Ch.5 三角函數解 Trigonometric Function

A1 $\because$ OA $\perp$ OD

$\therefore$ $\tan\angle OAD = \dfrac{OD}{OA} = \dfrac{5}{5\sqrt{3}} = \dfrac{1}{\sqrt{3}}$

$\therefore$ $\angle OAD = \dfrac{\pi}{6}$

$\because$ OC $\perp$ AD

$\therefore$ $\angle AOB = \dfrac{\pi}{3}$ 及 $\angle COD = \dfrac{\pi}{6}$

OABCD 的周界 $= OA + \overset{\frown}{AB} + BC + \overset{\frown}{CD} + OD$

$= 5\sqrt{3} + 5\sqrt{3}\left(\dfrac{\pi}{3}\right) + (5\sqrt{3} - 5) + 5\left(\dfrac{\pi}{6}\right) + 5$

$= 10\sqrt{3} + \dfrac{5\sqrt{3}}{3}\pi + \dfrac{5\pi}{6}$

$= 29.0$ cm (準確至 0.1 cm)

A2 $\cos\theta = -\dfrac{4}{5} < 0,\quad \sin\theta < 0$

$\therefore$ θ 在第三象限。

$\therefore$ $\sin\theta = -\dfrac{3}{5},\quad \csc\theta = -\dfrac{5}{3},\quad \tan\theta = \dfrac{3}{4}$

$\tan\theta - \csc\theta = -\dfrac{3}{4} - \left(-\dfrac{5}{3}\right)$

$= \dfrac{29}{12}$

A3 $\dfrac{\sec(-\alpha)+\sin(-\frac{\pi}{2}-\alpha)}{\csc(3\pi-\alpha)-\cos(\frac{3\pi}{2}+\alpha)} = \dfrac{\sec\alpha-\sin(\frac{\pi}{2}+\alpha)}{\csc(\pi-\alpha)-\sin\alpha}$

$= \dfrac{\sec\alpha-\cos\alpha}{\csc\alpha-\sin\alpha}$

$= \dfrac{\frac{1-\cos^2\alpha}{\cos\alpha}}{\frac{1-\sin^2\alpha}{\sin\alpha}}$

$= \tan^3\alpha$

A4 $\because$ $\sin\theta + \cos\theta = \dfrac{3}{2}$

$\therefore$ $\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta = \dfrac{9}{4}$

$\sin\theta\cos\theta = \dfrac{5}{8}$

$\sin^2\theta\cos^2\theta = \dfrac{25}{64}$

$\sin^4\theta + \cos^4\theta = (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta$

$= 1 - 2\times\dfrac{25}{64}$

$= \dfrac{7}{32}$

A5 左方 $= \sqrt{\dfrac{(1-\cos A)^2}{(1+\cos A)(1-\cos A)}}$

$= \sqrt{\dfrac{(1-\cos A)^2}{\sin^2 A}}$

$= \dfrac{1-\cos A}{\sin A}$

$= \dfrac{1}{\sin A} - \dfrac{\cos A}{\sin A}$

$= \csc A - \cot A$

= 右方

A6

$5\cos^2 x + 3\sin^2 x = 7\cos x$

$5\cos^2 x + 3(1-\cos^2 x) - 7\cos x = 0$

$2\cos^2 x - 7\cos x + 3 = 0$

$(2\cos x - 1)(\cos x - 3) = 0$

$\therefore$ $\cos x = \dfrac{1}{2}$ 或 3 (捨去)

$\therefore$ $x = 60°, 300°$

A7 $|\cos x| - |\sin x| = \sin x - \cos x$

情況一：當 $0 \le x \le \dfrac{\pi}{2}$

方程為

$\cos x - \sin x = \sin x - \cos x$

$\therefore$ $\tan x = 1$

$x = \dfrac{\pi}{4}$

情況二：當 $\dfrac{\pi}{2} < x \le \pi$

方程為

$-\cos x - \sin x = \sin x - \cos x$

$\sin x = 0$

$\therefore$ $x = \pi$

由此，方程的解為 $x = \dfrac{\pi}{4}$ 或 π。

A8 在 ΔABC 中，

$\dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$\therefore$ $\sin C = \dfrac{c\sin B}{b}$

