1. Find the equation of the tangent to the curve $x^2 + xy + y^2 = 7$ at the point (2,1). [1986 / 15M]

2. Let $y = x + \sin 2x$, where $0 \leqq x \leqq \pi$. Find

(a) $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$,

(b) the maximum and minimum values of y. [1991 / 15M]

3. Fig.2 shows a vessel in the shape of a right circular cone with semi-vertical angle 30 °. Water is flowing out of the cone through its apex at a constant rate of π cm^3s^{-1}.

(a) Let V cm^3 be the volume of water in the vessel when the depth of water is h cm. Express V in terms of h.

(b) How fast is the water level falling when the depth of water is 4 cm?

[1992 / 15M]

4. (a) Slimplify $[\sqrt{2(x+\Delta x)} - \sqrt{2x}][\sqrt{2(x+\Delta x)} + \sqrt{2x}]$

(b) Find $\frac{d}{dx}(\sqrt{2x})$ from first principles. [1993 / 15M]

5. Using the information in the following table, sketch the graph of y = f(x), where f(x) is a polynomial.

	x < 0	x = 0	0 < x < 1	x = 1	1 < x < 2	x = 2	x > 2
f (x)		1		2		1	
f '(x)	< 0	0	> 0	0	< 0	0	> 0

[1995 / 10M]

6. Fig.4 shows a rectangular picture of area A cm^2 mounted on a rectangular piece of cardboard of area 3600 cm^2 with sides of length x cm and y cm. The top, bottom and side margins are 12 cm, 13 cm and 8 cm wide respectively.

(a) Find A in terms of x.

(b) Show that the largest value of A is 1600.

(c) (i) Find the range of values of x for which A decreases as x increases.

(ii) If $x \geqq 50$, find the largest value of A.

(d) If $\frac{4}{9} \le \frac{x}{y} \le \frac{9}{16}$, find the range of values of x and the largest value of A. [1986 / 30M]

END