \frac{6}{8} = \frac{3}{4}$

$AF = 8 \times \sin \theta$

$= 8 \times \frac{3}{5} = \frac{24}{5}$



Area of rectanlge BCDE

$= 10 \times \frac{24}{5} = 48$

4. Area $= \frac{1}{2} \times AP \times BC + \frac{1}{2} \times PB \times BC$

$= \frac{1}{2} \times (AP + PB) \times BC$

$= \frac{1}{2} \times AB \times BC$

5. Area of ΔABM = Area of ΔBCN

$= \frac{1}{2} \times 2a \times a = a^2$

Area of $\Delta MND = \frac{1}{2} \times a \times a = \frac{1}{2} a^2$

$\therefore$ area of $\Delta MNB = (2a)^2 - \left(2a^2 + \frac{1}{2} a^2\right)$

$= \frac{3}{2} a^2$

6. Let h be the height

Area of $\Delta BCM = \frac{1}{2} \times MB \times h = a$

$MB \times h = 2a$

Area of $\Delta ADM = \frac{1}{2} \times AM \times h$

$= \frac{1}{2} \times 2 \times MB \times h$

$= MB \times h = 2a$

7. Let $AP = PB = x$

then $x^2 + x^2 = a^2$ $\quad x^2 = \frac{a^2}{2}$

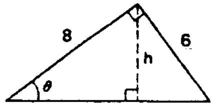
Area of $\Delta APB = \frac{1}{2} \times AP \times PB = \frac{1}{2} x^2 = a^2/4$

8. $\tan \theta = \frac{6}{8} = \frac{3}{4}$

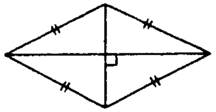
$h = 8 \times \sin \theta$

$= 8 \times \frac{3}{5}$

$= 4.8$

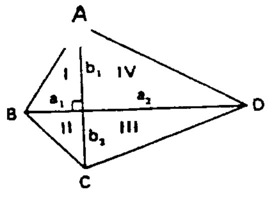


9. The diagonals may not be equal in length.



10. Area $= \frac{1}{2} \times AC \times BD \times \sin 30°$

11.



Let area of IV be A

Area of I $= \frac{1}{2} \times a_1 \times b_1 = 12$ ……… (1)

Area of II $= \frac{1}{2} \times a_1 \times b_2 = 6$ ……… (2)

$(1) \div (2) : \frac{b_1}{b_2} = \frac{2}{1}$

Area of III $= \frac{1}{2} \times a_2 \times b_2 = 10$ ……… (3)

Area of IV $= \frac{1}{2} \times a_2 \times b_1 = A$ ……… (4)

$(4) \div (3) : \frac{b_1}{b_2} = \frac{A}{10}$

$\because \frac{2}{1} = \frac{A}{10} \quad A = 20$

12. Let the sides of the rectangle be a, b, then

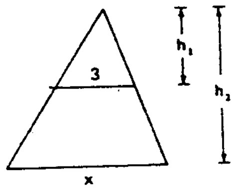
$a^2 + b^2 = m^2$

$ab = n$

$(a + b)^2 = a^2 + b^2 + 2ab = m^2 + 2n$

perimeter $= 2(a + b) = 2\sqrt{m^2 + 2n}$

13.



Let BC = x cm.

Area of BCED $= \frac{1}{2} \times (3 + x)(h_2 - h_1)$

We have: $\frac{1}{2}(3 + x)(h_2 - h_1) = 2 \times \frac{1}{2} \times 3 \times h_1$

$(3 + x)\left(\frac{h_2}{h_1} - 1\right) = 6$

But $\frac{h_2}{h_1} = \frac{x}{3}$

$\therefore (3 + x)\left(\frac{x}{3} - 1\right) = 6 \quad x^2 = 27$

$x = 3\sqrt{3}$

14. Area of XYDE $= 4 \times 6 = 24\ m^2$

Area of AFEX $= \frac{1}{2} \times (2 + 4) \times AX = 3AX$

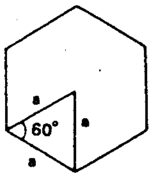
Area of BCDY $= \frac{1}{2} \times (2 + 4) \times BY = 3BY$

Total Area $= 24 + 3AX + 3BY$

$= 24 + 3(AX + BY)$

$= 24 + 3(10 - 6) = 36\ m^2$

15.