$= \dfrac{6\sin 30°}{2\sqrt{3}}$

$\angle C = 60°$ 或 $\angle C = 120°$

$\therefore$ $\angle A = 90°$ 或 $\angle A = 30°$

當 $\angle A = 90°$, $a^2 = b^2 + c^2$

$= (2\sqrt{3})^2 + 6^2$

$a = 4\sqrt{3}$

當 $\angle A = 30°$, $a = b = 2\sqrt{3}$

答案：$\angle A = 90°,\ \angle C = 60°,\ a = 4\sqrt{3}$

或 $\angle A = 30°,\ \angle C = 120°,\ a = 2\sqrt{3}$。

A9 $a = 7 = \sqrt{49}$

$b = 4\sqrt{3} = \sqrt{48}$

$c = \sqrt{13}$

$\therefore$ $c < b < a$

$\therefore$ $\angle C < \angle B < \angle A$

$\cos C = \dfrac{a^2 + b^2 + c^2}{2ab}$

$= \dfrac{7^2 + (4\sqrt{3})^2 - (\sqrt{13})^2}{2(7)(4\sqrt{3})}$

$\therefore$ $\angle C = 30°$

A10 $\because$ $\angle A : \angle B : \angle C = 5 : 10 : 21$

設 $\angle A = 5k,\ \angle B = 10k,\ \angle C = 21k$, 其中 k 為非零常數。

$\therefore$ $5k + 10k + 21k = 180°$
$k = 5°$
$\therefore$ $\angle A = 25°$, $\angle B = 50°$, $\angle C = 105°$
$\because$ $\angle C > \angle B > \angle A$
$\therefore$ $c > b > a$
根據正弦公式，

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$c = \frac{6\sin 105°}{\sin 25°}$$
$$= 13.71 \text{ cm} \quad (\text{二位小數})$$

A11 依題意，

$$(a + b + c)(b + c - a) = 3bc$$
$$(b + c)^2 - a^2 = 3bc$$
$$b^2 + 2bc + c^2 - a^2 = 3bc$$
$$\therefore \quad b^2 + c^2 - a^2 = bc$$
$$\therefore \quad \cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{bc}{2bc} = \frac{1}{2}$$
$$\therefore \quad \angle A = 60°$$

A12 左方

$= (b + c) \cos A + (c + a) \cos B + (a + b) \cos C$
$= (b \cos A + a \cos B) + (c \cos A + a \cos C) + (c \cos B + b \cos C)$

$$= (b \cdot \frac{b^2 + c^2 - a^2}{2bc} + a \cdot \frac{a^2 + c^2 - b^2}{2ac}) + (c \cdot \frac{b^2 + c^2 - a^2}{2bc} + a \cdot \frac{a^2 + b^2 - c^2}{2ab}) + (c \cdot \frac{a^2 + c^2 - b^2}{2ac} + b \cdot \frac{a^2 + b^2 - c^2}{2ab})$$

$$= \frac{b^2 + c^2 - a^2 + a^2 + c^2 - b^2}{2c} + \frac{b^2 + c^2 - a^2 + a^2 + b^2 - c^2}{2b} + \frac{a^2 + c^2 - b^2 + a^2 + b^2 - c^2}{2a}$$

$= c + b + a$
$=$ 右方

A13 設 AE = x, BE = y。
在 ΔAEB 中，
$AB^2 = AE^2 + BE^2 - 2(AE)(BE) \cos \angle AEB$
即 $2^2 = x^2 + y^2 - 2xy \cos 45°$
$\therefore$ $4 = x^2 + y^2 - \sqrt{2}\,xy$ . . . . (1)
在 ΔBEC 中，
$4^2 = x^2 + y^2 - 2xy \cos (180° - 45°)$
$\therefore$ $16 = x^2 + y^2 + \sqrt{2}\,xy$ . . . . (2)
(2) – (1),
$12 = 2\sqrt{2}\,xy$
$\therefore$ $xy = 3\sqrt{2}$
ABCD 的面積 = $4 \times \Delta$AEB 的面積
$= 4 (\frac{1}{2} xy \sin 45°)$
$= 6$

A14

$$\tan C = \tan [180° - (A + B)]$$
$$= \tan (A + B)$$
$$= -\frac{\tan A + \tan B}{1 - \tan A \tan B}$$
$$= -\frac{2 + 3}{1 - 2 \times 3}$$
$$= 1$$
$$\therefore \quad \angle C = 45°$$

