

**ECO B9502
Homework 4
Due April 3, 2006**

Solution

1. Consider a circular track with one lane that is 2 miles in length. There are 80 cars on this “track” traveling 50 miles per hour – speed (S) is 50.

(a) What is traffic density D?

$$D = \frac{\text{vehicles}}{\text{mile}} = \frac{80}{2} = 40$$

(b) What is traffic volume V?

$$V = SD = (50)(40) = 2000 \text{ vehicles/hour}$$

(c) What units of measurement did you use?

2. Consider an urban highway that is subject to traffic congestion. The average cost of travel per mile on that highway is (in cents)

$$AC = 10 + 4V$$

where V is traffic volume per hour, measured in hundreds of vehicles per hour. Assume that the demand function for traffic volume per hour (during rush hour) is

$$P = 46 - V$$

where P is the “price” paid by the driver.

(a) Assume that no toll is imposed. Compute V and P.

In absence of toll, driver “sees” AC function and makes decision accordingly.

Set $P=AC$ and solve for V.

$$46 - V = 10 + 4V$$

$$\mathbf{V=36/5 = 7.2 \text{ hundred vehicles per hour.}}$$

Plug into demand function to get price.

$$\mathbf{P = 46 - V = 46 - 7.2 = 38.8}$$

- (b) Assume it is possible to impose the efficient congestion toll. Find the toll and the efficient levels of V, P, and AC.

To solve this, need to first derive Marginal Cost function.

Since have AC and we know $TC = (AC)V$, we can use this to get MC.

$$TC = (10 + 4V)V = 10V + 4V^2$$

$$MC = \frac{\partial TC}{\partial V} = 10 + 8V$$

Efficient (i.e., optimal) V, P, and AC occur where $P = MC$

$$P = 46 - V$$

$$46 - V = 10 + 8V$$

$$9V = 36$$

$$\mathbf{V = 4}$$

$$\mathbf{P = 46 - 4 = 42}$$

$$\mathbf{AC = 10 + 4(4) = 26}$$

$$\mathbf{Toll = P - AC = MC - AC = 42 - 26 = 16}$$