

EXERCISES

Arithmetic Sequences & Series, 等差數列與級數

- (a) Find the 12th term of 3, 7, 11
求數列 3, 7, 11 的第十二項。

(b) Find the 13th term of $\frac{1}{2}, \frac{3}{4}, 1, \dots$
求數列 $\frac{1}{2}, \frac{3}{4}, 1, \dots$ 的第十三項。

(c) Find the nth term of 1, 3, 5
求數列 1, 3, 5 的第 n 項。

(d) Find the 23rd term of -7, -3, 1
求數列 -7, -3, 1 的第 23 項。
- The 5th term of an arithmetic sequences is -5 and the 11th term is -23, find the 30th term and the sum of 30 terms.
一等差數列的第五項是 -5 及第十一項是 -23。求該數列的第三十項及首三十項之和。
- The 7th term of an arithmetic sequence is 30 and the 13th is 42. Find the 1st term and the sum of 12 terms.
一等差數列的第七項是 30 及第十三項是 42。求該數列的首項及首十二項之和。
- Find the sequence in which the 5th and the 21st terms are $7x - 8y$ and $23x - 40y$.
一等差數列的第五項及第 21 項分別為 $7x - 8y$ 及 $23x - 40y$ 。寫出該數列的首三項。
- Insert 5 arithmetic means between 11 and 37.
試在 11 和 37 之間插入五個數，使這七個數成為等差數列。
- Insert 5 arithmetic means between $x + y$ and $x - y$.
試在 $x + y$ 和 $x - y$ 之間插入五個數，使這七個數成為等差數列。
- Find the sum of:
求下列各數列之總和

 - 12, 9, 6, to 25 terms.
12, 9, 6, 共 25 項。
 - 4.9, 5.6, 6.3, to 20 terms.
4.9, 5.6, 6.3, 共 20 項。
 - $\frac{1}{2}, \frac{1}{10}, \frac{9}{10}, \dots$ to 20 terms.
 $\frac{1}{2}, \frac{1}{10}, \frac{9}{10}, \dots$ 共 20 項。
- How many terms of the series $15 + 13 + 11 + \dots$ so that the sum may be 55?
若等差級數 $15 + 13 + 11 + \dots$ 首 n 項之和為 55，求 n 的值。
- How many terms must be taken of the series $(5x - y) + (3x - 2y) + (x - 3y) + \dots$ to make $(-280x - 210y)$?
若等差級數 $(5x - y) + (3x - 2y) + (x - 3y) + \dots$ 首 n 項之和為 $(-280x - 210y)$ ，求 n 的值。
- Find the arithmetic series, if $S_{10} = 465$, $9S_3 = 4S_6$.
若 $S_{10} = 465$, $9S_3 = 4S_6$ ；求該等差級數。
- The sum of the first 4 terms of an arithmetic series is 10 and the sum of the 4 terms from the 7th to the 10th inclusive is 42. Find the sum of the first 10 terms.
一等差級數首四項之和為 10 及由第七項至第十項之和為 42，求該級數首十項之和。
- Find the arithmetic series in which the first 10 terms together is 100 and the next 10 terms is 300.
一等差級數首十項之和為 100 及由第十一項至第二十項之和為 300。寫出該級數的首三項。
- Find the sum of n terms whose nth term is $4n - 5$.
一等差數列第 n 項是 $4n - 5$ ，求首 n 項之和。

