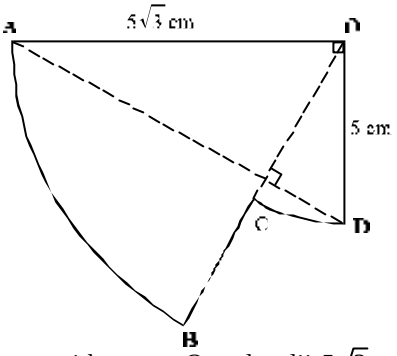
Ch.5 三角函數 Trigonometric Function

A1.



In the figure, the curves AB and CD are arcs with centre O and radii $5\sqrt{3}$ cm and 5 cm respectively, AO ⊥ OD and OB⊥ AD. Find the perimeter of OABCD correct to the nearest 0.1 cm.
如圖所示，曲線 AB **和** CD **是圓心為** O **，半徑分別為** $5\sqrt{3}$ cm **和** 5 cm **的弧，而** OA ⊥ OD **及** OB⊥ AD。**求** OABCD **的周界準確至** 0.1 cm。 (6 **分**)

A2. If $\cos\theta = -\frac{4}{5}$ and $\sin\theta < 0$, find the value of $\tan\theta - \csc\theta$.

若 $\cos\theta = -\frac{4}{5}$ **及** $\sin\theta < 0$，**求** $\tan\theta - \csc\theta$ **的值**。 (4 **分**)

A3. Simplify **化簡** $\dfrac{\sec(-\alpha)+\sin(-\frac{\pi}{2}-\alpha)}{\csc(3\pi-\alpha)-\cos(\frac{3\pi}{2}+\alpha)}$。 (4 **分**)

A4. Given that $\sin\theta+\cos\theta = \frac{3}{2}$, find the value of $\sin^4\theta+\cos^4\theta$.

已知 $\sin\theta+\cos\theta = \frac{3}{2}$，**求** $\sin^4\theta+\cos^4\theta$ **的值**。 (5 **分**)

A5. If A is an acute angle, prove that $\sqrt{\dfrac{1-\cos A}{1+\cos A}} = \csc A - \cot A$.

若 A **為銳角，證明** $\sqrt{\dfrac{1-\cos A}{1+\cos A}} = \csc A - \cot A$。 (5 **分**)

A6. Solve the equation **解方程** $5\cos^2 x + 3\sin^2 x = 7\cos x$ **，** for **其中** $0° \le x < 360°$。 (5 **分**)

A7. Solve the equation **解方程** $|\cos x| - |\sin x| = \sin x - \cos x$, for **其中** $0 \le x \le \pi$。 (6 **分**)

A8. In **在** ΔABC **中，** $b = 2\sqrt{3}$， $c = 6$, ∠B = 30°，solve the triangle **解該三角形**。 (6 **分**)

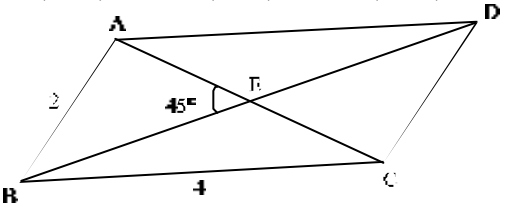
A9. The sides of ΔABC are $a = 7$, $b = 4\sqrt{3}$, $c = \sqrt{13}$. Find the smallest angle of ΔABC.
ΔABC **的邊長為** $a = 7$， $b = 4\sqrt{3}$， $c = \sqrt{13}$。**求** ΔABC **中最小的角**。 (5 **分**)

A10. The ratios of the angles of ΔABC are 5 : 10 : 21. The side opposite to the smallest angle is 6 cm. Find the length of the longest side correct to 2 decimal places.
ΔABC **三個內角之比為** 5 : 10 : 21，**其中最小的角的對邊之邊長為** 6 cm。**求最長一條邊的邊長。**
(答案須準確至二位小數) (6 **分**)

A11. In **在** ΔABC **中，** $(a+b+c)(b+c-a) = 3bc$， find **求**∠A。 (6 **分**)

A12. Prove that **證明** in **在** ΔABC **中，** $(b+c)\cos A + (c+a)\cos B + (a+b)\cos C = a+b+c$。 (6 **分**)

A13.



