

Ch.3 Mathematical Induction 數學歸納法

- A1. 利用數學歸納法，證明對所有正整數 n ， Prove, by mathematical induction, that
 $1^2 + 3^2 + 5^2 + \dots + (2n - 1)^2 = \frac{1}{3}n(2n + 1)(2n - 1)$. for all positive integers n . (7 分)
- A2. 利用數學歸納法，證明對所有正整數 n ， Prove, by mathematical induction, that for any positive integer n .
 $\frac{1}{2 \times 5} + \frac{1}{5 \times 8} + \frac{1}{8 \times 11} + \dots + \frac{1}{(3n - 1)(3n + 2)} = \frac{n}{6n + 4}$. (7 分)
- A3. 利用數學歸納法，證明對所有正整數 n ， $3^{2n+1} + 40n - 67$ 為 64 的倍數。
 Prove, by mathematical induction, that $3^{2n+1} + 40n - 67$ is a multiple of 64 for all positive integers n . (7 分)
- A4. 利用數學歸納法，證明對所有正整數 $n \geq 4$ ， $3^n > n^3$ 。
 Prove, by mathematical induction, that for all positive integers $n \geq 4$, $3^n > n^3$. (7 分)
- A5. 設 $S(n)$ 為命題 Let $S(n)$ be the statement
 “ $1 + 2 + 3 + \dots + n = \frac{1}{2}(n - 1)(n + 2)$ ”.
- (a) 證明若 $S(k)$ 成立，則 $S(k + 1)$ 成立。 Prove that if $S(k)$ is true then $S(k + 1)$ is true.
 (b) $S(n)$ 是否對所有自然數 n 皆成立？ Is $S(n)$ true for all natural numbers n ? (7 分)
- A6. 若 $S(n)$ 為一關於自然數的命題，其中 Suppose $S(n)$ is a statement about natural numbers n such that
 (1) $S(5)$ 成立，且 $S(5)$ is true, and
 (2) 若 $S(k)$ 成立，則 $S(k + 5)$ 成立。 if $S(k)$ is true then $S(k + 5)$ is true.
 試說出 $S(n)$ 對哪些 n 的數值成立。 What can you say about the values of n for which $S(n)$ is true? (4 分)
- A7. 利用數學歸納法證明 Prove by mathematical induction that
 $1^3 + 3^3 + 5^3 + \dots + (2n - 1)^3 = n^2(2n^2 - 1)$. (7 分)
- A8. 利用數學歸納法，證明對所有自然數 n ， $7^{n+1} + 3n + 2$ 可被 9 整除。
 Prove by mathematical induction that $7^{n+1} + 3n + 2$ is divisible by 9 for all natural numbers n . (7 分)
- A9. 利用數學歸納法，證明對所有正偶數 n ， $a^n - b^n$ 可被 $a + b$ 整除。
 Prove by mathematical induction that $a^n - b^n$ is divisible by $a + b$ for all even positive integers n . (7 分)
- A10. 利用數學歸納法，證明對所有自然數 $n \geq 2$ ， Prove by mathematical induction that
 $\frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{n+n} > \frac{13}{24}$. for all natural numbers $n \geq 2$. (7 分)
- A11. 平面上有 n 條直線，其中沒有任何兩條直線平行，亦沒有任何三條直線共點。證明該 n 條直線將自己分成 $L_n = n^2$ 條線段。
 There are n lines on a plane such that no two lines are parallel and no three lines are concurrent. Prove that they divide themselves into $L_n = n^2$ parts. (7 分)
- B1. 利用數學歸納法，證明對所有自然數 $n \geq 2$ ， Prove, by mathematical induction, that for all natural numbers $n \geq 2$,
 $(1 - \frac{1}{2^2})(1 - \frac{1}{3^2}) \dots (1 - \frac{1}{n^2}) = \frac{n+1}{2n}$. (10 分)
- B2. 已知三角形的內角和為 180° 。利用數學歸納法證明 n 邊形的內角和為 $(n - 2) \cdot 180^\circ$ 。
 It is known that the angle sum of a triangle is 180° . Prove, by mathematical induction, that the angle sum of an n -sided polygon is $(n - 2) \cdot 180^\circ$. (10 分)
- B3. 觀察以下的數字模式。 Study the following number pattern.
 $1 = 1$
 $3 + 5 = 8$
 $7 + 9 + 11 = 27$
 $13 + 15 + 17 + 19 = 64$
 $21 + 23 + 25 + 27 + 29 = 125$
- (a) 首 $(n - 1)$ 行的左方共有多少項？
 What is the total number of terms on the left side of the first $(n - 1)$ th rows? (1 分)
- (b) 求第 n 行的左方的第一項。 What is the first term of the left side of the n th row? (1 分)
- (c) 試以一數式表第 n 行的數字模式。 Postulate the expression for the n th row of the number pattern. (3 分)
- (d) 利用數學歸納法證明以上的推斷。 Prove your postulation by mathematical induction. (7 分)
- B4. 一序列 $\{a_n\}$ 滿足條件 A sequence $\{a_n\}$ satisfies the conditions that
 $a_1 = 2$ 及 and $a_n = 3a_{n-1} + 1$ 當 for $n = 2, 3, 4, \dots$
- (a) 求 a_2 、 a_3 的值。 What are the values of a_2 and a_3 ? (2 分)
- (b) 利用數學歸納法，證明對所有正整數 n ， Prove, by mathematical induction, that
 $a_n = \frac{5}{2}(3^{n-1}) - \frac{1}{2}$. for all positive integers n . (6 分)
- (c) 求該序列首 n 項的和。 Find the sum for the first n terms of the sequence. (4 分)
- B5. 利用數學歸納法證明 Prove, by mathematical induction, that
 $\log_7 2 + \log_7 14 + \log_7 98 + \dots + \log_7 (2 \times 7^n) = \frac{1}{2}(n + 1)(n + \log_7 4)$. [提示 Hint: $\log_a x^n = n \log_a x$] (10 分)