A15 $\because$ $\tan \alpha = 1 - \sqrt{2}$

$$\sin 2\alpha = \frac{2 \tan \alpha}{1 + \tan^2 \alpha} = \frac{2(1 - \sqrt{2})}{1 + (1 - \sqrt{2})^2} = -\frac{1}{\sqrt{2}}$$

$$\cos 2\alpha = \frac{1 - \tan^2 \alpha}{1 + \tan^2 \alpha} = \frac{1 - (1 - \sqrt{2})^2}{1 + (1 - \sqrt{2})^2} = \frac{1}{\sqrt{2}}$$

$$\tan 2\alpha = \frac{\sin 2\alpha}{\cos 2\alpha}$$
$$= \mathbf{-1}$$

A16
左方 $= \dfrac{2 \sin \theta - \sin 2\theta}{2 \sin \theta + \sin 2\theta}$

$$= \frac{2 \sin \theta - 2 \sin \theta \cos \theta}{2 \sin \theta + 2 \sin \theta \cos \theta}$$
$$= \frac{1 - \cos \theta}{1 + \cos \theta}$$
$$= \frac{2 \sin^2 \frac{\theta}{2}}{2 \cos^2 \frac{\theta}{2}}$$
$$= \tan^2 \frac{\theta}{2}$$
$=$ 右方

A17 $\cos^2 (\theta + 15°) + \cos^2 (\theta - 15°) - \dfrac{\sqrt{3}}{2} \cos 2\theta$

$$= \frac{1 + \cos(2\theta + 30°)}{2} + \frac{1 + \cos(2\theta - 30°)}{2} - \frac{\sqrt{3}}{2} \cos 2\theta$$
$$= 1 + \frac{1}{2} [\cos(2\theta + 30°) + \cos(2\theta - 30°)] - \frac{\sqrt{3}}{2} \cos 2\theta$$
$$= 1 + \cos 2\theta \cos 30° - \frac{\sqrt{3}}{2} \cos 2\theta$$
$= 1$

A18

$$\sqrt{3} \sin \theta - \cos \theta = \sqrt{2}$$
$$2(\frac{\sqrt{3}}{2} \sin \theta - \frac{1}{2} \cos \theta) = \sqrt{2}$$
$$\sin(\theta - 30°) = \frac{\sqrt{2}}{2}$$
$$\therefore \quad \theta - 30° = 180n° + (-1)^n \cdot 45°$$
$$\theta = 180n° + (-1)^n \cdot 45° + 30°,$$
其中 n 為任意整數。

A19

$$\cos x - \cos 3x = \sin 2x$$
$$2 \sin 2x \sin x - \sin 2x = 0$$
$$\sin 2x(2 \sin x - 1) = 0$$

$\therefore\quad \sin 2x = 0 \quad$ 或 $\quad \sin x = \frac{1}{2}$

$2x = n\pi \quad$ 或 $\quad x = n\pi + (-1)^n \cdot \frac{\pi}{6}$,

即 $x = \frac{n\pi}{2} \quad$ 或 $\quad n\pi + (-1)^n \cdot \frac{\pi}{6}$,

其中 n 為任意整數。

A20

$$\begin{aligned} \cos x + \cos 2x + \cos 3x &= 0 \\ (\cos x + \cos 3x) + \cos 2x &= 0 \\ 2\cos\frac{x+3x}{2}\cos\frac{x-3x}{2} + \cos 2x &= 0 \\ \cos 2x(2\cos x + 1) &= 0 \end{aligned}$$

$\therefore\quad \cos 2x = 0 \quad$ 或 $\quad \cos x = -\frac{1}{2}$

$2x = 2n\pi \pm \frac{\pi}{2} \quad$ 或 $\quad x = 2n\pi \pm \frac{2\pi}{3}$

$\therefore\quad x = n\pi \pm \frac{\pi}{4} \quad$ 或 $\quad 2n\pi \pm \frac{2\pi}{3}$,

其中 n 為任意整數。

A21 **(a)**

$\because\quad t = \tan\frac{\pi}{12}$

$\tan\frac{\pi}{6} = \frac{2t}{1-t^2}$

$\therefore\quad t^2 + 2\sqrt{3}\,t - 1 = 0$

(b) $t^2 + 2\sqrt{3}\,t - 1 = 0$

$$t = \frac{-2\sqrt{3} \pm \sqrt{(2\sqrt{3})^2 - 4(-1)}}{2} = -\sqrt{3} \pm 2$$

$\because\quad \tan\frac{\pi}{12} > 0$

$\therefore\quad \tan\frac{\pi}{12} = 2 - \sqrt{3}$

A22 **(a)** 依題意,

$$\begin{cases} \tan\alpha + \tan\beta = \frac{5}{6} \\ \tan\alpha\tan\beta = \frac{1}{6} \end{cases}$$

$\therefore\quad \tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = 1$

(b) 若 α、β 皆為銳角, 則 $0 < \alpha + \beta < 180°$。

而 $\tan(\alpha+\beta) = 1$

$\therefore\quad \alpha + \beta = \frac{\pi}{4}$

A23

$$\begin{aligned} \tan A + \tan B &= \frac{5}{3} \\ \tan A \tan B &= \frac{1}{3} \\ \tan(A+B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{5}{2} \end{aligned}$$