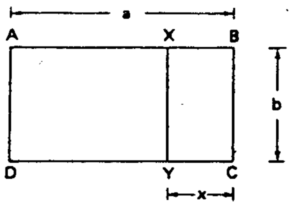


Area of each triangle

$= \frac{1}{2} \times a \times a \times \sin 60° = \frac{\sqrt{3}}{4} a^2$

Area of hexagon $= 6 \times \frac{\sqrt{3}}{4} a^2 = \frac{3\sqrt{3}}{2} a^2$

16.



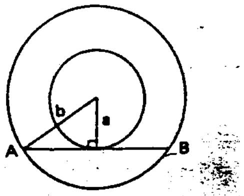
Since rectanlge ABCD ~ rectanlge BXYC

$\therefore \frac{a}{b} = \frac{b}{x}$ or $x = \frac{b^2}{a}$

Ratio $= \frac{ab}{bx} = \frac{a}{x} = \frac{a}{\frac{b^2}{a}} = \frac{a^2}{b^2}$

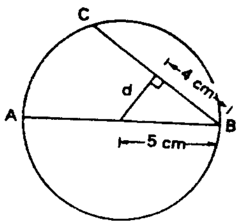
17. $\frac{AB}{2} = \sqrt{b^2 - a^2}$

$AB = 2\sqrt{b^2 - a^2}$



18. $d^2 = 5^2 - 4^2 = 9$

$d = 3$ cm



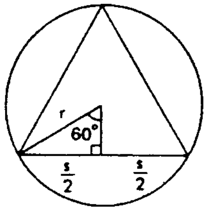
19. Let radius = r.

In ΔOAM: $r^2 = a^2 + (r - b)^2$

$r = \frac{a^2 + b^2}{2b}$

Diameter $= 2r = \frac{a^2 + b^2}{b}$

20.



The radius of the circle is given by

$r \times \sin 60° = \frac{s}{2}$

$r = \frac{s}{\sqrt{3}}$

Area of circle $= \pi r^2 = \frac{\pi s^2}{3}$

Area of $\Delta ABC = \frac{1}{2} \times s \times s \times \sin 60° = \frac{\sqrt{3}}{4} s^2$

Area of shaded region $= \frac{\pi s^2}{3} - \frac{\sqrt{3}}{4} s^2$

21. $\pi r^2 = 2\pi r$

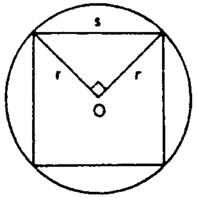
$r = 2$

22. From the figure,

$s^2 = r^2 + r^2$

$s^2 = 2r^2$

the ratio $= \frac{s^2}{\pi r^2} = \frac{2}{\pi}$



23. Let the radii of the circles with centres A, B, C be a, b, c respectively.

Then, $a + b = 9$

$b + c = 7$

$a + c = 8$

Solving: $a = 5$

24. Let the radii of the circles with centres R, P, Q be r, p, q respectively.

$p + q = 14$

$r - p = 12$

$r - q = 10$

Solving $r = 18$

25. $\pi r_1{}^2 = \pi r_2{}^2 - \pi r_1{}^2$

$2r_1{}^2 = r_2{}^2$

$\frac{r_2}{r_1} = \frac{\sqrt{2}}{1}$

26. Let the radius of the circle be r. AC = 2r. BC = πr.

Area of $\Delta ABC = \frac{1}{2} \times 2r \times \pi r = \pi r^2 = A$

27. $AC^2 = (2r)^2 + (2r)^2$

$AC = 2\sqrt{2}\,r$

Let the radius of the central circle be R.