In parallelogram ABCD, AB = 2, BC = 4, the angle between the diagonals AC and BD is 45°. Find the area of ABCD.
在平行四邊形 ABCD **中，** AB = 2，BC = 4，**對角線** AC **和** BD **的夾角為** 45°。**求** ABCD **的面積**。 (6 **分**)

A14. In ΔABC, tan A = 2 and tan B = 3. Find ∠C without using calculators.
在 ΔABC **中，** tan A = 2，tan B = 3。**試不用計算機，求**∠C **的值**。 (5 **分**)

A15. Given that $\tan\alpha = 1 - \sqrt{2}$, find the values of $\sin 2\alpha$, $\cos 2\alpha$ and $\tan 2\alpha$.
已知 $\tan\alpha = 1 - \sqrt{2}$，**求** $\sin 2\alpha$、$\cos 2\alpha$ **和** $\tan 2\alpha$ **的值**。 (5 **分**)

A16. Prove that **證明** $\dfrac{2\sin\theta - \sin 2\theta}{2\sin\theta + \sin 2\theta} = \tan^2\dfrac{\theta}{2}$。 (5 **分**)

A17. Simplify 化簡 $\cos^2(\theta + 15°) + \cos^2(\theta - 15°) - \frac{\sqrt{3}}{2}\cos 2\theta$。 (6 分)

A18. Find the general solution of the equation $\sqrt{3}\sin\theta - \cos\theta = \sqrt{2}$, giving the answer in degrees.
求方程 $\sqrt{3}\sin\theta - \cos\theta = \sqrt{2}$ 的通解，答案以度表示。 (5 分)

A19. Find the general solution of the equation $\cos x - \cos 3x = \sin 2x$, giving the answer in terms of π radians.
求方程 $\cos x - \cos 3x = \sin 2x$ 的通解，答案以 π 表示。 (5 分)

A20. Find the general solution of $\cos x + \cos 2x + \cos 3x = 0$.
求方程 $\cos x + \cos 2x + \cos 3x = 0$ 的通解。 (5 分)

A21. Let 設 $t = \tan\frac{\pi}{12}$。

(a) Using the double angle formula for tangent function, prove that $t^2 + 2\sqrt{3}\,t - 1 = 0$.
利用正切函數的二倍角公式，證明 $t^2 + 2\sqrt{3}\,t - 1 = 0$。

(b) Find the exact value of $\tan\frac{\pi}{12}$. (Express your answer in surd form.)
試求 $\tan\frac{\pi}{12}$。(答案以根式表示。) (6 分)

A22. It is given that $\tan\alpha$ and $\tan\beta$ are the roots of the equation $6x^2 - 5x + 1 = 0$.
已知 $\tan\alpha$ 及 $\tan\beta$ 為方程 $6x^2 - 5x + 1 = 0$ 的兩根

(a) Find the value of $\tan(\alpha + \beta)$.
試求 $\tan(\alpha + \beta)$。

(b) If α and β are acute angles, find $\alpha + \beta$.
若 α、β 皆為銳角，求 $\alpha + \beta$。 (6 分)

A23. If tan A and tan B are the roots of the equation $3x^2 - 5x + 1 = 0$, find the value of tan (A + B).
若 tan A 及 tan B 為方程 $3x^2 - 5x + 1 = 0$ 的根，求 tan (A + B) 的值。 (5 分)

A24. Given that $\sin\theta$ and $\cos\theta$ are the roots of the equation $x^2 + px + q = 0$, express the value of $2\sin^2\frac{\theta}{2}\left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right)^2$ in terms of p and q.

已知 $\sin\theta$ 及 $\cos\theta$ 為方程 $x^2 + px + q = 0$ 的根，試以 p 及 q 表 $2\sin^2\frac{\theta}{2}\left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right)^2$。
(5 分)

A25. Find the general solution of the equation $\cos x \cos 2x = \cos 3x \cos 4x$.
求方程 $\cos x \cos 2x = \cos 3x \cos 4x$ 的通解。 (5 分)

A26. Using the identity $\sin 3\theta \equiv 3\sin\theta - 4\sin^3\theta$, find the general solution of the equation $\sin 3\theta + 4\cos^2\theta - 4\sin\theta - 5 = 0$.
利用恆等式 $\sin 3\theta \equiv 3\sin\theta - 4\sin^3\theta$，求方程 $\sin 3\theta + 4\cos^2\theta - 4\sin\theta - 5 = 0$ 的通解。 (6 分)

A27. Let $s = a\cos\theta + b\sin\theta = r\cos(\theta - \alpha)$, where $r > 0$ and $\alpha < \frac{\pi}{2}$. It is given that when $\theta = \frac{\pi}{3}$, s has the maximum value 4.

