1. (a) Prove, by mathematical induction, that

$$1^3 + 2^3 + 3^3 + \ldots + (2n)^3 = n^2(2n+1)^2,$$

for all positive integers n.

(b) Hence, find the sum of the series

$$1^3 + 3^3 + 5^3 + \ldots + (2n-1)^3.$$

2. Prove, by mathematical induction, that $2^{n+3} > 2n+7$ for all positive integers n.

3. Prove, by mathematical induction, that $2^n > n^2$ for n is an integer not less than 5.

4. Prove, by mathematical induction, that, $3^{2n+2} - 8n - 9$ is divisible by 64 for all positive integers n.

5. Prove, by mathematical induction, that, $x^n - y^n$ is divisible by x + y for all positive ***even*** integers n.

6. (a) Prove, by mathematical induction, that for all positive integers n,

$$1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}.$$

(b) By considering the identity

$$(k+1)^3 - k^3 = 3k^2 + 3k + 1$$

and the result obtained in (a), show that

$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$