14. In the sequence whose n th term is $2n + 3$, find the ratio of $(n - 1)$ th term to the $(n + 1)$ th term.
一等差數列第 n 項是 $2n + 3$ ，求第 $n - 1$ 項與第 $n + 1$ 項之比。
15. Sum the series to 20 terms $3(2a+3b) + 3(3a+2b) + 3(4a+b) + \dots$
求級數 $3(2a+3b) + 3(3a+2b) + 3(4a+b) + \dots$ 首 20 項之和。
16. An arithmetic series consists of 21 terms, the sum of the three terms in the middle is 129, and the sum of the last three terms is 237, find the series.
一等差級數共 21 項。而中間三項之和是 129，最後三項之和是 237；寫出該級數的首三項。
17. 4 numbers are in arithmetic sequence. The product of the first and last is less than the product of the other two by 72, and the product of the second and fourth exceeds the product of the other two by 120. Find the numbers.
一等差數列共 4 項。第一項及第四項之積少於其餘兩項之積 72，第二項及第四項之積多於其餘兩項之積 120；求該數列。
18. A man saves \$100 more than he saved in the preceding year, and he saves \$500 the first year. How many years would it take for his saving, excluding interest, to amount \$9500?
一男子每年的存款金額比往年多 \$100，而第一年存入 \$500。問多少年後他的存款(不包括利息)共 \$9500。
19. How long will it take to pay a debt of \$2400 by weekly payments increasing by \$6 per week and beginning with \$24.
若某人每星期的還款比上星期增加 \$6，而第一星期還 \$24；求多少星期才可還清 \$2400 欠款。
20. A travels 2 km the first hour, $2\frac{1}{4}$ the second, $2\frac{1}{2}$ the third and so on. B travels 4 km an hour. When and where does A overtake B if they start together?
A 與 B 一起步行。A 於首小時行 2 km，第二小時行 $2\frac{1}{4}$ ，第三小時行 $2\frac{1}{2}$ ，如此類推。而 B 每小時行 4 km。求何時及於何地 A 超越 B。
21. A and B set out to meet each other from two places 330 km apart. A travels 30 km the first day, 28 the second, 26 the third and so on. The other travels 20 km the first day, 24 the second, 28 the third and so on. When will they meet?
A、B 位於相距 330 km 的兩地。A 於首天行 30 km，第二天行 28，第三天行 26，如此類推。而 B 於首天行 20 km，第二天行 24，第三天行 28，如此類推。求他們於何時相遇。

Geometric Sequences & Series, and Sum to Infinity 等比數列與級數及無限項之和

22. Find the 8th term and the sum of the first 8 terms of the geometric sequence 4, 12, 36,
- 求數列 4, 12, 36, 的第八項及首八項之和。
23. Find the common ratio, the 6th term and the n th term of
- 求下列各級數的公比，第六項及通項
- (a) $3 + 9 + 27 + \dots$
- (b) $3 - \frac{3}{2} + \frac{3}{4} - \frac{3}{8} + \dots$
- (c) $1 - \frac{1}{2^2} + \frac{1}{2^4} - \dots$
24. The 3rd term of a geometric sequence is 36 and the 5th term is 16. Find the 10th term.
一等比數列的第三項是 36 及第五項是 16，求該數列的第十項。

25. Sum the following
求下列數列的總和
- (a) $-4, 8, -16, \dots$ to 9 terms.
 $-4, 8, -16, \dots$ 共 9 項。
- (b) $\frac{1}{2}, -1\frac{1}{2}, 4\frac{1}{2}, \dots$ to 8 terms.
 $\frac{1}{2}, -1\frac{1}{2}, 4\frac{1}{2}, \dots$ 共 8 項。
- (c) $-3, -6, -12, \dots$ to 9 terms.
 $-3, -6, -12, \dots$ 共 9 項。
26. (a) Insert 4 geometric means between 32 and 1.
試在 32 和 1 之間插入四個數，使這六個數成為等比數列。
- (b) Insert 5 geometric means between 6 and 384.
試在 6 和 384 之間插入五個數，使這七個數成為等比數列。
- (c) Insert 5 geometric means between 8 and $\frac{1}{8}$.
試在 8 和 $\frac{1}{8}$ 之間插入五個數，使這七個數成為等比數列。
27. Find the geometric means between
求下列各題的等比中項
- (a) 3 and 147.
3 和 147。
- (b) $\frac{a+b}{a-b}$ and $\frac{a-b}{a+b}$.
 $\frac{a+b}{a-b}$ 和 $\frac{a-b}{a+b}$ 。
- (c) $x^2 - 6ax + 9a^2$ and $9x^2 + 6ax + a^2$.
 $x^2 - 6ax + 9a^2$ 和 $9x^2 + 6ax + a^2$ 。
28. The sum of the first 3 terms of a geometric sequence is 336 and the sum of the 5th, 6th and 7th terms is 21. Find the two possible values of the 3rd term.
一等比數列首三項之和是 336 及第五項至第七項之和是 21。求該數列的第三項的兩個可能值。
29. Find the sum of the series to n terms $2 + 4 + 10 + 28 + \dots$
求級數 $2 + 4 + 10 + 28 + \dots$ 首 n 項之和。
30. Find the sum of the series $(a + \frac{1}{3}) + (3a - \frac{1}{6}) + (5a + \frac{1}{12}) + \dots$ to p terms.
求級數 $(a + \frac{1}{3}) + (3a - \frac{1}{6}) + (5a + \frac{1}{12}) + \dots$ 首 p 項之和。
31. Find the sum of the series to n terms $(a+p) + (2a+px) + (3a + px^2) + \dots$
求級數 $(a+p) + (2a+px) + (3a + px^2) + \dots$ 首 n 項之和。
32. Find the sum to r terms the series whose rth term is $4^{2r} - 3(r+2)$.
若數列的 r 項為 $4^{2r} - 3(r+2)$ ，求首 r 項之和。
33. Sum to infinite geometric series
求下列等比級數無限項之和
- (a) $18 + 12 + 8 + \dots$
- (b) $0.6 + 0.06 + 0.006 + \dots$
- (d) $5.6 - 2.24 + 0.896 - \dots$
34. In a certain colony of bacteria each divides into two every hour. How many will be produced from a single bacillus if the rate of division continues for 12 hours?
某一種細菌每小時分裂二次。若開始時只有一個細菌，問 12 小時後共有多少細菌。