設 $s = a\cos\theta + b\sin\theta = r\cos(\theta - \alpha)$，其中 $r > 0$ 及 $\alpha < \frac{\pi}{2}$。已知當 $\theta = \frac{\pi}{3}$ 時，s 有最大值 4。

(a) Express r and $\tan\alpha$ in terms of a and b.
試以 a 及 b 表 r 及 $\tan\alpha$。

(b) Find the values of a and b.
求 a 及 b 的值。 (6 分)

A28. It is given that $\tan\alpha + \tan\beta = 2$ and $\cot\alpha + \cot\beta = 4$, where $\frac{\pi}{2} < \alpha + \beta < \frac{3\pi}{2}$. Find the values of

已知 $\tan\alpha + \tan\beta = 2$ 及 $\cot\alpha + \cot\beta = 4$，其中 $\frac{\pi}{2} < \alpha + \beta < \frac{3\pi}{2}$。試求

(a) $\tan(\alpha + \beta)$,

(b) $\cos(\alpha + \beta)$。 (6 分)

B1. Let 設 $f(x) = 2\sin^2 x - 3\cos x + 5$。

(a) Express $f(x)$ in terms of $\cos x$.
試以 $\cos x$ 表 $f(x)$。 (2 分)

(b) Find the least and the greatest values of $f(x)$. Find also the corresponding values of $\cos x$ in such cases.
求 $f(x)$ 的最小值及最大值，並求相應的 $\cos x$ 的值。 (7 分)

B2. **(a)** Using the formula $\sin 2\theta = 2\sin\theta\cos\theta$, find the range of values of $\sin\theta\cos\theta$.
利用公式 $\sin 2\theta = 2\sin\theta\cos\theta$，求 $\sin\theta\cos\theta$ 的值域。 (3 分)

(b) Given $\sin\theta + \cos\theta = \sin\theta\cos\theta$, find the value of $\sin\theta\cos\theta$.
已知 $\sin\theta + \cos\theta = \sin\theta\cos\theta$，試求 $\sin\theta\cos\theta$ 的值。 (6 分)

B3. It is given that $\sin\alpha$ and $\cos\alpha$ are the roots of the equation $25x^2 - 5x + k = 0$, where $\frac{3\pi}{2} \le \alpha \le 2\pi$.

已知 $\sin\alpha$ 和 $\cos\alpha$ 為方程 $25x^2 - 5x + k = 0$ 的根，其中 $\frac{3\pi}{2} \le \alpha \le 2\pi$。

(a) Find the value of k.
求 k 的值。 (4 分)

(b) Find the value of $\dfrac{\sin\alpha + \cos\alpha}{\sin\alpha - \cos\alpha}$.
求 $\dfrac{\sin\alpha + \cos\alpha}{\sin\alpha - \cos\alpha}$ 的值。 (4 分)

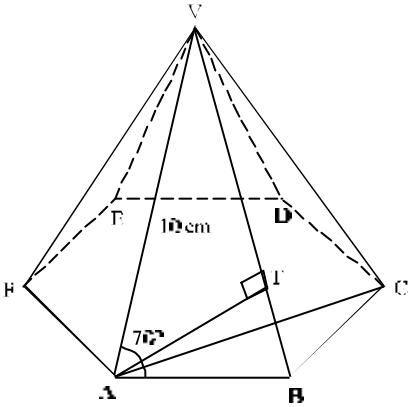
(c) Find the value of α.
求 α 的值。 (2 分)

B4. **(a)** Draw the graphs of $y = 2\cos x + 1$ and $y = 2\sin\frac{x}{2}$ for $0^\circ \le x \le 360^\circ$ on the same diagram.

在同一圖中描繪 $y = 2\cos x + 1$ 及 $y = 2\sin\frac{x}{2}$ 的圖像，其中 $0° \le x \le 360°$。 (6 分)

(b) Hence, solve graphically the equation $2\cos x - 2\sin\frac{x}{2} + 1 = 0$ for $0^\circ \le x \le 360^\circ$. Give your answers correct to the nearest degree.
由此，用圖像法解方程 $2\cos x - 2\sin\frac{x}{2} + 1 = 0$，其中 $0° \le x \le 360°$。答案準確至度。 (4 分)

B5.