A24

$$\begin{aligned} &2\sin^2\frac{\theta}{2}\left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right)^2 \\ &= (1-\cos\theta)\left(\cos^2\frac{\theta}{2} - 2\cos\frac{\theta}{2}\sin\frac{\theta}{2} + \sin^2\frac{\theta}{2}\right) \\ &= (1-\cos\theta)(1-\sin\theta) \\ &= 1 - (\sin\theta + \cos\theta) + \sin\theta\cos\theta \\ &= 1 + p + q \end{aligned}$$

A25

$$\begin{aligned} \cos x\cos 2x &= \cos 3x\cos 4x \\ \frac{1}{2}(\cos 3x + \cos x) &= \frac{1}{2}(\cos 7x + \cos x) \\ \therefore\quad \cos 7x - \cos 3x &= 0 \\ -2\sin 5x\sin 2x &= 0 \end{aligned}$$

$\sin 5x = 0 \quad$ 或 $\quad \sin 2x = 0$

$5x = n\pi \quad$ 或 $\quad 2x = n\pi$

$\therefore\quad x = \frac{n\pi}{5} \quad$ 或 $\quad \frac{n\pi}{2}$,

其中 n 為整數。

A26

$$\begin{aligned} \sin 3\theta + 4\cos^2\theta - 4\sin\theta - 5 &= 0 \\ 3\sin\theta - 4\sin^3\theta + 4 - 4\sin^2\theta - 4\sin\theta - 5 &= 0 \\ 4\sin^3\theta + 4\sin^2\theta + \sin\theta + 1 &= 0 \\ 4\sin^2\theta(\sin\theta + 1) + (\sin\theta + 1) &= 0 \\ (\sin\theta + 1)(4\sin^2\theta + 1) &= 0 \end{aligned}$$

$\therefore\quad \sin\theta = -1 \quad$ 或 $\quad 4\sin^2\theta = -1$ (捨去)

$\therefore\quad \theta = n\pi + (-1)^{n+1}\frac{\pi}{2}$, 其中 n 為任意整數。

A27 **(a)** $s = a\cos\theta + b\sin\theta = r\cos(\theta - \alpha)$

$\therefore\quad a\cos\theta + b\sin\theta = r\cos\theta\cos\alpha - r\sin\theta\sin\alpha$

比較等式兩邊得

$$\begin{cases} r\cos\alpha = a \\ r\sin\alpha = b \end{cases}$$

$\therefore\quad r = \sqrt{a^2 + b^2},\quad \tan\alpha = \frac{b}{a}$

(b) $\because\quad s = r\cos(\theta - \alpha)$

$\therefore$ 當 $\cos(\theta - \alpha) = 1$, 即 $\theta = \alpha$ 時, s 有最大值 r。

依題意, 當 $\alpha = \theta = \frac{\pi}{3}$ 時, s 有最大值 4。

$$\therefore\quad \begin{cases} \sqrt{a^2 + b^2} = 4 \\ \frac{b}{a} = \tan\frac{\pi}{3} = \sqrt{3} \end{cases}$$

解之得: $a = 2$, $b = 2\sqrt{3}$。

A28 **(a)**

$$\begin{aligned} \because\quad \cot\alpha + \cot\beta &= \frac{1}{\tan\alpha} + \frac{1}{\tan\beta} \\ &= \frac{\tan\beta + \tan\alpha}{\tan\alpha\tan\beta} \\ 4 &= \frac{2}{\tan\alpha\tan\beta} \end{aligned}$$

因此, $\tan\alpha\tan\beta = \frac{1}{2}$。

$$\begin{aligned} \text{則}\quad \tan(\alpha+\beta) &= \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} \\ &= \frac{2}{1 - \frac{1}{2}} \\ &= 4 \end{aligned}$$

(b)

$$\begin{aligned} \cos(\alpha+\beta) &= \frac{1}{\sec(\alpha+\beta)} \\ &= \frac{1}{-\sqrt{1 + \tan^2(\alpha+\beta)}} \end{aligned}$$

($\because\ -\frac{\pi}{2} < \alpha + \beta < \frac{3\pi}{2}$, 故 $\cos(\alpha+\beta) < 0$。)

$$= \frac{1}{-\sqrt{1 + 4^2}} = -\frac{\sqrt{17}}{17}$$

B1 **(a)**

$f(x) = 2\sin^2 x - 3\cos x + 5$

$= 2(1-\cos^2 x) - 3\cos x + 5$

$= -2\cos^2 x - 3\cos x + 7$

(b)