$2R + 2r = 2\sqrt{2}\,r$

$R = (\sqrt{2} - 1)r$

28. Area of the square $= (2r)(2r) = 4r^2$.

The area of the 4 quadrants sum up to a circle with area $= \pi r^2$.

The area of the shaded region $= 4r^2 - \pi r^2$

29. $\frac{\text{Area of } \Delta AMN}{\text{Area of } \Delta BMN} = \frac{AM}{MB} = \frac{1}{1}$

$\frac{\text{Area of } \Delta ABN}{\text{Area of } \Delta CBN} = \frac{AN}{NC} = \frac{2}{1}$

Area of $\Delta CBN = \frac{1}{2} \times$ Area of ΔABN

$= \frac{1}{2}$ (Area of ΔAMN + Area of ΔBMN)

$= \frac{1}{2}$ (2 × Area of ΔBMN)

= Area of ΔBMN

30. The cross-sectio a

$= \frac{1}{2} \times (7 + 2) \times 12 = 54\ m^2$

The volume $= 54 \times 10 = 540\ m^3$

31. Height of the block under water $= 6 \times \frac{3}{4} = \frac{9}{2}$ cm

Area under water $= 10 \times 4 + 2 \times 10 \times \frac{9}{2}$

$+ 2 \times 4 \times \frac{9}{2}$

$= 166\ cm^2$

32. Let the dimensions of the cuboid be a, b and c.

Then $ab = 2$, $bc = 8$, $ac = 16$

volume $= abc = \sqrt{2 \times 8 \times 16} = 16\ cm^3$

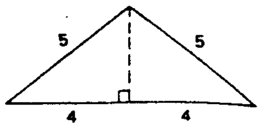
33. Let A be the cross-sectional area.

$\pi r^2 h = Ah$

$A = \pi r^2$

34. The area of the square base $= 8 \times 8$

$= 64\ cm^2$



The height of each triangular face

$= \sqrt{5^2 - 4^2} = 3$

Area of the triangular faces

$= \frac{1}{2} \times 8 \times 3 \times 4 = 48$

Total area $= 64 + 48 = 112\ cm^2$

35. $V_1 = \pi r^2 r = \pi r^3$

$V_2 = \frac{1}{3} \pi r^2 r = \frac{1}{3} \pi r^3$

$V_3 = \frac{4}{3} \pi r^3$

$\therefore\ V_3 > V_1 > V_2$

36. Let R be the radius of the sphere.

$\frac{4}{3} \pi R^3 = \pi (2r)^2 \frac{3}{4} r + \frac{1}{3} \pi \left(\frac{3}{2}\right)^2 (2r)$

$R^3 = \frac{27}{8} r^3$

$R = \frac{3}{2} r$

37. The new radius R is given by

$\frac{4}{3} \pi R^3 = 8 \times \frac{4}{3} \pi r^3$

$R = 2r$

Original surface area $= 8 \times 4\pi r^2 = 32\pi r^2$

New surface area $= 4\pi (2r)^2 = 16\pi r^2$

% change $= \frac{16\pi r^2 - 32\pi r^2}{32\pi r^2} \times 100\% = -50\%$

38. Volume of box $= a^3$

Volume of cylinder $= \pi \left(\frac{a}{2}\right)^2 a = \frac{\pi a^3}{4}$

Volume of space $= a^3 - \frac{\pi a^3}{4}$

39. Volume of I $= \frac{1}{3} \pi r^2 h$

Volume of II $= \pi r^2 h - \frac{1}{3} \pi r^2 h = \frac{2}{3} \pi r^2 h$

Ratio $= \frac{\frac{1}{3} \pi r^2 h}{\frac{2}{3} \pi r^2 h} = \frac{1}{2}$

40. $\frac{A_A}{A_A + A_B} = \left(\frac{1}{2}\right)^2$ $\quad 3A_A = A_B$

$\frac{A_A}{A_A + A_B + A_C} = \left(\frac{1}{3}\right)^2$ $\quad 8A_A = A_B + A_C$

$\therefore\ \frac{5}{3} A_B = A_C$

$\frac{A_B}{A_C} = \frac{3}{5}$