35. Find the value in vulgar fractions
以分數表示下列循環小數

(a) $0.\dot{8}$

(b) $0.12\dot{3}$

(c) $0.24\dot{1}\dot{2}$

36. A ball is dropped from a height of 50 metres rebounds to a height of 30 metres, and continues to fall and rebound, rising after each rebound to three-fifths of the height it previously fell from. Through what distance will it move before it comes to rest on the ground?

一皮球從 50 m 高處落下，著地後反彈回 30 m 高；並繼續此運動直至停下來。若每次反彈的高度為之前下跌的高度的五分四，求皮球於停下前共行了多少距離。

37. A swinging pendulum bob tranverses the following distances, 40, 30, $22\frac{1}{2}$cm.

Find the distance which it travels before coming to rest.

一單擺移動的距離為 40, 30, $22\frac{1}{2}$ cm。求它停頓前共行了多少距離。

Numerical Answers 數值答案

1. (a) 47 (b) $3\frac{1}{2}$ (c) $2n - 1$ (d) 81

3. 18, 1410

5. $15\frac{1}{3}$, $19\frac{2}{3}$, 24, $28\frac{1}{3}$, $32\frac{2}{3}$

7. (a) -600 (b) 231 (c) 48

9. 20

11. 65

13. $n(2n - 3)$

15. $690a - 390b$

17. 1, 7, 13, 17 ... or -1, -7, -13, -17

19. 25 weeks

21. 6 days

23. (a) 3 ; 729 ; 3^n (b) $-\frac{1}{2}$; $-\frac{3}{2^5}$; $3x(-\frac{1}{2})^{n-1}$ or $(-1)^{n-1}\frac{3}{2^{n-1}}$

(c) $-\frac{1}{2^2}$ or $-\frac{1}{4}$; $-\frac{1}{2^{10}}$; $-\frac{1}{2^{2(n-1)}}$ or $\frac{(-1)^{n-1}}{2^{2n-2}}$

24. $\pm\frac{512}{243}$

25. (a) -684 (b) -820 (c) -1533

26. (a) 16, 8, 4, 2 (b) 12, 24, 48, 96, 192 (c) -4, 2, -1, $\frac{1}{2}$, $-\frac{1}{4}$ or 4, 2, 1, $\frac{1}{2}$, $\frac{1}{4}$

27. (a) ± 21 (b) ± 1 (c) $\pm(3x^2 - 8ax - 3a^2)$

28. 192, 448

29. $n + \frac{3^n - 1}{2}$

30. $ap^2 + \frac{2}{9}[1 - (-\frac{1}{2})^p]$

31. $\frac{an}{2}(1+n) + \frac{p(x^n - 1)}{x - 1}$

32. $\frac{16(2^{4n} - 1)}{15} - \frac{3n}{2}(n+5)$

33. (a) 54 (b) $\frac{2}{3}$ (c) 4

34. 4096

35. (a) $\frac{8}{9}$ (b) $\frac{61}{495}$ (c) $\frac{199}{825}$

36. 160 m

37. $32(2 + \sqrt{2})$ cm