$f(x) = -2(\cos^2 x + \frac{3}{2}\cos x - \frac{7}{2})$

$= -2[(\cos x + \frac{3}{4})^2 - \frac{9}{16} - \frac{7}{2}]$

$= -2(\cos x + \frac{3}{4})^2 + \frac{65}{8}$

$\because (\cos x + \frac{3}{4})^2 \ge 0$ 而 $-1 \le \cos x \le 1$,

$\therefore$ 當 $\cos x = -\frac{3}{4}$ 時, $f(x)$ 有最大值 $\frac{65}{8}$。

當 $\cos x = 1$ 時, $f(x)$ 有最小值 2。

B2 **(a)**

$\because \sin 2\theta = 2\sin\theta\cos\theta$

而 $-1 \le \sin 2\theta \le 1$

$\therefore -1 \le 2\sin\theta\cos\theta \le 1$

故 $\sin\theta\cos\theta$ 的值域為 $[-\frac{1}{2}, \frac{1}{2}]$。

(b)

$\because \sin\theta + \cos\theta = \sin\theta\cos\theta$

$\therefore \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta = (\sin\theta\cos\theta)^2$

即 $(\sin\theta\cos\theta)^2 - 2(\sin\theta\cos\theta) - 1 = 0$

$\therefore \sin\theta\cos\theta = \frac{2 \pm \sqrt{4+4}}{2}$

$= 1 \pm \sqrt{2}$

由 **(a)** 的結果, $\sin\theta\cos\theta = 1 - \sqrt{2}$。

B3 **(a)**

$$\begin{cases} \sin\alpha + \cos\alpha = \frac{1}{5} \ . \ . \ . \ . \ . \ . \ . \ . \ (1) \\ \sin\alpha + \cos\alpha = \frac{k}{25} \ . \ . \ . \ . \ . \ . \ . \ . (2) \end{cases}$$

由 (1),

$(\sin\alpha + \cos\alpha)^2 = \frac{1}{25}$

$\therefore \sin\alpha \ \cos\alpha = -\frac{12}{25}$

$\therefore \frac{k}{25} = -\frac{12}{25}$

$k = -12$

(b)

$\because k = -12$

$25x^2 - 5x - 12 = 0$

$x = -\frac{3}{5}$ 或 $\frac{4}{5}$

$\because \frac{3\pi}{2} \le \alpha \le 2\pi$

$\therefore \sin\alpha = -\frac{3}{5}$ 及 $\cos\alpha = \frac{4}{5}$

$\frac{\sin\alpha + \cos\alpha}{\sin\alpha - \cos\alpha} = \frac{-\frac{3}{5} + \frac{4}{5}}{-\frac{3}{5} - \frac{4}{5}}$

$= -\frac{1}{7}$

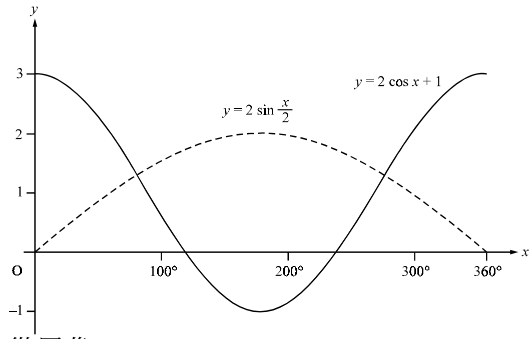
(c)

$\because \sin\alpha = -\frac{3}{5}$, $\cos\alpha = \frac{4}{5}$ 及

$\frac{3\pi}{2} \le \alpha \le 2\pi$

$\therefore \alpha = 5.64$ (三位有效數字)

B4 **(a)**



(b) 從圖像,

$x = 81°$ 或 $279°$。

B5 **(a)** $\because \Delta$VAB 為等腰三角形, 而 $\angle$VAB = 70°。

$\therefore \angle AVB = 180° - 2 \times 70°$

$= 40°$

在 ΔATV 中,

AT = VA sin $\angle$AVT

= 10 sin 40°

= 6.428

= 6.43 cm (三位有效數字)

在 ΔABV 中,

AB = 2VA cos $\angle$VAB

= 2 × 10 cos 70°

= 6.840

= 6.84 cm

$\because$ ABCDEF 為正六邊形,

$\therefore \angle$ABC = 120°

由此, AC = 2AB sin 60°

$= 2 \times 6.840 \times \frac{\sqrt{3}}{2}$

= 11.848

= 11.8 cm (三位有效數字)