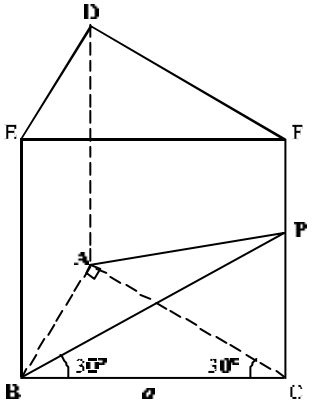
The figure shows a right pyramid with vertex V and a regular hexagon base ABCDEF, VA = 10 cm, ∠VAB = 70°. T is a point on VB such that AT⊥VB. Find
如圖所示，V–ABCDEF 為一直立角錐體，其底 ABCDEF 為一正六邊形，VA = 10 cm，∠VAB = 70°。T 為 VB 的一點，且 AT ⊥ VB。求

(a) the lengths of AT and AC;
AT 和 AC， (5 分)

(b) the angle between the faces VAB and VBC.
面 VAB 和面 VBC 的夾角。 (5 分)

Give your answers correct to 3 significant figures.
答案須準確至三位有效數字。

B6.



The figure shows a right prism whose base is a triangle ABC with ∠BAC = 90°, ∠ACB = 30° and BC = a. P is a point on the edge CF such that ∠PBC = 30°.

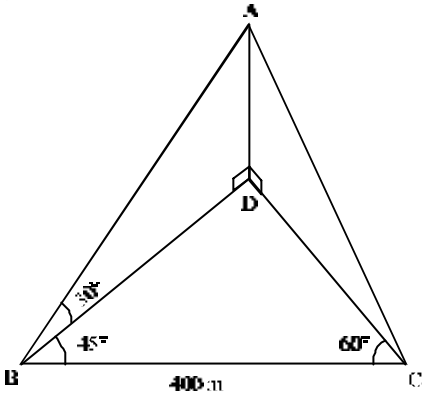
圖中所示為一正棱柱體，其底為 ΔABC，其中∠BAC = 90°，∠ACB = 30° 而 BC = a。P 為邊 CF 上的一點且∠PBC = 30°。

(a) Express BP and PC in terms of a.
以 a 表 BP 和 PC。 (2 分)

(b) Find ∠APB correct to the nearest 0.1°.
求∠APB 準確至最接近的 0.1°。 (6 分)

(c) Find the angle between the planes ABP and ABC correct to the nearest 0.1°.
求平面 ABP 和 ABC 的夾角，準確至最接近的 0.1°。 (2 分)

B7.

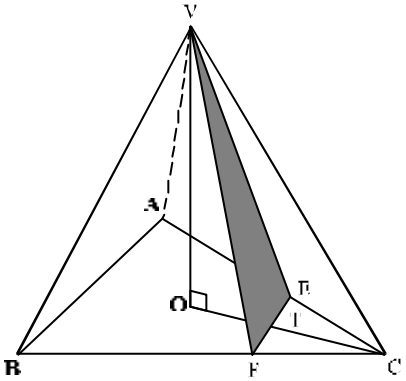


In the figure, A is a balloon in air and D, B, C are three points on the same level ground. B and C are 400 m apart, ∠DBC = 45° and ∠DCB = 60°. The angle of elevation of the balloon from B is 30°.
圖中所示，A 為一氣球，D、B、C 為同一水平面上的三點。B 和 C 相距 400 m，∠DBC = 45°，∠DCB = 60°。由 B 測得氣球的仰角為 30°。

(a) Find the altitude of the balloon correct to the nearest metre.
求氣球的高度，準確至最接近的 m。 (5 分)

(b) Find the angle of elevation of the balloon from C correct to the nearest 0.1°.
求由 C 至氣球的仰角，準確至最接近的 0.1°。 (5 分)

B8.



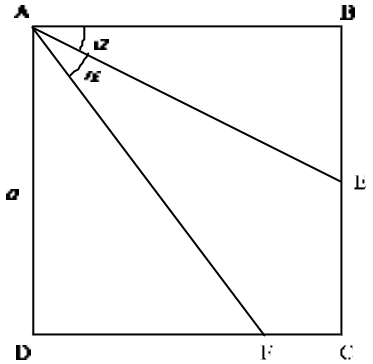
In the figure, VABC is a regular tetrahedron of side 3 units, VO is an altitude, plane VEF cuts AC at E and BC at F such that CE = $\frac{1}{3}$CA and CF = $\frac{1}{3}$CB, EF intersects CO at T.
如圖所示，VABC 為一個六條棱長皆等於 3 的四面體 (三角錐體)，VO 為其高。平面 VEF 交 AC 於 E，交 BC 於 F，且 CE = $\frac{1}{3}$CA，CF = $\frac{1}{3}$CB，EF 交 CO 於 T。

(a) Find 求∠EVF。 (5 分)

(b) Find the angle between the planes VEF and ABC.
求平面 EVF 與平面 ABC 的交角。 (5 分)

Give your answers correct to the nearest 0.1°.
(答案準確至最接近的 0.1°)

B9.



