1. Find the indefinite integral
求不定積分

$\int \frac{3t+1}{\sqrt[3]{t}}\,dt$. (4 分)

2. The slope at any point (x, y) of a curve is given by
一曲線上任意點 (x, y) **的斜率為**

$\frac{dy}{dx} = (3x^2+4)(x^3+4x+1)^{\frac{1}{5}}$.

If the curve passes through (0, 2), find the equation of the curve.
若曲線通過點 (0, 2)**，求曲線的方程。**
[Hint **提示**: Put **設** $x^3 + 4x + 1 = u$.] (6 分)

3. Find the indefinite integral
求不定積分

$\int \sec^4 x \tan^3 x\, dx$. (5 分)

4. Using the substitution **利用代換** $u = \sin^2 x$, find **求**

$\int \frac{\sin x \cos x}{\sqrt{25\sin^2 x + 9\cos^2 x}}\,dx$. (6 分)

5. **(a)** Let a be a constant.
設 a **為一常數。**
Find $\int \frac{dx}{x^2+a^2}$ using the substitution $x = a\tan\theta$.
利用代換 $x = a\tan\theta$**，求** $\int \frac{dx}{x^2+a^2}$ 。 (3 分)

(b) Find $\int \frac{1}{13+12\cos\theta}\,d\theta$ using the substitution $t = \tan\frac{\theta}{2}$.
利用代換 $t = \tan\frac{\theta}{2}$**，求** $\int \frac{1}{13+12\cos\theta}\,d\theta$。 (5 分)

(c) Show that **證明** $\frac{d}{d\theta}\left(\frac{q\sin\theta}{p+q\cos\theta}\right) = \frac{p}{p+q\cos\theta} - \frac{p^2-q^2}{(p+q\cos\theta)^2}$, where p and q are constants. **其中** p **及** q **為常數。** (4 分)

(d) Hence **由此**, or otherwise **或用其他方法**, find **求** $\int \frac{d\theta}{(13+12\cos\theta)^2}$. (4 分)

完

答案：

1. $\frac{9}{5}t^{\frac{5}{3}} + \frac{3}{2}t^{\frac{2}{3}} + C$

2. $y = \frac{5}{6}(x^3+4x+1)^{\frac{6}{5}} + \frac{7}{6}$。

3. $\frac{1}{4}\tan^4 x + \frac{1}{6}\tan^6 x + C$

4. $\frac{1}{16}(16\sin^2 x + 9)^{\frac{1}{2}} + C$

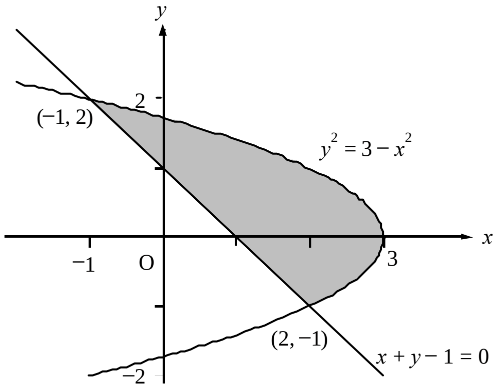
5. (a) $\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

(b) $\frac{2}{5}\tan^{-1}\left(\frac{1}{5}\tan\frac{\theta}{2}\right) + C$

(d) $\frac{2}{25}\left[\frac{13}{5}\tan^{-1}\left(\frac{1}{5}\tan\frac{\theta}{2}\right) - \frac{6\sin\theta}{13+12\cos\theta}\right] + C_2$

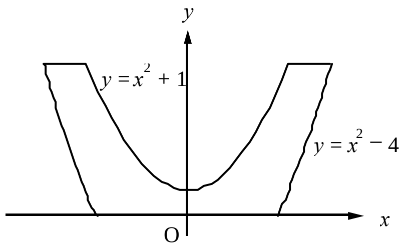
其中 C_2 **為一任意常數。**

1.



Find the area bounded by the parabola $y^2 = 3 - x$ and the line $x + y - 1 = 0$.
求拋物線 $y^2 = 3 - x$ **及直線** $x + y - 1 = 0$ **所圍成的區域的面積。** (8 **分**)

2.



A clay pot is formed by rotating the region bounded by the curves $y = x^2 - 4$, $y = x^2 + 1$ and the lines $y = 0$, $y = k$ through four right angles about the y-axis. The capacity of the pot is 8π units^3.
由曲線 $y = x^2 - 4$**、** $y = x^2 + 1$ **及直線** $y = 0$**、** $y = k$ **所圍成的區域繞** y **軸旋轉** $360°$**，形成一瓶。該瓶的容量為** 8π **單位** ^{3}**。**

(a) Find the value of k.
求 k **的值。** (5 **分**)

(b) Find the volume of material required in making the pot.
求製造此瓶所需原料的體積。 (4 **分**)

完

答案：

1. $\frac{9}{2}$

2. (a) $k = 5$

(b) $\frac{49\pi}{2}$ **單位** ^{3}