(b) 連結 CT, 則$\angle$ATC 為面 VAB 與面 VBC 的夾角。

在 ΔATC 中,

$\sin\frac{\angle ATC}{2} = \frac{\frac{1}{2}AC}{AT}$

$= \frac{\frac{1}{2} \times 11.848}{6.428}$

$\frac{\angle ATC}{2} = 67.16°$

$\therefore \angle ATC = 134.32°$

故所求兩面之夾角為 134° (三位有效數字)

B6 **(a)** 在 ΔBPC 中,

BP = $a \sec 30°$

$= \frac{2}{\sqrt{3}}a$

PC = $a \tan 30°$

$= \frac{1}{\sqrt{3}}a$

(b) 在 ΔABC 中,

AB = $a \sin 30°$

$= \frac{1}{2}a$

AC = $a \cos 30°$

$= \frac{\sqrt{3}}{2}a$

在 ΔACP 中,

$\angle ACP = 90°$

$AP^2 = AC^2 + CP^2$

$= \left(\frac{\sqrt{3}}{2}a\right)^2 + \left(\frac{1}{\sqrt{3}}a\right)^2$

$\therefore \quad AP = \sqrt{\frac{13}{12}}a$

在 ΔAPB 中,

$\cos\angle APB = \frac{AP^2 + BP^2 - AB^2}{2(AP)(BP)}$

$= \frac{\frac{13}{12}a^2 + \frac{4}{3}a^2 - \frac{1}{4}a^2}{2\left(\sqrt{\frac{13}{12}}a\right)\left(\frac{2}{\sqrt{3}}a\right)}$

$\angle APB = 25.7°$ (準確至最接近的 0.1°)

(c)

$\tan\angle PAC = \frac{\frac{1}{\sqrt{3}}a}{\frac{\sqrt{3}}{2}a}$

$\angle PAC = 33.7°$ (準確至最接近的 0.1°)

$\therefore$ 所求的角 $= 33.7°$

B7 **(a)** 在 ΔBCD 中,

$\angle BDC = 180° - 45° - 60° = 75°$

$\frac{BD}{\sin 60°} = \frac{400}{\sin 75°}$

$\therefore \quad BD = \frac{400\sin 60°}{\sin 75°}$

在 ΔABD 中,

$AD = BD\tan 30°$

$= \frac{400\sin 60°\tan 30°}{\sin 75°}$

$= 207.06$

$\therefore$ 氣球的高度 $= 207$ m (準確至最接近的 m)

(b) 在 ΔBCD 中,

$\frac{CD}{\sin 45°} = \frac{400}{\sin 75°}$

$\therefore \quad CD = \frac{400\sin 45°}{\sin 75°}$

$= 292.82$

在 ΔACD 中,

$\tan\angle ACD = \frac{AD}{CD}$

$= \frac{207.06}{292.82}$

$= 207.06$

$\angle ACD = 35.3°$ (準確至最接近的 0.1°)