The figure shows a square ABCD of side a units. It is given that $\angle BAE = \angle EAF = \alpha$ and AF = BC = CF.
圖中所示為一邊長為 a 單位的正方形ABCD。已知$\angle BAE = \angle EAF = \alpha$，AF = BC + CF。

(a) Express AF and DF in terms of functions of 2α.
試以 2α 表 AF 和 DF。 (2 分)

(b) Prove that 證明 $\tan\alpha = \frac{1}{2}$。 (6 分)

(c) Hence 由此，prove that 證明 BE = CE。 (2 分)

B10. It is given that $2 - \sqrt{3}$ is a root of the equation $x^2 - (\tan\theta + \cot\theta)x + c = 0$, and $\tan\theta + \cot\theta$ is a rational number.
已知 $2 - \sqrt{3}$ 為方程 $x^2 - (\tan\theta + \cot\theta)x + c = 0$ 其中的一個根，而 $\tan\theta + \cot\theta$ 為一有理數。

(a) Find the value of $\tan\theta + \cot\theta$.
求 $\tan\theta + \cot\theta$ 的值。 (3 分)

(b) Hence, find the value of $\sin 2\theta$.
由此，求 $\sin 2\theta$ 的值。 (4 分)

(c) Find the general solution of θ, giving the answer in terms of π radians.
求θ的通解，答案以π表示。 (3 分)

B11. (a) Express $\cos 3\theta$ in terms of $\cos\theta$.
試以 $\cos\theta$ 表 $\cos 3\theta$。 (3 分)

(b) Hence, find the three roots of the equation $8x^3 - 6x - \sqrt{3} = 0$. Give your answers correct to 3 significant figures.
由此求方程 $8x^3 - 6x - \sqrt{3} = 0$的三個根。答案須準確至三位有效數字。 (8 分)

B12. (a) Using the identity 利用恆等式 $2\sin\alpha\sin\beta = \cos(\alpha - \beta) - \cos(\alpha + \beta)$, prove that 證明
$2[\sin\theta + \sin(\theta + 2\phi) + \sin(\theta + 4\phi) + \sin(\theta + 6\phi) + \sin(\theta + 8\phi)]\sin\phi = \cos(\theta - \phi) - \cos(\theta - 9\phi)$。 (6 分)

(b) Hence 由此 prove that 證明

$$\sin\left(\theta + \frac{2\pi}{5}\right) + \sin\left(\theta + \frac{4\pi}{5}\right) + \sin\left(\theta + \frac{6\pi}{5}\right) + \sin\left(\theta + \frac{8\pi}{5}\right) = 0。$$ (4 分)

B13. α and β are acute angles such that $\alpha > \beta$. They satisfy the equation $6\cos^2\theta - 5\cos\theta + 1 = 0$. Without solving the equation,
若 α、β 為兩銳角 ($\alpha > \beta$)，且滿足方程 $6\cos^2\theta - 5\cos\theta + 1 = 0$。試不解此方程，

(a) find 求 $\cos\alpha - \cos\beta$, (4 分)

(b) prove that 證明 $\sin\frac{\alpha + \beta}{2} + \sin\frac{\alpha - \beta}{2} = 1$。 (5 分)

B14. In ΔABC, it is given that $\cos A = \frac{7}{25}$ and $\cos B = \frac{4}{5}$.

在 ΔABC 中，$\cos A = \frac{7}{25}$，$\cos B = \frac{4}{5}$。

(a) Prove that $\cos C = -\cos(A + B)$. Hence, without using calculators, find the value of $\cos C$.
證明 $\cos C = -\cos(A + B)$。由此，不使用計算機，求 $\cos C$ 的值。 (5 分)

(b) If the length of the longest side of ΔABC is 40, find its perimeter.
若 ΔABC 最長的邊長為 40，試求其周長。 (5 分)

完