Rectangular Coordinate System

1. The distance between $A(x_1,y_1)$ and $B(x_2,y_2)$ is

   $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

2. Given the line segment joining $A(x_1,y_1)$ and $B(x_2,y_2)$, the point of division in the ratio $m : n$ is

   $\left(\dfrac{nx_1 + mx_2}{m+n}, \dfrac{ny_1 + my_2}{m+n}\right)$

   The ratio is negative for external point of division.

   The midpoint of AB is $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

3. If the vertices of a triangle taken in anticlockwise direction are (x_1,y_1), (x_2,y_2) and (x_3,y_3), then

   $$\text{area of triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix}$$

   $$= \frac{1}{2}(x_1y_2 + x_2y_3 + x_3y_1 - x_1y_3 - x_3y_2 - x_2y_1)$$

4. If the vertices of a n-sided polygon taken in anticlockwise direction are (x_1,y_1), (x_2,y_2), $\cdots$, (x_n,y_n), then

   $$\text{area of polygon} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ \vdots & \vdots \\ x_n & y_n \\ x_1 & y_1 \end{vmatrix}$$

5. The inclination q of a line is the angle that it makes with the positive direction of the x-axis. The range of q is $0° \leq$ è $< 180°$.

   Slope of a line = m = tan q

   If (x_1,y_1) and (x_2,y_2) are two points on a line, then

   $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

6. Given two lines L_1 and L_2 of slope m_1 and m_2 respectively,

   (a) $\tan f = \left|\dfrac{m_1 - m_2}{1 + m_1m_2}\right|$ where f is the acute angle between L_1 and L_2.

   (b) $L_1 // L_2$ if and only if $m_1 = m_2$.

   (c) $L_1 \perp L_2$ if and only if $m_1m_2 = -1$.

Straight Lines

1. Equations of straight lines:

   (a) Point-slope form: $y - y_1 = m(x - x_1)$

   (b) Slope-intercept form: $y = mx + c$

   (c) Two-point form: $\dfrac{y - y_1}{x - x_1} = \dfrac{y_2 - y_1}{x_2 - x_1}$

   (d) Intercept form: $\dfrac{x}{a} + \dfrac{y}{b} = 1$

   (e) General form: $Ax + By + C = 0$

   (f) Normal form: $x \cos q + y \sin q - p = 0$

2. The equation of a line in general form $Ax + By + C = 0$ can be converted to the normal form $\dfrac{Ax + By + C}{\pm\sqrt{A^2 + B^2}} = 0$.

   Here, the rule of taking the sign $\pm$ before $\sqrt{A^2 + B^2}$ is as follows:

   (a) If $C \neq 0$, the sign is opposite to that of C.

   (b) If $C = 0$ and $B \neq 0$, the sign is the same as that of B.

3. The distance d from $P(x_1,y_1)$ to the line $Ax + By + C = 0$ is

   $d = \dfrac{Ax + By + C}{\pm\sqrt{A^2 + B^2}}$

   The rule of taking the sign before $\sqrt{A^2 + B^2}$ is the same as that stated in pont 2 above. If the line does not pass through the origin, d is positive if P and the origin are on opposite sides of the line; and d is negative if P and the origin are on the same side of the line.

4. The family of straight lines through the intersection of the lines

   $L_1 : A_1x + B_1y + C_1 = 0$ and $L_2 : A_2x + B_2y + C_2 = 0$

   is $(A_1x + B_1y + C_1) + k(A_2x + B_2y + C_2) = 0$ where k is an arbitrary constant.

5. The general steps for finding the locus of a variable points are:

   (a) Let the coordinates of the variable point or a point on the locus be (x, y).

   (b) Write down an equation from the given conditions and hence, set up an equation connecting x and y.

   (c) Simplify the equation to obtain the required equation of the locus.