$\therefore$所求的角 $= 35.3°$

B8 **(a)** 在 ΔCEF 中,

$CF = \frac{1}{3}CB = 1,\quad CE = \frac{1}{3}CA = 1,\quad \angle ACB = 60°$。

因此, ΔCEF 為等邊三角形, 故 $EF = 1$。

在 ΔVCF 中,

$VC = 3,\quad CF = 1,\quad \angle VCB = 60°$。

$\therefore \quad VF^2 = CF^2 + CV^2 - 2CF\cdot CV\cdot\cos\angle VCF$

$= 1^2 + 3^2 - 2\cdot 1\cdot 3\cdot\cos 60°$

$VF = \sqrt{7}$

同理, $VE = \sqrt{7}$。

在 ΔEVF 中,

$\cos\angle EVF = \frac{VE^2 + VF^2 - EF^2}{2VE\cdot VF}$ 或

$\sin\frac{\angle EVF}{2} = \frac{ET}{VE}$

$= \frac{2\times 7 - 1}{2\times 7}$

$= \frac{13}{14}$

$\therefore \quad \angle EVF = 21.8°$

(b) 連結 VT, 則$\angle VTO$ 為平面 EVF 與平面 ABC 的交角。

$\because$ O 為底 ABC 的形心,

$CO = \frac{2}{3}(\frac{\sqrt{3}}{2}\cdot 3)$

$= \sqrt{3}$

$\therefore$ 在 ΔVOC 中,

$VO = \sqrt{VC^2 - CO^2}$

$= \sqrt{3^2 - (\sqrt{3})^2}$

$= \sqrt{6}$

$TO = CO - CT$

$= \sqrt{3} - \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{2}$

在 ΔVOT 中,

$\tan\angle VTO = \frac{VO}{TO}$

$= \frac{\sqrt{6}}{\frac{\sqrt{3}}{2}}$

$= 2\sqrt{2}$

$\angle VTO = 70.5°$

即平面 EVF 與平面 ABC 的交角為 70.5°。

B9 **(a)** 在 ΔADF 中,

$AF = a\sec\angle DAF$

$= a\sec(90° - 2\alpha)$

$= a\csc 2\alpha$

$DF = a\tan\angle DAF$

$= a\tan(90° - 2\alpha)$

$= a\cot 2\alpha$

(b) 依題意,

$AF = BC + CF = a + CF$

$DF = DC - CF = a - CF$

$\therefore \quad AF + DF = 2a$

則 $a\csc 2\alpha + a\cot 2\alpha = 2a$

$\therefore \quad \csc 2\alpha + \cot 2\alpha = 2$

$\frac{1}{\sin 2\alpha} + \frac{\cos 2\alpha}{\sin 2\alpha} = 2$

$\frac{2\cos^2\alpha}{2\sin\alpha\cos\alpha} = 2$

$\cot\alpha = 2$

$\therefore \quad \tan\alpha = \frac{1}{2}$

(c) 在 ΔABE 中，

$$\tan\alpha = \frac{BE}{AB}$$

$$\therefore \quad \frac{1}{2} = \frac{BE}{a}$$

$$BE = \frac{1}{2}a$$

$$\therefore \quad CE = a - \frac{1}{2}a = \frac{1}{2}a$$

即 $BE = CE$

B10 (a) 由於所給方程的係數是有理數，故當其中一根為 $2-\sqrt{3}$ 時，另一根必為 $2+\sqrt{3}$。

$$\therefore \text{ 兩根之和 } = (2-\sqrt{3})(2+\sqrt{3})$$

$$= 4$$

$$\tan\theta + \cot\theta = 4$$

(b)

$$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\frac{1}{2}\sin 2\theta}$$

$$\therefore \quad \frac{2}{\sin 2\theta} = 4$$

$$\sin 2\theta = \frac{1}{2}$$

(c)

$$\because \quad \sin 2\theta = \frac{1}{2}$$

$$\therefore \quad 2\theta = n\pi + (-1)^n\frac{\pi}{6}$$

$$\theta = \frac{n\pi}{2} + (-1)^n\frac{\pi}{12},$$

其中 n 為任意整數。

B11 (a) $\cos 3\theta$

$= \cos(2\theta + \theta)$

$= \cos 2\theta\cos\theta - \sin 2\theta\sin\theta$

$= (2\cos^2\theta - 1)\cos\theta - (2\sin\theta\cos\theta)\sin\theta$

$= 2\cos^3\theta - \cos\theta - 2\cos\theta(1-\cos^2\theta)$

$= 4\cos^3\theta - 3\cos\theta$

(b) 設 $x = \cos\theta$。

則 $8x^3 - 6x - \sqrt{3} = 0$ 可寫成

$$8\cos^3\theta - 6\cos\theta - \sqrt{3} = 0$$

$$4\cos^3\theta - 3\cos\theta = \frac{\sqrt{3}}{2}$$

$$\cos 3\theta = \frac{\sqrt{3}}{2}$$

$$3\theta = 2n\theta \pm \frac{\pi}{6}$$

$$\theta = \frac{2n\pi}{3} \pm \frac{\pi}{18}$$

$$x = \cos\frac{\pi}{18}, \cos\frac{13\pi}{18} \text{ 或 } \cos\frac{25\pi}{18}$$

$= 0.985, -0.643$ 或 -0.342

(三位有效數字)

B12 (a) $2[\sin\theta + \sin(\theta+2\phi) + \sin(\theta+4\phi) + \sin(\theta+6\phi) + \sin(\theta+8\phi)]\sin\phi$

$= 2\sin\theta\sin\phi + 2\sin(\theta+2\phi)\sin\phi + 2\sin(\theta+4\phi)\sin\phi + 2\sin(\theta+6\phi)\sin\phi + 2\sin(\theta+8\phi)\sin\phi$

$= [\cos(\theta-\phi) - \cos(\theta+\phi)] + [\cos(\theta+\phi) - \cos(\theta+3\phi)] + [\cos(\theta+3\phi) - \cos(\theta+5\phi)] + [\cos(\theta+5\phi) - \cos(\theta+7\phi)] + [\cos(\theta+7\phi) - \cos(\theta+9\phi)]$

$= \cos(\theta-\phi) - \cos(\theta-9\phi)$

(b) 上式中取 $\phi = \frac{\pi}{5}$，

則

$$2\left[\sin\theta + \sin\left(\theta+\frac{2\pi}{5}\right) + \sin\left(\theta+\frac{4\pi}{5}\right) + \sin\left(\theta+\frac{6\pi}{5}\right) + \sin\left(\theta+\frac{8\pi}{5}\right)\right]\sin\frac{\pi}{5}$$

$$= \cos\left(\theta-\frac{\pi}{5}\right) - \cos\left(\theta-\frac{9\pi}{5}\right)。$$

$$\because \quad \cos\left(\theta-\frac{9\pi}{5}\right) = \cos\left(\theta+\frac{\pi}{5}-2\pi\right)$$

$$= \cos\left(\theta+\frac{\pi}{5}\right)$$

$$\therefore \sin\theta + \sin\left(\theta+\frac{2\pi}{5}\right) + \sin\left(\theta+\frac{4\pi}{5}\right) + \sin\left(\theta+\frac{6\pi}{5}\right) + \sin\left(\theta+\frac{8\pi}{5}\right)$$

$$= \frac{\cos\left(\theta-\frac{\pi}{5}\right) - \cos\left(\theta+\frac{\pi}{5}\right)}{2\sin\frac{\pi}{5}}$$

$$= \frac{2\sin\theta\sin\frac{\pi}{5}}{2\sin\frac{\pi}{5}}$$

$$= \sin\theta$$

$$\therefore \sin\left(\theta+\frac{2\pi}{5}\right) + \sin\left(\theta+\frac{4\pi}{5}\right) + \sin\left(\theta+\frac{6\pi}{5}\right) + \sin\left(\theta+\frac{8\pi}{5}\right) = 0$$

B13 (a) 依題意，

$$\begin{cases} \cos\alpha + \cos\beta = \frac{5}{6} \\ \cos\alpha\cos\beta = \frac{1}{6} \end{cases}$$

$$\therefore \quad (\cos\alpha - \cos\beta)^2$$

$$= (\cos\alpha + \cos\beta)^2 - 4\cos\alpha\cos\beta$$

$$= \left(\frac{5}{6}\right)^2 - 4\left(\frac{1}{6}\right) = \frac{1}{36}$$

$$\because \quad 0 < \beta < \alpha < \frac{\pi}{2}$$

$$\therefore \quad \cos\alpha < \cos\beta$$

由此，$\cos\alpha - \cos\beta = -\frac{1}{6}$。

(b) $\left(\sin\frac{\alpha+\beta}{2}+\sin\frac{\alpha-\beta}{2}\right)^2$

$=\sin^2\frac{\alpha+\beta}{2} + 2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} +$

$\sin^2\frac{\alpha-\beta}{2}$

$= \frac{1-\cos(\alpha+\beta)}{2}+2\left[-\frac{1}{2}(\cos\alpha-\cos\beta)\right]+$

$\frac{1-\cos(\alpha-\beta)}{2}$

$= 1-\frac{[\cos(\alpha+\beta)+\cos(\alpha-\beta)]}{2} - (\cos\alpha - \cos\beta)$

$= 1 - \cos\alpha\cos\beta - (\cos\alpha - \cos\beta)$

$= 1 - \frac{1}{6} - \left(-\frac{1}{6}\right)$

$= 1$

$\therefore \sin\frac{\alpha+\beta}{2}+\sin\frac{\alpha-\beta}{2} = 1$

$(\sin\frac{\alpha+\beta}{2} > 0,\ \sin\frac{\alpha-\beta}{2} > 0)$

B14 **(a)** 在 ΔABC 中, $\angle A + \angle B + \angle C = 180°$

$\therefore \cos C = \cos[180° - (A + B)]$
$= -\cos(A + B)$

$\because \cos A = \frac{7}{25}$

$\therefore \sin A = \frac{24}{25}$

$\because \cos B = \frac{4}{5}$

$\therefore \sin B = \frac{3}{5}$

$\therefore \cos C = -\cos(A + B)$
$= -(\cos A\cos B - \sin A\sin B)$
$= -\left(\frac{7}{25}\cdot\frac{4}{5}-\frac{24}{25}\cdot\frac{3}{5}\right)$
$= \frac{44}{125}$

(b) $\because \cos C = \frac{44}{125}$

$\therefore \sin C = \frac{117}{125}$

由 **(a)**,

$\sin A = \frac{24}{25} = \frac{120}{125}$, $\sin B = \frac{3}{5} = \frac{75}{125}$。

故 ΔABC 中, $\angle A$ 為最大角而 a 為最長的邊長。

由正弦公式得,

$$\frac{40}{\frac{120}{125}} = \frac{b}{\frac{75}{125}} = \frac{c}{\frac{117}{125}}$$

$\therefore b = 25$, $c = 39$

$\therefore$ ΔABC 的周長 $= 40 + 25 + 39$
$= 104$